Implementing the Core Chase for the Description Logic ALC

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INRIA

2021

Factbase

We consider variables $(x, y, \ldots)$, constants $(a, b, \mathit{Pierre}, \ldots)$ and predicates $(P, R, \mathit{IsTheSonOf}, \ldots)$

term = variable or constant.

An *atom* is of the form $P(t_1, \ldots, t_n)$ and is ground if $t_1, \ldots, t_n$ are constants.

Example: *IsTheSonOf(Pierre,Francis)* is a ground atom.

Factbase = existentially closed formula of the form $\exists x_1, \ldots, x_n . P_1(t_1^1, \ldots, t_{k_1}^1) \wedge \ldots \wedge P_m(t_1^m, \ldots, t_{k_m}^m)$ identify with:

$\{P_1(t_1^1, \ldots, t_{k_1}^1), \ldots, P_m(t_1^m, \ldots, t_{k_m}^m)\}$

Homomorphism

A *substitution* $\sigma : X \to \textbf{Terms}$ is a function where X is a set of variables. For example $\{x \mapsto z, y \mapsto a\}$ is a substitution from $\{x, y\}$ to **Terms**.

- if $c \in \textbf{Csts}$, then $\sigma(c) = c$;
- if $x \in \textbf{Vars} \setminus X$, $\sigma(x) = x$;
- if $f = P(t_1, \ldots, t_n)$ is an atom, then $\sigma(f) = P(\sigma(t_1), \ldots, \sigma(t_n))$; and
- if $F = \{f_1, \ldots, f_n\}$ is a factbase, then $\sigma(F) = \{\sigma(f_1), \ldots, \sigma(f_n)\}$.

For two factbases F and F', a *homomorphism* from F to F' is a substitution $\sigma : \textbf{Vars}(F) \to \textbf{Terms}$ where $\sigma(F) \subseteq F'$.

An *isomorphism* h from F to F' is a bijective homomorphism where its inverse is a homomorphism from F' to F.

Entailment

A factbase F *entails* another factbase Q if there exists a homomorphism from Q to F.

Example: The factbase $F = \{P(b, a), A(x)\}$ entails the factbase $Q = \{P(x, a), P(y, z)\}$ due to the homomorphism $\{x \mapsto b, y \mapsto b, z \mapsto a\}$.

A factbase G is a *retract* of another factbase F if $G \subseteq F$ and $G \models F$

If a factbase F does not contain a strict retract, then we say that F is a *core*. A *core* of a factbase F (noted $core(F)$) is a subset of F that is a core.

The cores of a finite factbase F are unique up to isomorphism.

The conjunctive query entailment