

# Contention Management for 5G

DB,CC,MG,OM,YS

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## Abstract

This article treats about Contention Management for 5G.

The evolutions proposed in next generations of mobile network architectures aim at evolving toward centralized radio network architectures (C-RAN, for Cloud Radio Access Network) to reduce consumption costs and power at the base stations [1]. This type of architecture faces the challenge of mastering the latency in the transfer process between the Remote Radio Heads (RRHs) on the field and baseband units (BBUs) in the cloud. Low latency is already critical for the deployment of C-RAN approach. The standard requires meeting time constraints for functions like HARQ (Hybrid Automatic Repeat reQuest) that needs to be processed in less than from 1 to 10ms [2] (depending on targeted services). One specificity in the C-RAN context is not only the latency constraint, but also the periodicity of the data transfer between RRH and BBU (this HARQ constraints must be enforced for each frame emitted every millisecond).

Thus in this article, we work on latency and periodicity constraints in fronthaul. The best current solution is to rely on an almost full optical approach, where each end-points (RRH on one side, BBU on the other side) are connected through direct fiber or full optical switches [3]. This architecture is very expensive and hardly scales in the case of a mobile network composed of about 10,000 base stations. It is then needed to find a solution to offer low latency over commoditized packet based networks. Indeed, dynamical optical bypass and dynamical management of the emission should be considered to guarantee latency constraints, as it is considered in the french ANR project N-GREEN. This project proposes a new type of switching/routing node and a specific network architecture exploiting WDM packets thanks to a new generation of optical add/drop multiplexers (WSADM: WDM slotted add/drop multiplexer). These packets having a fixed duration close to  $1\mu\text{s}$  are transported in a transparent way, to better exploit the switching matrix of the node; their headers will be transported over one dedicated wavelength at a lower bit rate, to reduce the physical constraints of the electronic processing and scheduler.

Thus, new scheduling and routing paradigms and new technologies have to be considered to guarantee delay constrained periodic data transfers. One of the most promising approaches relies on the concept of Deterministic Networking (DN) such that one get rid of statistical multiplexing. The traditional queue managements are replaced by time based forwarding. Solutions for Deterministic Networking are under standardization in IEEE 802.1 TSN group [4], as well

at IETF DetNet working group [5]. Several patents on concepts and mechanisms for DetNet have been already published, see for example [6, 7]. To make DN working over a network composed of several nodes, it is required to manage the time at which the packets of deterministic paths are crossing each nodes. The major difficulty of this problem is the periodicity of the process. Indeed, a deterministic sending for the messages between each pair BBU/RRH must not collide with the other messages sent by the others BBU/RRH in the same period, but also in the previous and following periods.

Considering a graph, modeling the network topology, and a set of routes from source nodes (modeling connections to the BBU) and destination nodes (modeling the RRH) in this graph, the purpose is to select, for each destination node a route from one source node to it and a periodic routing scheme allowing to periodically sent a packet to each base station without congestion conflicts between all such packets and to insure a minimum latency. In a slotted time model, the aim is here to minimize the duration of the period, with a constraint of the maximum length of routes to be selected. Even if the selected set of routes is given this optimization problem has been shown to be NP-hard. From an algorithmic point of view, the purpose of this project is first, to study the complexity and the approximability of this problem when the length of the routes is small (which corresponds to realistic cases), and secondly, to propose and implement some heuristics to solve this problem on realistic topologies.

This problem may look like wormhole problem [8], very popular few years ago, but here, we want to minimize the time lost in buffers and not just avoid the deadlock, and the wormhole does not treat about the periodicity. Several graph colorings have been introduced to model similar problems such as the allocation of frequencies [9], bandwidths [10][17] or routes [8][18] in a network or train schedules [11][19]. Unfortunately, they do not take into account the periodicity and the associated problems are also NP-complete. The only model which incorporates some periodicity is the circular coloring [12, 13, 14] but is not expressive enough to capture our problem.

## Paper organisation

In the next section, ...

# 1 Model

## 1.1 Definitions

We consider a directed graph  $G = (V, A)$  modeling a network. Each arc  $(u, v)$  in  $A$  is labeled by an integer  $dl(u, v) \geq 1$  that we call the delay and which represents the number of time slots taken by a signal to go from  $u$  to  $v$  using this arc.

A **route**  $r$  in  $G$  is a sequence of consecutive arcs  $a_0, \dots, a_{k-1}$ , with  $a_i = (u_i, u_{i+1}) \in A$ . We will often refer to the first element of the route as a source

and the last as a target.

The **latency** of a vertex  $u_i$  in  $r$ , with  $i \geq 1$ , is defined by

$$\lambda(u_i, r) = \sum_{0 \leq j < i} dl(a_j)$$

We also define  $\lambda(u_0, r) = 0$ . The latency of the route  $r$  is defined by  $\lambda(r) = \lambda(u_k, r)$ .

A **routing function**  $\mathcal{R}$  in  $G$  associates to each pair of vertices  $(u, v)$  a route from  $u$  to  $v$ . Let  $\mathcal{C}$  be an **assignment** in  $G$ , i.e., a set of couples of different vertices of  $G$ . We denote by  $\mathcal{R}_{\mathcal{C}}$  the set of routes  $\mathcal{R}(u, v)$  for any  $(u, v)$  in  $\mathcal{C}$ . We call  $\mathcal{R}_{\mathcal{C}}$  a **routage graph**, it contains all the informations needed in the forthcoming problems (assignment, routes and delays of the arcs).

**TODO:** Si on s'en sert, ajouter ici que le routage est cohérent.

## 1.2 Slotted time Model

Consider now a positive integer  $P$  called the **period**. In our problem, we send messages in the network with period  $P$ . The time will thus be cut into slices of  $P$  discrete slots. Assume we send a message at the source  $s_i$  of the route  $r_{s_i}$ , at the time slot  $m_{s_i}$  in the first period, then a message will be sent at time slot  $m_{s_i}$  at each new period. We define the first time slot at which the message reaches a vertex  $v$  in this route by  $t(v, r_{s_i}) = m_{s_i} + \lambda(v, r) \bmod P$ .

A message usually cannot be transported in a single time slot. We denote by  $\tau$  the number of consecutive slots necessary to transmit a message. Let us call  $[t(v, r_{s_i})]_{P, \tau}$  the set of time slots used by  $r_{s_i}$  at a vertex  $v$  in a period  $P$ , that is  $[t(v, r_{s_i})]_{P, \tau} = \{t(v, r_{s_i}) + k \bmod P \mid 0 \leq k < \tau\}$ . Usually  $P$  and  $\tau$  will be clear from the context and we will denote  $[t(v, r_{s_i})]_{P, \tau}$  by  $[t(v, r_{s_i})]$ .

A  $(P, \tau)$ -**periodic affectation** of a routage graph  $\mathcal{R}_{\mathcal{C}}$  is a sequence  $\mathcal{M} = (m_{s_0}, \dots, m_{s_{n-1}})$  of  $n$  integers that we call **offsets**, with  $n$  the number of routes in  $\mathcal{R}_{\mathcal{C}}$ . The number  $m_{s_i}$  represents the index of the first slot used in a period by the route  $r_{s_i} \in \mathcal{R}_{\mathcal{C}}$  at its source  $s_i$ . A  $(P, \tau)$ -periodic affectation must have no **collision** between two routes in  $\mathcal{R}_{\mathcal{C}}$ , that is  $\forall (r_{s_i}, r_{s_j}) \in \mathcal{R}_{\mathcal{C}}^2, i \neq j$ , we have

$$[t(u, r_{s_i})] \cap [t(u, r_{s_j})] = \emptyset.$$

As an example of a  $(2, 1)$ -periodic affectation, let consider a routage graph with routes  $\{r_{s_i}\}_{i=0, \dots, n-1}$ , such that all pairs of routes intersect at a different edge. We set  $\tau = 1$  and the delays are chosen so that if  $r_{s_i}$  and  $r_{s_j}$  have  $v$  as first common vertex then we have  $\lambda(v, r_{s_i}) - \lambda(v, r_{s_j}) = 1$ . There is a  $(2, 1)$ -periodic affectation by setting all  $m_{s_i}$  to 0.

## 1.3 Problems

We want to ensure that there is an affectation which allows to send periodic messages from sources to target without collisions. The problem we need to solve is thus the following:



Figure 1: A routage graph with  $(0,0,0)$  as a  $(2,1)$ -periodic affectation

### Periodic Routes Assignment (PRA)

**Input:** a routage graph  $\mathcal{R}_C$ , an integer  $\tau$  and an integer  $P$ .

**Question:** does there exist a  $(P, \tau)$ -periodic affectation of  $\mathcal{R}_C$  ?

We will prove in Sec. 2 that the problem PRA is NP-complete, even in restricted settings. Even approximating the smallest value of  $P$  for which there is a  $(P, \tau)$ -periodic affectation is hard.

An unusual property of affectation is that given a routage graph, we may have a  $(P, \tau)$ -periodic affectation but no  $(P', \tau)$ -periodic affectation with  $P' > P$ : the existence of an affectation is not monotone with regards to  $P$ .

**Lemma 1.** *For any odd  $P$ , there is a routage graph such that there is  $(2, 1)$ -periodic affectation but no  $(P, 1)$ -periodic affectation.*

*Proof.* Consider the routage graph  $\mathcal{R}_C$  given in the previous subsection. We change the delays so that for  $v$ , the first vertex which belongs to  $r_i$  and  $r_j$ , we have  $\lambda(v, r_i) - \lambda(v, r_j) = P$ , where  $P$  is an odd number smaller than  $n$ , the number of routes in  $\mathcal{R}_C$ . In such a graph, there is no  $(P, \tau)$ -periodic affectation, since the problem reduces to finding a  $P$ -coloring in a complete graph with  $n > P$  vertices.

If we consider a period of 2, for all  $i \neq j$ ,  $\lambda(v, r_i) - \lambda(v, r_j) \bmod 2 = 1$ . Therefore  $(0, \dots, 0)$  is a  $(2, 1)$ -periodic affectation of  $\mathcal{R}_C$ .

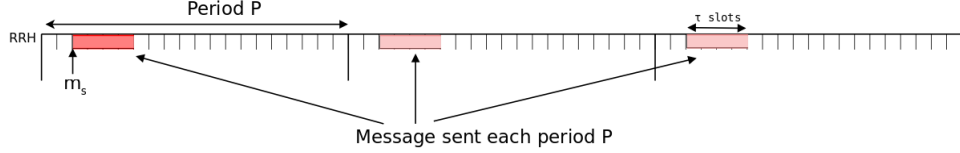
□

In the context of cloud-RAN applications, we consider here the digraph  $G = (V, A)$  modeling the target network and two disjoint subsets of vertices  $S$  and  $T$  of equal cardinality, where  $S$  is the set of RRHs and  $T$  is the set of BBUs. A **symmetric** assignment  $\mathcal{C}$ , is an involutive function from  $S$  to  $T$ , which maps each element  $s \in S$  to  $\mathcal{C}(s)$  and  $\mathcal{C}(s)$  to  $s$ . It can also be seen as the set of pairs  $(s, \mathcal{C}(s))$  and  $(\mathcal{C}(s), s)$ .

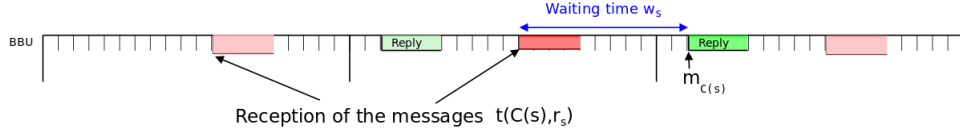
The routing function  $\mathcal{R}$  associates to each couple  $(s, \mathcal{C}(s))$  a route called  $r_s$  and

to each couples  $(\mathcal{C}(s), s)$  a route called  $r_{\mathcal{C}(s)}$ .

We are given a period  $P$ , a routing function  $\mathcal{R}$ , and we consider a  $(P, \tau)$ -periodic affectation of  $\mathcal{R}_{\mathcal{C}}$  which associates  $m_s$  to the route  $r_s$  which begins by  $s$ , and  $m_{\mathcal{C}(s)}$  to the routes  $r_{\mathcal{C}(s)}$  which begins by  $\mathcal{C}(s)$ . This affectation represents the following process: first a message is sent at  $s$ , through the route  $r_s$ , at time  $m_s$ .



This message is received by  $\mathcal{C}(s)$  at time  $t(\mathcal{C}(s), r_s)$ . It is then sent back to  $s$  in the same period at time  $m_{\mathcal{C}(s)}$  if  $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$ , otherwise at time  $m_{\mathcal{C}(s)}$  in the next period. The time between the arrival of the message and the time it is sent back is called the **waiting time** and is defined by  $w_s = m_{\mathcal{C}(s)} - t(\mathcal{C}(s), r_s)$  if  $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$  and  $w_s = m_{\mathcal{C}(s)} + P - t(\mathcal{C}(s), r_s)$  otherwise.



When a BBU receives a message, it must compute the answer before sending it back to the RRH. This time can be encoded in the last arc leading to the BBU and thus we need not to consider it explicitly in our model.

Thus, the whole process time for a message sent at vertex  $s$  is equal to

$$PT(s) = \lambda(r_s) + w_s + \lambda(r_{\mathcal{C}(s)}).$$

In the process time, we count the time between the first time at which a message is emitted and the first time at which the message comes back. Alternatively we could consider the time between the emission of the first slot and the reception of the last slot of the message, which would add  $\tau$  to the process time. However, since all messages are of size  $\tau$ , it will not change the problem we solve in the rest of the article and that we now introduce.

The **maximum process time** of the  $(P, \tau)$ -periodic affectation  $\mathcal{M}$  is defined by  $MT(\mathcal{M}) = \max_{s \in S} PT(s)$ . The problem we want to solve is the following.

### Periodic Assignment for Low Latency(PALL)

**Input:** A routage graph  $\mathcal{R}_{\mathcal{C}}$  with  $\mathcal{C}$  a symmetric assignment, a period  $P$ , an integer  $\tau$ , an integer  $T_{max}$ .

**Question:** does there exist a  $(P, \tau)$ -periodic affectation  $\mathcal{M}$  of  $\mathcal{R}_{\mathcal{C}}$  such that  $MT(\mathcal{M}) \leq T_{max}$ ?

TODO: Si on veut, on peut parler du temps moyen aussi ici, seulement si on fait quelque chose dessus dans la suite

In this article, we work on a given routage, but we could imagine a problem, in which, considering a given graph  $G = (V, A)$  and a period  $P$ , the question is to find, if it exists, a routage graph  $\mathcal{R}_C$ , in which there is  $(P, \tau)$ -periodic affectation.

**TODO:** Dire si on règle le problème de trouver un bon routage dans un cas particulier après

## 2 Solving PRA

### 2.1 NP-Hardness

In this section we assume that the size of a message  $\tau$  is equal to one. We will prove the hardness of PRA and PALL for this parameter, which implies the hardness of the general problems. Consider an instance of the problem PRA, i.e., a routage graph  $\mathcal{R}_C$  and a period  $P$ .

The **conflict depth** of a route is the number of other routes which share an edge with it. The conflict depth of a routage graph  $\mathcal{R}_C$  is the maximum of the conflict depth of the routes in  $\mathcal{R}_C$ .

The **load** of a routage graph is the maximal number of routes sharing the same arc. It is clear that a  $(P, 1)$ -periodic affectation must satisfy that  $P$  is larger or equal to the load.

We give two alternate proofs that PRA is NP-complete. The first proof works already for conflict depth two. Remark that for conflict depth one, the graph can be seen as a set of disjoint pair of routes, on which PRA and PALL can be solved in linear time. The second proof reduces the problem to graph coloring and implies inapproximability when one tries to find the smallest possible  $P$ .

**Proposition 1.** *Problem PRA is NP-complete, when the routing is of conflict depth two.*

*Proof.* The problem PRA is in NP since given an offset for each route in an affectation, it is easy to check in linear time in the number of edges whether there are collisions.

Let  $H = (V, E)$  be a graph and let  $d$  be its maximal degree. We consider the problem to determine whether  $H$  is edge-colorable with  $d$  or  $d + 1$  colors. The edge coloring problem is NP-hard [15] and we reduce it to PRA to prove its NP-hardness. We define from  $H$  an instance of PRA as follows. For each  $v$  in  $V$ , the graph  $G$  has two vertices  $v_1, v_2$ , and for each  $(u, v) \in E$ , the graph  $G$  has two vertices  $s_{u,v}, t_{u,v}$ . Set an arbitrary orientation of the edge  $(u, v)$ , such that the following route is directed from  $u$  to  $v$ .

For each edge  $(u, v) \in E$ , there is a route  $s_{u,v}, u_1, u_2, v_1, v_2, t_{u,v}$  in  $\mathcal{R}$ . All these arcs are of weight 0. The set of arcs of  $G$  is the union between all the arcs of the previous routes. The affectation  $\mathcal{C}$  is the set of pair of vertices  $(s_{u,v}, t_{u,v})$ .

Observe that the existence of a  $d$ -coloring of  $H$  is equivalent to the existence of a  $(d, 1)$ -periodic affectation of  $\mathcal{R}_C$ . Indeed, a  $d$ -coloring of  $H$  can be seen as a labeling of its edges by the integers in  $\{0, \dots, d - 1\}$  and we have a bijection

between  $d$ -colorings of  $H$  and offsets of the routes of  $\mathcal{R}_C$ . By construction, the constraint of having no collision between the routes is equivalent to the fact that no two adjacent edges have the same color. Therefore we have reduced edge coloring to PRA which concludes the proof.  $\square$

**TODO: Faire un dessin d'illustration ?**

Remark that we have used zero weight in the proof. If we ask the weights to be strictly positive, which makes sense in our model since they represent the latency of physical links, it is easy to adapt the proof. We just have to set them so that in any route the delay at  $u_1$  is equal to  $d$  and thus equal to 0 modulo  $d$ . We now lift this hardness result to the problem PALL.

**Corollary 1.** *Problem PALL is NP-complete for routing of conflict depth two.*

*Proof.* We consider  $(\mathcal{R}_C, P, \tau)$  an instance of PRA. We assume that no vertex appears both in the first and second position in a pair of  $\mathcal{C}$ . Remark that this condition is satisfied in the previous proof, which makes the problem PRA restricted to this kind of instances NP-complete. Let us define  $T_{max} = 2 \times \max_{r \in \mathcal{R}} \lambda(r) + P$ . We consider  $\mathcal{C}'$  and  $\mathcal{R}'_{C'}$ , symmetrized versions of  $\mathcal{C}$  and  $\mathcal{R}_C$  where for every route there is a symmetric route with new arcs and the same weights. The instance  $(\mathcal{R}'_{C'}, P, \tau, T_{max})$  is in PALL if and only if  $(\mathcal{R}_C, P, \tau)$  is in PRA. Indeed the waiting time of each route is by definition less than  $P$  and thus the maximal process time is always less than  $T_{max}$ . Moreover a  $(P, \tau)$ -affectation of  $\mathcal{R}_C$  can be extended into a  $(P, \tau)$ -affectation of  $\mathcal{R}'_{C'}$  in the following way. For each route  $r_{u,v}$ , the time at which the message arrives is  $t(v, r_{u,v})$ , then we choose as offset for  $r_{v,u}$ ,  $-t(v, r_{u,v}) \bmod P$ . The symmetry ensures that each new route  $r_{v,u}$  in  $\mathcal{R}'_{C'}$  uses the same times slot as  $r_{u,v}$  and thus avoid collisions.  $\square$

Let MIN-PRA be the problem, given a routage graph and an assignment, to find the minimal period  $P$  such that there is a  $P$ -periodic affectation.

**Theorem 2.** *The problem MIN-PRA cannot be approximated in polynomial time within a factor  $n^{1-o(1)}$ , with  $n$  the number of routes, unless  $P = NP$  even when the load is two.*

*Proof.* We reduce graph coloring to PRA. Let  $H$  be a graph instance of the  $k$ -coloring problem. We define  $\mathcal{R}$  in the following way: for each vertex  $v$  in  $H$ , there is a route  $r_v$  in  $\mathcal{R}$ . Two routes  $r_v$  and  $r_u$  share an arc if and only if  $(u, v)$  is an edge in  $H$ ; this arc is the only one shared by these two routes. All arcs are of delay 0.

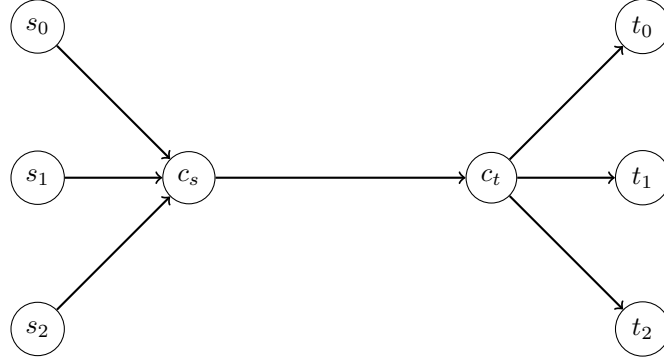
Observe that the existence of a  $k$ -coloring of  $H$  is equivalent to the existence of a  $(k, 1)$ -periodic affectation in  $G$ , by converting an offset of a route into a color of a vertex and reciprocally. Therefore if we can approximate the minimum value of  $P$  within some factor, we could approximate the minimal number of colors needed to color a graph within the same factor. The proof follows from the hardness of approximability of finding a minimal coloring [16].  $\square$

In particular, this reduction shows that even with small maximal load, the minimal period can be large.



### 3 The Star Topology

In this section, we consider graphs with a very simple topology that we call the **star topology**. First, for each arc  $(u, v)$ , there is also an arc  $(v, u)$  with the same weight. Moreover, there is a special arc, the central arc, which is shared by all routes. All routes consist from an arc from its source to the central source node, denoted by  $c_s$ , then the central arc to  $c_t$ , the central target node, and an arc to its target. In addition to those central nodes, there are two sets of vertices,  $S = \{s_0, \dots, s_{n-1}\}$  and  $T = \{t_0, \dots, t_{n-1}\}$ , of cardinality  $n$  and  $\mathcal{C}$  a symmetric assignment from  $S$  to  $T$ . The routes are the directed paths of the form  $s_i, c_s, c_t, t_i$  and  $t_i, c_t, c_s, s_i$ .



We assume the weight of the central arc to be 0 since it appears in every route, its value does not matter when considering affectations.

Collisions between messages can only appear at node  $c_s$  on the way forward and at node  $c_t$  on the way back. Thus, we only have to check if there is no collisions in a period at those two points. We consider two periods  $P$  at these vertices, the **forward period** at  $c_s$ , and the **backward period** at  $c_t$ . A message issued by the source  $s_1$  at time  $m_{s_1}$  will reach  $c_s$  at time  $m_{s_1} + t(c_s, r_{s_1}) \bmod P$  in the forward period, and  $c_t$  at time  $m_{t_1} + t(c_t, r_{t_1}) \bmod P$  in the backward period. A  $(P, \tau)$ -periodic affectation consist in choosing for all  $i < n$ ,  $m_i$  and  $m_{\mathcal{C}(s_i)}$  such that, for any couple  $(j, k)$ , we have  $[t(c_s, r_{s_j})] \cap [t(c_s, r_{s_k})] = \emptyset$  and  $[t(c_t, r_{t_j})] \cap [t(c_t, r_{t_k})] = \emptyset$ .



### 3.1 Solving PALL without waiting times

In this subsection, we deal with a simpler version of the problem PALL. We ask for a  $(P, \tau)$ -periodic affectation **with all waiting times equal to 0** and we call this restricted problem **Periodic Assignment for Zero Latency** or PAZL. In that case  $MT(\mathcal{M})$  is equal to twice the delay of the longest route, thus  $T_{max}$  is not relevant anymore. Since  $w_i = 0$ , choosing  $m_i$ , the offset of the route from  $s_i$  to  $t_i$ , also sets the offset of the route from  $t_i$  to  $s_i$  to  $m_i + \lambda(r_i) \bmod P$ . We assume from now on that the weight of the arcs from  $s_i$  to  $c_s$  are all equal to 0, since we can change all  $m_i$  into  $m_i - t(c_s, r_i) \bmod P$  and obtain the same affectation.

#### 3.1.1 Shortest-longest policy

We present a simple policy, which works when the period is large with regards to the delay of the routes. The message are sent in order from the shortest route to the longest route, without any gap between two messages in the forward period. In other words, we assume that the route  $r_i$  are sorted by increasing  $\lambda(r_i)$  and we set  $m_{s_i} = i\tau$ . We call this algorithm **Shortest-Longest**.

By definition, there are no collision in the forward period and if the period is long enough, it is easy to see that in the backward period the order of the messages are the same as in the forward period and that no collision can occur.

**Proposition 2.** *When  $n\tau + 2(\lambda(r_{s_{n-1}}) - \lambda(r_{s_0})) \leq P$ , the algorithm Shortest-Longest solves PAZL.*

*Proof.* Since  $m_{s_i} = i\tau$ ,  $[t(c_s, r_{s_i})] = \{i\tau, \dots, (i+1)\tau - 1\}$  and there are no collision on the forward period.

We assume that  $\lambda(r_{s_0}) = 0$ , by removing  $\lambda(r_{s_0})$  to every route, it does not change the order on the routes nor the result of this algorithm. We have  $[t(c_s, r_{t_i})] = \{2\lambda(r_{s_i}) + i\tau, \dots, 2\lambda(r_{s_i}) + (i+1)\tau - 1\}$  since  $2\lambda(r_{s_{n-1}}) + n\tau \leq P$ . Since  $\lambda(r_{s_i}) \leq \lambda(r_{s_{i+1}})$  by construction, we have  $2\lambda(r_{s_i}) + (i+1)\tau - 1 < 2\lambda(r_{s_{i+1}}) + (i+1)\tau$  which proves that there are no collision on the backward period.  $\square$

If the period is slightly smaller than the bound, a collision will occur on the route in the backward period, making this policy not useful even as a heuristic for longer routes.

#### 3.1.2 Greedy Algorithm

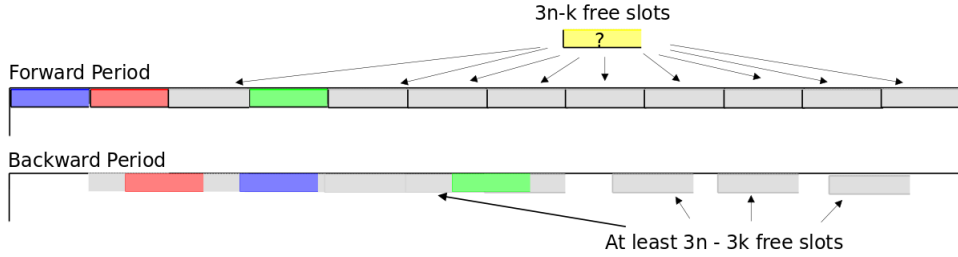
We propose a greedy algorithm to build a  $(P, \tau)$ -periodic affectation, which provably works when the period is large enough with regards to the number of routes and size of messages. In other words, we can always find a solution when the network is far from saturated.

**Proposition 3.** *Let  $n$  be the number of routes and let  $3n\tau \leq P$ , then there is an algorithm which finds a  $(P, \tau)$ -periodic affectation in time  $O(n^2)$ .*

*Proof.* We consider the forward period and cut it into consecutive macro slots of  $\tau$  time slots. The algorithm works by choosing an offset for each route in the following way: try all offsets which put the message in a yet not used macro slot in the forward period. The choice of an offset also fixes the position of the message in the backward period, chose the first one which does not create a collision. We now prove that this algorithm always find an offset for all routes when  $P \geq 3n\tau$ .

Assume we are choosing the offset of the route  $r_{k+1}$ , we have at least  $P - k \geq 3n - k$  free macro-slots, since  $P \geq 3n\tau$ . Each of these  $3n - k$  possible offset values translates into  $3n - k$  positions of messages in the backward period. All these positions are separated by at least  $\tau$  slots. There are already  $k$  messages of size  $\tau$  in the backward period. One such message can intersect at most 2 potential positions since they are disjoint intervals. Therefore amongst the possible  $3n - k$  positions, there are  $3n - k - 2k$  which are without collision. Since  $k < n$ , one of the choice is without collision, which proves that the algorithm terminates and find a  $(P, \tau)$ -periodic affectation.

This algorithm works with a complexity  $O(n^2)$ , since for the  $k^{\text{th}}$  route we have to try at most  $2k$  offsets before finding a correct one. We can test the  $2k$  offsets of the backward period in time  $O(k)$  by maintaining an ordered list of the intervals used by already set routes.  $\square$



**TODO:** J'ai commencé à modifier l'algo pour qu'il colle, mais ça n'est pas encore ça. pas sur que ça vaille la peine de le mettre, je le commente pour l'instant.

This algorithm, contrarily to the previous one, may work well, even when the condition  $P \geq 3n\tau$  is not true. In fact, experimental data in Subsection 3.1.4 suggest that the algorithm works well as soon as  $P \geq 1.5\tau$  for small routes.

**TODO:** vérifier cette affirmation en faisant une expérience et une régression linéaire

### 3.1.3 Exhaustive search

We now present an exhaustive search algorithm, which tries to set the offsets in all possible ways until it has found a  $(P, \tau)$ -periodic affectation. We have  $n$  routes denoted by  $\{0, \dots, n-1\}$ . A partial solution  $S$  is a partial function from  $[n]$  to  $\{0, 1, \dots, P - \tau - 1\}$  which sets a starting time for a subset of the routes  $R(S) \subseteq [n]$ , such that there are no collisions for these routes. A partial solution  $S'$  extends  $S$ , if it sets one more routes than  $S$  to a larger starting time than all

routes in  $S$ , that is  $R(S') = R(S) \cup \{r'\}$  and for all  $r \in R(S)$ ,  $S(r) + \tau \leq S'r'$ . We define a tree whose nodes are partial solutions and such that the root is the empty partial solution and the children of a partial solution are the partial solutions which extend it. All solutions to our problem will be leaves in this tree, therefore our algorithm is simply a depth-first search of the tree.

Remark that a node  $S$  with  $|R(S)| = k$  can have as many as  $(n - k)P$  children. Since the tree is of depth  $n$ , the tree may have as many as  $n!P^n$  elements and while  $n$  is often small,  $P$  may be large which makes its traversal intractable. Therefore we have to find cuts in the tree to avoid to explore it entirely and henceforth make the algorithm practical. Cuts correspond to the detection of subtrees which contain no solutions and can thus be skipped. We now propose three cuts, the first two being particularly useful when the network is loaded ( $n\tau$  is not much smaller than  $P$ ).

1. We consider the number of slots which can be used by routes not yet fixed by a partial solution in the *forward period*. When we extend a solution into  $S$  by a route at offset  $m$ , then at most  $(P - m)/\tau$  routes can still be set without collisions in the forward period. If that value is less than the number of routes which are not in  $R(S)$ , it is a failure and the algorithm backtracks.
2. For the next two cuts, we need to define the notion of the useful slots of a partial solution  $S$  in the *backward period*: a slot is said to be useful, if it is not used by a message set by  $S$  in the backward period and it belongs to an interval of at least  $\tau$  such slots. Useful slots are positions which can be used when extending  $S$ . We will denote by  $([a_i, b_i])_{i \leq l}$  the ordered sequence of intervals of useful slots of  $S$ . Without loss of generality we can assume that all  $a_i, b_i \leq l$ . The number of message of size  $\tau$  which can be set in the useful slots of  $S$  is thus  $\sum_{i=0}^l (b_i - a_i)/\tau$ . If that value is less than the number of routes which are not in  $R(S)$ , it is a failure and the algorithm backtracks. Notice that the list of intervals of useful slots and the value  $\sum_{i=0}^l (b_i - a_i)/\tau$  can be maintained in constant time, since each time a route is added, we only need to split an interval of useful slots into at most two such intervals.
3. Let  $S_1$  and  $S_2$  be two partial solutions with  $R(S_1) = R(S_2)$ . Let  $US_1$  (respectively  $US_2$ ) be the set of useful slots of  $S_1$  (resp.  $S_2$ ). We say that  $S_1$  dominates  $S_2$  if the largest offset fixed in  $S_1$  is smaller than the one in  $S_2$  and  $US_2 \subseteq US_1$ . Remark that any valid sequence of extension of  $S_2$  (fixing offsets of routes in the complementary of  $R(S_2)$ ) is also a valid sequence of extensions of  $S_1$ . Therefore if the tree rooted at  $S_2$  contain a solution, then  $S_1$  contains one too. Hence, in our exhaustive search of the tree of partial solutions, we can skip the tree rooted at  $S_2$ .

We now explain how we can detect some partial solutions which are dominated so that we do not explore their subtrees. Consider a partial solution  $S$  which we extend into  $S'$  by setting the offset of the route  $r$  to be the smallest possible. The offset of  $r$  in the backward period is  $S'(R) + \lambda$  and

we denote the end of the message before by  $a$ . Then all extensions of  $S$  into  $S''$  with  $S'(r) < S''(r) < a + \tau - \lambda$  are dominated by  $S'$ . Therefore when computing the extension of  $S$ , we first build  $S'$  and then  $S''$  with  $S''(r) = a + \tau - \lambda$ , skipping all values in between.

The third cut works well in conjunction with the first one since it makes the offsets grow quickly and which makes the first cut more likely to apply. A last cut could be implemented, by computing whether for every route not in  $R(S)$  the set of possible positions in the backward period and checking whether one is contained in the useful slots of  $S$ .

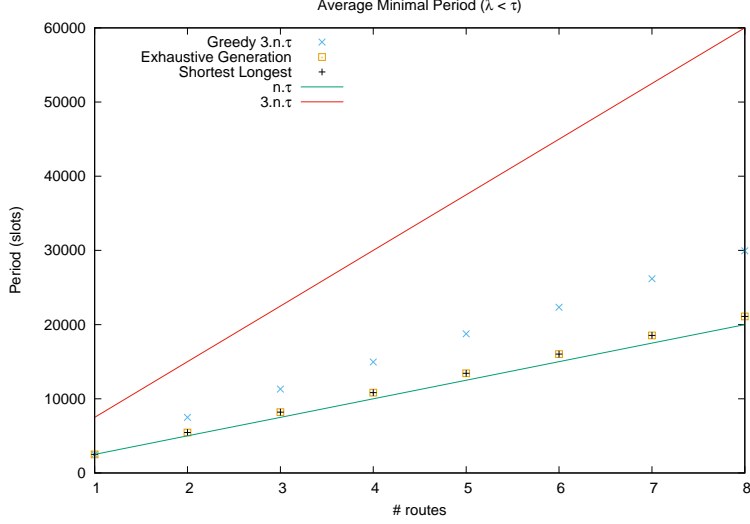
### 3.1.4 Experimental Results

The following performance evaluation results compares the three presented algorithms. Notice that both Greedy algorithm and the Shortest-Longest are polynomial time algorithm but are not always able to find optimal solutions. On the other hand, the exhaustive search will find an optimal solution, if it exists, but works in exponential time. We will compare the performance of the algorithms in two different regimes: the routes are either smaller than the size of a message or they are arbitrary long.

**TODO:** Mettre les données concrètes plus proprement et éventuellement ne pas les mettre si on envoie pas à un truc de télécom

**Short routes** We chose parameters coming from our C-RAN settings. A message is of 2500 slots, while the values  $\lambda(c_t, r_{t_i})$  are drawn uniformly between 0 and 700 slots. For each of the three different algorithms and for a given instance we look for the minimal period  $P$  for which the problem has a  $(P, \tau)$  periodic affectation. We then compute the average of these minimal periods for 1000 random star topologies for different number of routes. To find the minimum period, we need to do a linear search on the value  $P$ , a dichotomy would not work because of Lemma 1.

Since in this setting the exhaustive generation was always finishing in less than ten seconds, the results corresponding to this algorithms are optimal and are a lower bounds for the period found for the other algorithms. As an information, we draw the  $f(n) = n\tau$ , which is an absolute lower bound on the period and  $g(n) = 3n\tau$  lines which is an upper bound on the period found by the greedy algorithm.



With those parameters, we can observe that the average period of the Shortest-Longest algorithm, which is determined by the difference between the longest and the shortest route, is always equal to the Exhaustive generation, that is, the lower bound. The next results is to theoretically prove that, with those parameters, the Shortest-Longest algorithm gives optimal solutions, like this experimental study suggests it does. About the Greedy Algorithm, with those parameters, it always gives us bigger periods than the Shortest longest. However, the results show that this algorithm gives average periods lower theoretical upper bound of this algorithm ( $3n\tau$ ). We can also observe that, for any algorithm, the average minimal periods increases linearly with the number of routes.

**TODO: regression linéaire**

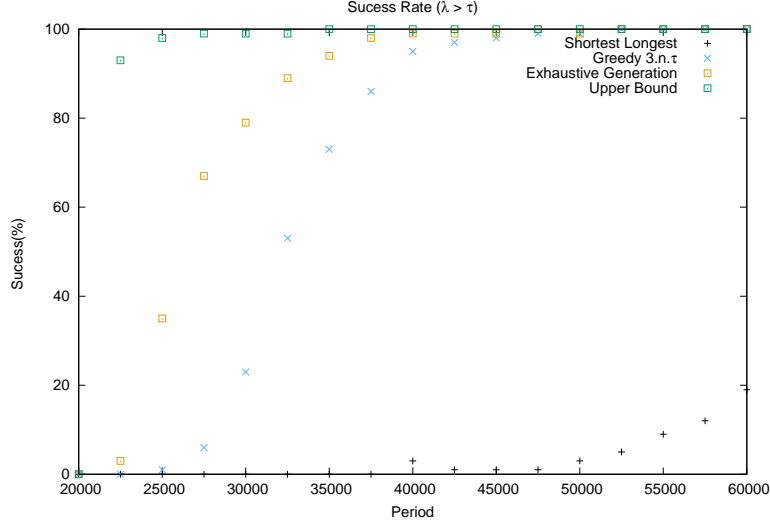
**Long routes** Then, we looked at the evolution of the minimal period required with the different algorithm if we significantly increase the size of the route. The following figure shows us the success rate of the algorithms, according to the period, for 8 routes in which  $\lambda(c_s, r_{s_i})$  and  $\lambda(c_t, r_{t_i})$ , are drawn following an uniform distribution between 0 and 30000 slots, that correspond to have the size of the route and the period approximately in the same order of magnitude.

By increasing the size of the routes, the cuts of the exhaustive search, are less efficient. Thus, we defined a counter that allow the search to end if no solution are found. On the following figure, the upper bound represents the cases in which the exhaustive search found a solution plus the number of *early* halts of the search. The real upper bound is probably between those two curves.

The reason why the shortest longest were optimal in the previous experiences (i.e. having routes shorter than message size) is no longer reviewed. We can also imagine that sending the shortest route first on long routes significantly

increases the period, because there will be a lot of lost size in the backward and forward periods.

On the other hand, one can imagine that the greedy  $3.n.\tau$  algorithm finds reasonable solutions, because the algorithm does not consider the size of the route, and thus, is not impacted by this parameter.



As we can see, the Shortest longest has a very low success rate, because of the distribution of the size of the routes is now broader, so is the required period. Whereas, the Greedy 3NT gives us solutions with much lower period than Shortest Longest. The exhaustive search keeps finding good solutions in short periods but it fails a lot for periods until  $2.n.\tau$ .

So, those experiences shows us that for real topologies with small routes, the Shortest-Longest algorithm finds some affectations with small Periods, but, the longer the routes are, the better it is to use the Greedy policy to find a solution without waiting times.

Nevertheless, if we want to consider networks, the period is fixed. As we just saw, if the load increases, the needed period increases too, and it is not possible anymore to find a solution without waiting times with a small period. The solution is to allow some messages to wait on target nodes, as described in the next section.

## 3.2 Allowing waiting times

### 3.2.1 Intro

We now consider waiting times. The weights on links from sources to  $c_s$  are now relevant because the goal is to minimize  $MT(\mathcal{M})$ , with  $\mathcal{M}$  the  $(P - \tau)$ -periodic affectation of  $\mathcal{R}_C$ . To build such a  $(P - \tau)$ -periodic affectation of  $\mathcal{R}_C$ , we must

choose the offsets  $m_{s_i}$  and  $m_{t_i}$ , such that there is no  $PT(s)$  greater than a given  $T_{max}$ . As a reminder,  $PT(s) = \lambda(r_s) + w_s + \lambda(r_{t_i})$ .

### 3.2.2 LSG

To find a solution with waiting times, the following heuristic is suggested:

On the forward period, the messages are scheduled so that they are following each others, from the one using the longest route, to the one using the shortest route. Thus, we set the  $m_{s_i}$  of the routes such that  $t(c_s, r_{s_i}) = \tau \cdot i$ , if the routes  $r_i$  are ordered from the longest to the shortest.

On  $c_t$ , the policy is the following one, after setting the clock to 0:

1. To be eligible, a job needs to be able to come back on the switch at the current clock (if clock = 0, take the first message able to come back).
2. Between the eligible jobs, schedule first the one which has the lowest deadline.
3. Repeat until all the messages had been scheduled.

For any route  $i$ , the deadline correspond to  $m_{s_i} + T_{max}$ .

---

#### Algorithm 1 LSG

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**Input:** routage graph  $\mathcal{R}_C$ , period  $P$ , packet size  $\tau$

**Output:**  $(P - \tau)$ -periodic affectation of  $\mathcal{R}_C$

clock  $\leftarrow 0$

**for all** route  $i$  in  $r_{s_i}$  from the longest to the shortest **do**

$m_{s_i} \leftarrow \text{clock}$

    clock  $\leftarrow \text{clock} + \tau$

**end for**

clock  $\leftarrow 0$

take  $i$  such that  $r_{\mathcal{C}(f_j)}$  is the first route to come back in sources switch

$m_{\mathcal{C}(f_j)} \leftarrow \text{clock};$

clock  $\leftarrow t(c_t, r_{\mathcal{C}(f_j)}) + \tau$

**while** there is a route which has not been scheduled **do**

    take  $i$  such that  $r_{\mathcal{C}(f_j)}$  is the eligible route.

$m_{\mathcal{C}(f_j)} \leftarrow \text{clock} - t(c_t, r_{\mathcal{C}(f_j)});$

    clock  $\leftarrow \text{clock} + \tau$

**end while**

---

**Analysis** One of the closest approach to real networks is the case in which all the weight on the arcs going from  $c_t$  to targets are the, this algorithm gives an optimal solution. Indeed, all the same weight can be simplified by 0, thus the scheduling in the forward and backward period is the same, that is, all message following each others, so the period is minimal :  $n \cdot \tau$ , where  $n$  is the number of couples  $(s, \mathcal{C}(s))$ . Furthermore, all the waiting times are set to 0 i.e.  $w_i = 0, \forall i \in n$ , thus,  $MT(\mathcal{M})$  is equal to twice the longer delay of the routes, that is not alterable.

Another case of this topology is the case in which all the weight on the arcs between  $c_s$  and the sources are the same. In this case, the algorithm gives optimal solutions too:

We consider only the delay between  $c_t$  and the targets, that are  $\lambda(r_s)$ . Order the routes such that  $\lambda(r_{s_i}) > \lambda(r_{s_j})$ ,  $i, j \in n$ ,  $i < j$ , i.e. the route 1 is the longest one, and the route  $n$  is the shortest. The messages are sent in the forward period from 0 to  $n - 1$ , that is  $m_{s_i} = \tau * n$ .

The first message has totally reach the backward period at time  $2.\lambda(r_{s_0}) + \tau$ . The process time of the first message is so  $PT(s_0) = 2.\lambda(r_{s_0}) + \tau$ . The second message is eligible in the backward period at time  $\tau + 2.\lambda(r_{s_1})$ . Thus, the waiting time of the second route is  $w_{s_1} = 2.\lambda(r_{s_0}) + \tau - (\tau + 2.\lambda(r_{s_1})) = 2.(\lambda(r_{s_0}) - \lambda(r_{s_1}))$ . Thus  $PT(s_1) = 2.\lambda(r_{s_1}) + 2.(\lambda(r_{s_0}) - \lambda(r_{s_1})) + \tau = 2.\lambda(r_{s_0}) + \tau = PT(s_0)$ . By induction,  $PT(s_i) = PT(s_0)$ ,  $\forall i \in n$ . Consequently, for this case, the heuristic is optimal, because the longest route has no waiting time all the other routes will take exactly the same time, thus,  $MT(M) = PT(s_0) = 2.\lambda(r_{s_0}) + \tau$ , which is not alterable

### 3.2.3 Results

#### Random

#### Distributions

## 4 Conclusion

Open questions. Can we improve the results if:

- The routes are smaller than the size of a message.
- The routes are smaller than the period.
- The largest difference between two routes is smallest than one of these parameters

On a general graph:

- The routing is coherent
- The graph is symmetric

## References

- [1] C. Mobile, "C-RAN: the road towards green RAN," *White Paper*, ver, vol. 2, 2011.



- [2] Y. Bouguen, E. Hardouin, A. Maloberti, and F.-X. Wolff, *LTE et les réseaux 4G*. Editions Eyrolles, 2012.
- [3] G. P. A. W. Group, “View on 5G architecture,” *White Paper, ver*, vol. 1.0, 2016.
- [4] N. Finn and P. Thubert, “Deterministic Networking Architecture,” Internet-Draft draft-finn-detnet-architecture-08, Internet Engineering Task Force, Sept. 2016. Work in Progress.
- [5] “Time-sensitive networking task group.” <http://www.ieee802.org/1/pages/tsn.html>. Accessed: 2016-09-22.
- [6] W. Howe, “Time-scheduled and time-reservation packet switching,” Mar. 17 2005. US Patent App. 10/947,487.
- [7] B. Leclerc and O. Marcé, “Transmission of coherent data flow within packet-switched network,” June 15 2016. EP Patent App. EP20,140,307,006.
- [8] R. J. Cole, B. M. Maggs, and R. K. Sitaraman, “On the benefit of supporting virtual channels in wormhole routers,” in *Proceedings of the eighth annual ACM symposium on Parallel algorithms and architectures*, pp. 131–141, ACM, 1996.
- [9] R. Borndörfer, A. Eisenblätter, M. Grötschel, and A. Martin, “Frequency assignment in cellular phone networks,” *Annals of Operations Research*, vol. 76, pp. 73–93, 1998.
- [10] T. Erlebach and K. Jansen, “The complexity of path coloring and call scheduling,” *Theoretical Computer Science*, vol. 255, no. 1, pp. 33–50, 2001.
- [11] C. Strotmann, *Railway scheduling problems and their decomposition*. PhD thesis, PhD thesis, Universität Osnabrück, 2007.
- [12] B. Zhou, “Multiple circular colouring as a model for scheduling,” 2013.
- [13] X. Zhu, “Circular chromatic number: a survey,” *Discrete mathematics*, vol. 229, no. 1, pp. 371–410, 2001.
- [14] X. Zhu, “Recent developments in circular colouring of graphs,” in *Topics in discrete mathematics*, pp. 497–550, Springer, 2006.
- [15] I. Holyer, “The np-completeness of edge-coloring,” *SIAM Journal on computing*, vol. 10, no. 4, pp. 718–720, 1981.
- [16] D. Zuckerman, “Linear degree extractors and the inapproximability of max clique and chromatic number,” in *Proceedings of the thirty-eighth annual ACM symposium on Theory of computing*, pp. 681–690, ACM, 2006.