Implementing the Core Chase for the Description Logic ALC

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1 Introduction

Definition 1. *We define a first-order language as a set of constants (often noted a,b,c,c_1,...), predicates (P,Q,R,P_1,...) and variables (x,y,x_1,...).*
A term is a variable or a constant (often noted t,t_1,...). We note Ar(P) the arity of P.
$P(t_1, ..., t_n)$ is an atom.
A fact is a variable-free atom.
A factbase $F := \exists x_1, ..., x_n.P_1(t_1^1, ..., t_{k_1}^1), ..., P_m(t_1^m, ..., t_{k_m}^m)$ is a existentially quantified conjunctions of atoms which is closed (it means that every variable of F is quantified). var(F) (respectively cst(F) and term(F) is the set of variables (resp. constants and terms) that occur in F.

Remark We will often see factbases as sets of atoms. For example, the factbase $\exists x, x_1, x_2, x_3.P(x) \wedge Q(x, a) \wedge R(x_1, x_2, x_3, b)$ can be represented by $\{P(x), Q(x, a), R(x_1, x_2, x_3, b)\}$.

Definition 2. *A substitution $\sigma : X \rightarrow Terms$ is a function where X is a set of variables. For example $\{x \mapsto z, y \mapsto a\}$ is a substitution from {x,y} to Terms.*
A homomorphism from F to F' is a substitution $\sigma : var(F) \rightarrow term(F')$ where $\sigma(F) \subseteq F'$

Proposal 1. *A factbase F entails a factbase F' (often noted $F \rightarrow F'$) $\Leftrightarrow$ there exists a homomorphism from F' to F.*
For example, F = {P(b,a),Q(x)} entails F' = {P(x,a)} thanks to the homomorphism $\{x \mapsto b\}$

Definition 3. *An isomorphism is a bijective homomorphism.*
A subset F' $\subseteq$ F is a retract of F if there exists a subsitution σ such that $\sigma(F) = F'$ and $\sigma_{|F'} = id$ (σ is called a retractation from F to F'.
A factbase is a core if its strict subset are not retracts.
A core of a factbase F is a minimal subset of F that is a core.

Proposal 2. *A factbase F is a core $\Leftrightarrow$ every homomorphism $\sigma : F \rightarrow F$ is a bijection.*

Example of core