# Implementing the Core Chase for the Description Logic ALC

Maël Abily

31 mai 2021

## 1 Introduction

**Definition 1.** *We define a first-order language as a set of constants (often noted a,b,c,$c_1$,...), predicates (P,Q,R,$P_1$,...) and variables (x,y,$x_1$,...).*
*A term is a variable or a constant (often noted t,$t_1$,...). We note Ar(P) the arity of P.*
*$P(t_1, ..., t_n)$ is an atom.*
*A fact is a variable-free atom.*
*A factbase $F := \exists x_1, ..., x_n.P_1(t_1^1, ..., t_{k_1}^1) \wedge ... \wedge P_m(t_1^m, ..., t_{k_m}^m)$ is a existentially quantified conjunctions of atoms which is closed (it means that every variable of F is quantified). var(F) (respectively cst(F) and term(F) is the set of variables (resp. constants and terms) that occur in F.*

**Remark** We will often see factbases as sets of atoms. For example, the factbase $\exists x, x_1, x_2, x_3.P(x) \wedge Q(x, a) \wedge R(x_1, x_2, x_3, b)$ can be represented by $\{P(x), Q(x, a), R(x_1, x_2, x_3, b)\}$.

**Definition 2.** *A substitution $\sigma : X \rightarrow Terms$ is a function where X is a set of variables. For example $\{x \mapsto z, y \mapsto a\}$ is a substitution from {x,y} to Terms.*
*A homomorphism from F to F' is a substitution $\sigma : var(F) \rightarrow term(F')$ where $\sigma(F) \subseteq F'$*

**Proposal 1.** *A factbase F entails a factbase F' (often noted $F \rightarrow F'$) $\Leftrightarrow$ there exists a homomorphism from F' to F.*
*For example, F = {P(b,a),Q(x)} entails F' = {P(x,a)} thanks to the homomorphism $\{x \mapsto b\}$*

**Definition 3.** *An isomorphism is a bijective homomorphism.*
*A subset F' $\subseteq$ F is a retract of F if there exists a subsitution $\sigma$ such that $\sigma(F) = F'$ and $\sigma_{|F'} = id$($\sigma$ is called a retractation from F to F').*
*A factbase is a core if all of its strict subsets are not retracts.*
*A core of a factbase F is a minimal subset of F that is a core.*

**Proposal 2.** *A factbase F is a core $\Leftrightarrow$ every homomorphism $\sigma : F \rightarrow F$ is a bijection.*

Use emph instead of underline.

No need to add a break line after each line; let Latex take care of the formatting. If you want to separate things use the itemize environment.

What are $t_i$ ?

Simply say that we identify factbases as sets of atoms

Use the latex command textit around terms.

Explain how homomosphisms can be applied to sets of atoms. Also, what are $F$ and $F'$ here ?

Use the $\models$

**Example** F = $\{R(a,x)\}$ is the core of F' = $\{R(a,x), R(y,z)\}$

**Definition 4.** *An (existential) rule $R$ is a first-order formula of the form :$\forall\vec{x}.\forall\vec{y}.A($* No need for colon here.
*$\exists\vec{z}.B(\vec{x},\vec{z})$where $A$ and $B$ are conjunctions of atoms. We define body(R) = $A$, head(R) = $B$ and the frontier of $R$ fr(R) = $\vec{x}$ (the set of variables shared by the body and the head of $R$).* Define $\vec{x}$, $\vec{y}$, and $\vec{z}$/
*An ontology $O$ is a pair (T,F) where $T$ is a set of existential rules and $F$ is a factbase.*

**Definition 5.** *Let $T$ be a ruleset, $\alpha$ be a rule, $\sigma$ be a subsitution and $F$ be a factbase.*
*The tuple $(\alpha, \sigma)$ is a trigger for $F$ if :*

- *the domain of $\sigma$ is the set of all variables occurring in $Body(\alpha)$.*
- *$\sigma(Body(\alpha)) \subseteq F$.*
- *For all $\hat{\sigma}$ which extends $\sigma$ and has its domain equals to the set of all variables occurring in $Body(\alpha)$ and $Head(\alpha)$ ,$\hat{\sigma}(Head(\alpha)) \nsubseteq F$*

*$Tr_\alpha(F)$ is the set of all trigers $(\alpha, \sigma)$ for $F$.*
*$Tr_T(F)$ is the set of all trigers for $F$.* Some stuff is missing from this definition.
*The rule $\alpha$ is satisfied by $F$ if $Tr_\alpha(F) = \emptyset$.*
*$T$ is satisfied by $F$ if $Tr_T(F) = \emptyset$.*

**Example** If $\alpha = A(x,y) \rightarrow B(x,z)$, F = $\{A(b,c)\}$ and $\sigma = \{x \mapsto b, y \mapsto c\}$ then $(\alpha, \sigma)$ is a trigger for F. Shouldn't $z$ be existentially quantified here?

**Definition 6.** *a ruleset $T$ is satisfied by a factbase $M$ if every rule of $T$ is satisfied by $M$.*
*A factset $M$ is a model for an ontology $O$ = (T,F) if $F \subset M$and $T$ is satisfied by $M$.* Use subseteq instead.
*A Boolean conjunctive query (BCQ) is a closed formula of the form $\exists x_1, ..., x_n.F(x_1, ..., x_n)$ where $F$ is a conjunction of atoms.*
*A fact set $F$ entails a BCQ $B = \exists x_1, ..., x_n.F(x_1, ..., x_n)$ (noted $F \vDash B$) if there exists a substitution $\sigma$ such that $\sigma(B) \subset F$. An ontology $O$ entails a BCQ $B = \exists x_1, ..., x_n.F(x_1, ..., x_n)$ (noted $O \vDash B$) if for every model $M$ of $O$, $M \vDash B$.* Use subseteq instead.
*A model $U$ for an ontology $O$ is universal if for every model $M$ of $O$, there exists an homomorphism $h : U \rightarrow M$.* Write a instead.