Implementing the Core Chase for the Description Logic ALC

Maël Abily

29 avril 2021

1 Introduction

Definition 1. *We define a first-order language as a set of constants (often noted a,b,c,c_1,...), predicates (P,Q,R,P_1,...) and variables (x,y,x_1,...).*
A term is a variable or a constant (often noted t,t_1,...). We note Ar(P) the arity of P.
$P(t_1,...,t_n)$ is an atom.
A fact is a variable-free atom.
A factbase $F := \exists x_1,...,x_n.P_1(t^1_1,...,t^1_{k_1}) \wedge ... \wedge P_m(t^m_1,...,t^m_{k_m})$ is a existentially quantified conjunctions of atoms which is closed (it means that every variable of F is quantified). var(F) (respectively cst(F) and term(F) is the set of variables (resp. constants and terms) that occur in F.

Remark We will often see factbases as sets of atoms. For example, the factbase $\exists x, x_1, x_2, x_3.P(x) \wedge Q(x,a) \wedge R(x_1,x_2,x_3,b)$ can be represented by $\{P(x), Q(x,a), R(x_1,x_2,x_3,b)\}$.

Definition 2. *A substitution $\sigma : X \rightarrow Terms$ is a function where X is a set of variables. For example $\{x \mapsto z, y \mapsto a\}$ is a substitution from {x,y} to Terms.*
A homomorphism from F to F' is a substitution $\sigma : var(F) \rightarrow term(F')$ where $\sigma(F) \subseteq F'$

Proposal 1. *A factbase F entails a factbase F' (often noted $F \rightarrow F'$) $\Leftrightarrow$ there exists a homomorphism from F' to F.*
For example, F = {P(b,a),Q(x)} entails F' = {P(x,a)} thanks to the homomorphism $\{x \mapsto b\}$

Definition 3. *An isomorphism is a bijective homomorphism.*
A subset F' $\subseteq$ F is a retract of F if there exists a subsitution σ such that $\sigma(F) = F'$ and $\sigma_{|F'} = id$ (σ is called a retractation from F to F').
A factbase is a core if its strict subset are not retracts.
A core of a factbase F is a minimal subset of F that is a core.

Proposal 2. *A factbase F is a core $\Leftrightarrow$ every homomorphism $\sigma : F \rightarrow F$ is a bijection.*

Example F = $\{R(a,x)\}$ is the core of F' = $\{R(a,x), R(y,z)\}$

Definition 4. *An (existential) rule R is a first-order formula of the form :* $\forall\vec{x}.\forall\vec{y}.A(\vec{x},\vec{y}) \rightarrow \exists\vec{z}.B(\vec{x},\vec{z})$ *where A and B are conjonctions of atoms. We define body(R) = A, head(R) = B and the frontier of R fr(R) =* $\vec{x}$ *(the set of variables shared by the body and the head of R).*
An ontology O is a pair (T,F) where T is a set of existential rules and F is a factbase.

Definition 5. *Let T be a ruleset,* α *be a rule,* σ *be a subsitution and F be a factbase.*
The tuple (α, σ) *is a trigger for F if :*
- *the domain of* σ *is the set of all variables occurring in Body(*α*).*
- $\sigma(Body(\alpha)) \subseteq F$.
- *For all* $\hat{\sigma}$ *which extends* σ *and has its domain equals to the set of all variables occurring in Body(*α*) and Head(*α*) ,*$\hat{\sigma}(Head(\alpha)) \nsubseteq F$

$Tr_\alpha(F)$ *is the set of all trigers* (α, σ) *for F.*
$Tr_T(F)$ *is the set of all trigers for F.*
The rule α *is satisfied by F if* $Tr_\alpha(F) = \emptyset$.
T is satisfied by F if $Tr_T(F) = \emptyset$.

Example If $\alpha = A(x,y) \rightarrow B(x,z)$, F = $\{A(b,c)\}$ and $\sigma = \{x \mapsto b, y \mapsto c\}$ then (α, σ) is a trigger for F.

Definition 6. *a ruleset T is satisfied by a factbase M if every rule of T is satisfied by M.*
A factset M is a model for an ontology O = (T,F) if $F \subset M$ *and T is satisfied by M.*
A Boolean conjunctive query (BCQ) is a closed formula of the form $\exists x_1, ..., x_n.F(x_1, ..., x_n)$ *where F is a conjunction of atoms.*
A fact set F entails a BCQ $B = \exists x_1, ..., x_n.F(x_1, ..., x_n)$ *(noted* $F \vDash B$*) if there exists a substitution* σ *such that* $\sigma(B) \subset F$*. An ontology O entails a BCQ* $B = \exists x_1, ..., x_n.F(x_1, ..., x_n)$ *(noted* $O \vDash B$*) if for every model M of O,* $M \vDash B$*.*
A model U for an ontology O is universal if for every model M of O, there exists an homomorphism $h : U \rightarrow M$.