Contention Management for 5G

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Abstract

This article treats about Contention Management for 5G.

1 Introduction

- Context and problematic
- Related works
- Article contribution

2 Model

2.1 Definitions

We consider a directed graph $G = (V, A)$ modelling a network. Each arc (u, v) in A is labeled by an integer $dl(u, v) \geq 1$ that we call the delay and which represents the number of time slots taken by a signal to go from u to v using this arc.

A **route** r in G is a sequence of consecutive arcs $a_0, \ldots, a_{k-1}$, with $a_i = (u_i, u_{i+1}) \in A$. We will often refer to the first element of the route as a source and the last as a target.

The **latency** of a vertex u_i in r, with $i \geq 1$, is defined by

$$\lambda(u_i, r) = \sum_{0 \leq j < i} dl(a_j)$$

We also define $\lambda(u_0, r) = 0$. The latency of the route r is defined by $\lambda(r) = \lambda(u_k, r)$.

A **routing function** $\mathcal{R}$ in G associates to each pair of vertices (u, v) a route from u to v. Let $\mathcal{C}$ be an **assignment** in G, i.e., a set of couples of different vertices of G. We denote by $\mathcal{R}_\mathcal{C}$ the set of routes $\mathcal{R}(u, v)$ for any (u, v) in $\mathcal{C}$. We call $\mathcal{R}_\mathcal{C}$ a **routage graph**, it contains all the informations needed in the forthcoming problems (assignment, routes and delays of the arcs).

TODO: Dire après la définition des problèmes qu'on pourrait demander de trouver l'assignement et même le routage pour optimiser, mais pas dans cet

article et qu'on travail avec des réseaux déjà constitués TODO: Si on s'en sert, ajouter ici que le routage est cohérent.

2.2 Slotted time Model

Consider now a positive integer P called the **period**. In our problem, we send in the network messages with period P. The time will thus be cut into slices of P discrete slots. Assume we send a message at the source of the route r, at the time slot m in the first period, then a message will be sent at time slot m at each new period. We define the first time slot at which the message reaches a vertex v in this route by $t(v,r) = m + \lambda(v,r) \mod P$.

A message usually cannot be transported in a single time slot. We denote by τ the number of consecutive slots necessary to transmit a message. Let us call $[t(v,r)]_{P,\tau}$ the set of time slots used by r at a vertex v in a period P, that is $[t(v,r)]_{P,\tau} = \{t(v,r) + i \mod P \mid 0 \leq i < \tau\}$. Usually P and τ will be clear from the context and we will denote $[t(v,r)]_{P,\tau}$ by $[t(v,r)]$.

A (P,τ)**-periodic affectation** of a routage graph $\mathcal{R}$ is a sequence $\mathcal{M} = (m_0, \ldots, m_{c-1})$ of c integers that we call **offsets**, with c the number of routes in $\mathcal{R}$. The number m_i represents the index of the first slot used in a period by the route $r_i \in \mathcal{R}$ at its source. A P-periodic affectation must have no **collision** between two routes in $\mathcal{R}$, that is $\forall (r_i, r_j) \in \mathcal{R}^2, i \neq j$, we have

$$[t(u,r_i)] \cap [t(u,r_j)] = \emptyset.$$

As an exemple of a $(2,1)$-periodic affectation, let consider a routage graph with routes $\{r_i\}_{i=1,\ldots,c}$, such that all pairs of routes intersect at a different edge. We set $\tau = 1$ and the delays are chosen so that if r_i and r_j have v as first common vertex then we have $\lambda(v,r_i) - \lambda(v,r_j) = 1$.There is a $(2,1)$-periodic affectation by setting all m_i to 0.

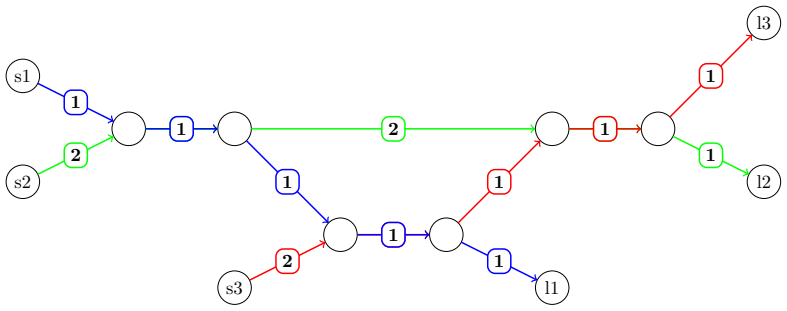


Figure 1: A routage graph with $(0,\ldots,0)$ as a $(2,1)$-periodic affectation

2.3 Problems

We want to ensure that there is an affectation which allows to send periodic messages from sources to target without collisions. The problem we need to

solve is thus the following:

Periodic Routes Assignment (PRA)

Input: a routage graph $\mathcal{R}$, an integer τ and an integer P.

Question: does there exist a (P, τ)-periodic affectation of $\mathcal{R}$?

We will prove in Sec. 3 that the problem PRA is NP-complete, even in restricted settings. Even approximating the smallest value of P for which there is a (P, τ)-periodic affectation is hard.

An unusual property of affectation is that given a routage graph, we may have a (P, τ)-periodic affectation but no (P', τ)-periodic affectation with $P' > P$: the existence of an affectation is not monotone with regards to P.

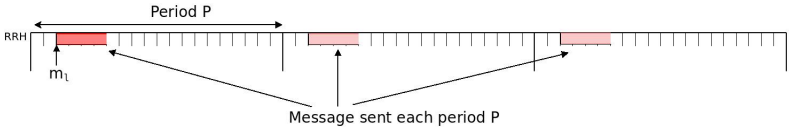
Lemma 1. *For any odd P, there is a routage graphe such that there is $(2, 1)$-periodic affectation but no $(P, 1)$-periodic affectation.*

Proof. Consider the routage graph $\mathcal{R}$ given in the previous subsection. We change the delays so that for v, the first vertex which belongs to r_i and r_j, we have $\lambda(v, r_i) - \lambda(v, r_j) = P$, where P is an odd number smaller than c the number of routes in $\mathcal{R}$. If we consider a period of 2, for all $i \neq j$, $\lambda(v, r_i) - \lambda(v, r_j) = 1$ modulo 2. Therefore $(0, \ldots, 0)$ is a $(2, 1)$-periodic affectation of $\mathcal{R}$.

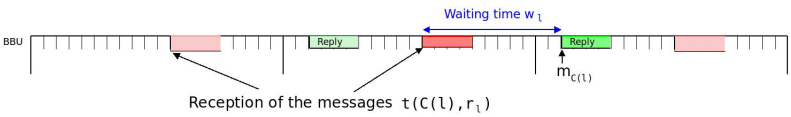
□

In the context of cloud-RAN applications, we consider here the digraph $G = (V, A)$ modeling the target network and two disjoint subsets of vertices S and T of equal cardinality, where S is the set of RRHs and T is the set of BBUs. A **symmetric** assignment $\mathcal{C}$, is an involutive function from S to T, which maps each element $s \in S$ to $\mathcal{C}(s)$ and $\mathcal{C}(s)$ to s. It can also be seen as the set of pairs $(s, \mathcal{C}(s))$ and $(\mathcal{C}(s), s)$.

We are given a period P, a routing function $\mathcal{R}$, and we consider a (P, τ)-periodic affectation of $\mathcal{R}_{\mathcal{C}}$ which associates m_s to the route r_s which begins by s. This affectation represents the following process: first a message is sent at s, through the route r_s, at time m_s.



This message is received by $\mathcal{C}(s)$ at time $t(\mathcal{C}(s), r_s)$. It is then sent back to s in the same period at time $m_{\mathcal{C}(s)}$ if $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$, otherwise at time $m_{\mathcal{C}(s)}$ in the next period. The time between the arrival of the message and the time it is sent back is called the **waiting time** and is defined by $w_s = m_{\mathcal{C}(s)} - t(\mathcal{C}(s), r_s)$ if $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$ and $w_s = m_{\mathcal{C}(s)} + P - t(\mathcal{C}(s), r_s)$ otherwise.



When a BBU receives a message, it must compute the answer before sending it back to the RRH. This time can be encoded in the last arc leading to the BBU and thus we need not to consider it explicitly in our model.

Thus, the whole process time for a message sent at vertex s is equal to

$$PT(s) = \lambda(r_s) + w_s + \lambda(r_{\mathcal{C}(s)}).$$

The **maximum process time** of the (P, τ)-periodic affectation $\mathcal{M}$ is defined by $MT(\mathcal{M}) = \max_{s \in S} PT(s)$. The problem we want to solve is the following.

Periodic Assignment for Low Latency(PALL)

Input: A routage graph $\mathcal{R}_{\mathcal{C}}$ with $\mathcal{C}$ a symmetric assignment, a period P, an integer τ, an integer T_{max}.

Question: does there exist a (P, τ)-periodic affectation $\mathcal{M}$ of $\mathcal{R}_{\mathcal{C}}$ such that $MT(\mathcal{M}) \leq T_{max}$?

TODO: Si on veut, on peut parler du temps moyen aussi ici, seulement si on fait quelque chose dessus dans la suite

3 Solving PRA

3.1 NP-Hardness

In this section we assume that the size of a message τ is equal to one. We will prove the hardness of PRA and PALL for this parameter, which implies the hardness of the general problems. Consider an instance of the problem PRA, i.e., a routage graph $\mathcal{R}$, a message size τ and a period P. The **conflict depth** of a route is the number of other routes which share an edge with it. The conflict depth of a routage graph $\mathcal{R}$ is the maximum of the conflict depth of the routes in $\mathcal{R}$. The **load** of a routage graph is the maximal number of routes sharing the same arc. It is clear that a (P, τ)-periodic affectation must satisfy that P is larger or equal to the load times τ.

We give two alternate proofs that PRA is NP-complete. The first one works for conflict depth 2 and is minimal in this regards since we later prove that for conflict depth one, it is easy to solve PRA. The second one reduces the problem to graph coloring and implies inapproximability when one tries to minimize the parameter P.

Proposition 1. *Problem PRA is NP-complete, when the routing is of conflict depth two.*

Proof. The problem PRA is in NP since given an offset for each route in an affectation, it is easy to check in linear time in the number of edges whether there are collisions.

Let $H = (V, E)$ be a graph and let d be its maximal degree. We consider the problem to determine whether H is edge-colorable with d or $d + 1$ colors. The edge coloring problem is NP-hard [1] and we reduce it to PRA to prove its

NP-hardness. We define from H an instance of PRA as follows. The graph G has for vertices v_1, v_2 for each v in V and $s_{u,v}, t_{u,v}$ for each $(u,v) \in E$. The set A of arcs of G is:

$$\{(v_1, v_2) \mid v \in V\} \cup \{(u_2, v_1) \mid u \neq v \in V^2\} \cup \{(s_{u,v}, u_1), (v_2, t_{u,v}) \mid (u,v) \in E\}.$$

All these arcs are of weight 0. For each edge $(u,v) \in E$, there is a route $s_{u,v}, u_1, u_2, v_1, v_2, t_{u,v}$ in $\mathcal{R}$. The affectation $\mathcal{C}$ is the set of pair of vertices $(s_{u,v}, t_{u,v})$.

Observe that the existence of a d-coloring of H is equivalent to the existence of a $(d,1)$-periodic affectation of $\mathcal{R}_{\mathcal{C}}$. Indeed, a d-coloring of H can be seen as a labeling of its edges by the integers in $\{0, \ldots, d-1\}$ and we have a bijection between d-colorings of H and offsets of the routes of $\mathcal{R}_{\mathcal{C}}$. By construction, the constraint of having no collision between the routes is equivalent to the fact that no two adjacent edges have the same color. Therefore we have reduced edge coloring to PRA which concludes the proof. □

TODO: Faire un dessin d'illustration ?

Remark that we have used zero weight in the proof. If we ask the weights to be strictly positive, which makes sense in our model since they represent the latency of physical links, it is easy to adapt the proof. We just have to set them so that in any route the delay at u_1 is equal to d and thus equal to 0 modulo d. We now lift this hardness result to the problem PALL.

Corollary 1. *Problem PALL is* NP*-complete for routing of conflict depth two.*

Proof. We consider $(\mathcal{R}_{\mathcal{C}}, P, \tau)$ an instance of PRA. We assume that no vertex appears both in the first and second position in a pair of $\mathcal{C}$. Remark that this condition is satisfied in the previous proof, which makes the problem PRA restricted to this kind of instances NP-complete. Let us define $T_{max} = 2 \times \max_{r \in \mathcal{R}} \lambda(r) + P$. We consider $\mathcal{C}'$ and $\mathcal{R}'_{\mathcal{C}'}$ symmetrized versions of $\mathcal{C}$ and $\mathcal{R}_{\mathcal{C}}$ where for every route there is a symmetric route with new arcs and the same weights. The instance $(\mathcal{R}'_{\mathcal{C}'}, P, \tau, T_{max})$ is in PALL if and only if $(\mathcal{R}_{\mathcal{C}}, P, \tau)$ is in PRA. Indeed the waiting time of each route is by definition less than P and thus the maximal process time is always less than T_{max}. Moreover a (P,τ)-affectation of $\mathcal{R}_{\mathcal{C}}$ can be extended into a (P,τ)-affectation of $\mathcal{R}'_{\mathcal{C}'}$ in the following way. For each route $r_{u,v}$, the time at which the message arrives is $t(v, r_{u,v})$, then we choose as offset for $r_{v,u}$, $-t(v, r_{u,v}) \mod P$. The symmetry ensures that each new route $r_{v,u}$ in $\mathcal{R}'_{\mathcal{C}'}$ uses the same times slot as $r_{u,v}$ and thus avoid collisions. □

Let MIN-PRA be the problem, given a routage graph and an assignment, to find the minimal period P such that there is a P-periodic affectation.

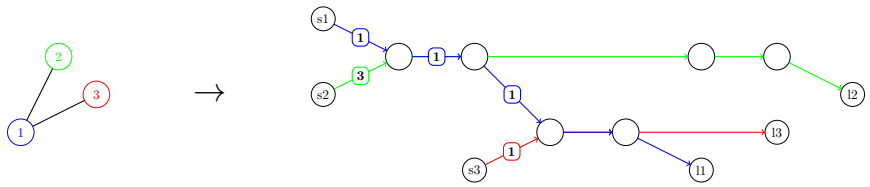
Theorem 2. *The problem MIN-PRA cannot be approximated in polynomial time within a factor $n^{1-o(1)}$, with n the number of routes, unless* P = NP *even when the load is two.*

Proof. We reduce graph coloring to PRA. Let H be a graph instance of the k-coloring problem. We define $\mathcal{R}$ in the following way: for each vertex v in H,

there is a route r_v in $\mathcal{R}$. Two routes r_v and r_u share an arc if and only if (u, v) is an edge in H; this arc is the only one shared by these two routes. All arcs are of delay 0.

Observe that the existence of a k-coloring of H is equivalent to the existence of a $(k, 1)$-periodic affectation in G, by converting an offset of a route into a color of a vertex and reciprocally. Therefore if we can approximate the minimum value of P within some factor, we could approximate the minimal number of colors needed to color a graph within the same factor. The proof follows from the hardness of approximability of finding a minimal coloring [2]. □

In particular, this reduction shows that even with small maximal load, the minimal period can be large.



H G

3.2 MIN-PRA

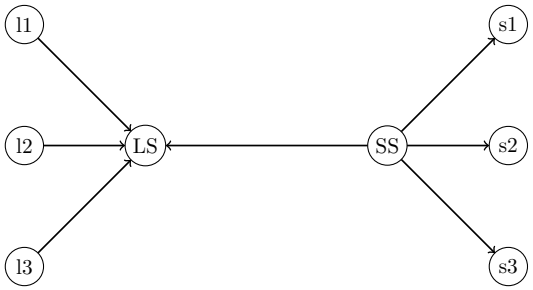
Exemple de cas polynomiaux

4 The Star Topolgy

4.1 Intro

PALL NP-Hard car PRA NP-Hard

In this section, we consider a particular case of the model, in which for each (u, v) , the route is the same in both directions. This means that $\forall (u, v) \in \mathcal{C}, \mathcal{R}_{\mathcal{C}}(u, v)$ uses the same arcs as $\mathcal{R}_{\mathcal{C}}(v, u)$ in the opposite direction.
Furthermore, the following algorithms and results were designed for a particular topology of network, in which all the routes uses a same common switch.



Let us call **LN** (leaves node) the central node connected to leaves, and **SN** (sources node), the one connected to sources. The period in those nodes are respectively called *forward period*, and *backward period*. The weight on the link between the two central nodes is simplified in the forthcoming calculations.

4.2 No waiting times

4.2.1 Shortest-longest

Algo

Period

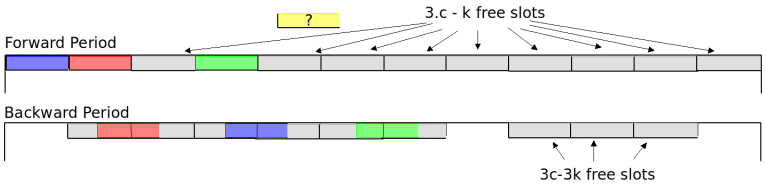
4.2.2 Greedy Algorithm with higher bound

Algorithm 1 Greedy with Higher bound Period(GHP)

Input: $\mathcal{R}_\mathcal{C}$, period P
Output: A P-periodic affectation in p $\leq P$, or FAILURE
  $P1[P]$ slots of size τ in first way period.
  $P2[P]$ slots of size τ in back way period.
  for all route i in $\mathcal{R}_\mathcal{C}$ **do**
    for all slot j in P1 **do**
      if $P1[j]$ is free AND $P2[j + \lambda(r_i)]$ is free **then**
        $o_i \leftarrow j$
      end if
      if No intervals are found for i **then**
        return FAILURE
      end if
    end for
  end for

Period This algorithm gives us a solution without waiting times in a maximum period $3.\tau.c$, if we have c routes.
Suppose that we have a period of $3.\tau.c$ slots, divided in τ macro-slots. Put a message in the first slot of size τ in the forward period, such that the corresponding area in the backward period is free.This message takes at most 2 slots

of time τ in the backward period.



When $k < c$ messages are put in the forward period, and we want to add another message, there is $3.c - k$ free slots of size τ in the forward period. Those $3.c - k$ gives us $3.c - k$ possible slots in the backward periods.
The k messages uses at most $2k$ slots of size τ used in the backward period. Since $k < c$, $2k < 3.c - k$, thus using the pigeonhole principle, there is at least one free slot in the backward period for the new message.

4.2.3 Exhaustive generation

Algorithm 2 Exhaustive Generation

Input: A routage graph $\mathcal{R}_\mathcal{C}$, period P, packet size τ
Output: (P, τ)-periodic affectation of $\mathcal{R}_\mathcal{C}$

```
Forward-budget ← P - c * τ
Backward-budget ← P - c * τ
Free-Intervals ← list of free intervals, init to [0; P[
for all route i in R_C do
  for all j in Free-Intervals do
    if Message of the route i does not collides with scheduled routes then
      m_i ← the first slot of Free-Intervals[j]
      Split the Free-Intervals considering the new packet
      Forward-budget ← Forward-budget - lost size
      Backward-budget ← Backward-budget - lost size
      call Exhaustive Generation on remaining routes
    end if
  end for
end for
```

Lost size, is the size lost when an interval is too few to put another packet. Ex : if the interval [250;3250[receives a packet of size 2500, there is 500 slots lost, because there is no way to uses the slots for another packet.

This algorithm enumerates all the solutions by traversing a tree. The leaves of the tree are the $(P\tau)$-periodic affectations, and the nodes are partial solutions. A partial solution is a choice of a starting time for a subset of the routes. For each route, the algorithm tries every possible starting offset until it finds one available (that allows the message to come back without collisions). When a route have been scheduled, it take another route that have not been scheduled yet. This creates a new sub-tree in which the algorithm will try every possible

starting offset too, and create a sub-tree on the following route too. If no offset are available for a route, this means that the partial solution is not good, so the algorithm backtracks over the tree that is, it goes back to the father of the current sub-tree. Once a leaf is obtained, the algorithm stops and return the scheduling.
To reduce the number of useless calculation, in each time we add another packet, we consider the lost size in both periods, thus, we know when a period has no many great enough free area to put a packet.

4.2.4 Results

Resultats des simulations : Shortest-longest optimal pour ces parametres.

4.3 Allowing waiting times

4.3.1 Intro

Importance des waiting times quand la période est donnée (Résultats D'éxepriences et preuve avec l'exemple)

4.3.2 LSG

Algorithm

Analysis Parler de LSO et expliquer pourquoi LSG mieux avec nos params

4.3.3 Results

Random

Distributions

5 Conclusion

Open questions. Can we improve the results if:

- The routes are smaller than the size of a message.
- The routes are smaller than the period.
- The largest difference between two routes is smallest than one of these parameters

On a general graph:

- The routing is coherent
- The graph is symmetric

References

[1] Ian Holyer. The np-completeness of edge-coloring. *SIAM Journal on computing*, 10(4):718–720, 1981.

[2] David Zuckerman. Linear degree extractors and the inapproximability of max clique and chromatic number. In *Proceedings of the thirty-eighth annual ACM symposium on Theory of computing*, pages 681–690. ACM, 2006.