Contention Management for 5G

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Abstract

This article treats about Contention Management for 5G.

1 Introduction

- Context and problematic
- Related works
- Article contribution

2 Model

2.1 Definitions

We consider a symmetric directed graph G = (V, A) modelling a network. Each arc (u, v) in A is labeled by an integer $Dl(u, v) \ge 1$ that we call the delay and which represents the number of time slots taken by a signal to go from u to v using this arc. Note that for any arc (u, v), Dl(u, v) = Dl(v, u).

A **route** r in G is a sequence of consecutive arcs a_0, \ldots, a_{k-1} , with $a_i = (u_i, u_{i+1}) \in A$.

The **latency** of a vertex u_i in r, with $i \geq 1$, is defined by

$$\lambda(u_i, r) = \sum_{0 \le j < i} Dl(a_j)$$

We also define $\lambda(u_0, r) = 0$. The latency of the route r is defined by $\lambda(r) = \lambda(u_k, r)$.

A routing function \mathcal{R} in G associates to each pair of vertices (u, v) a route from u to v. Let \mathcal{C} be an **assignment** in G, i.e., a set of couples of different vertices of G. We denote by $\mathcal{R}_{\mathcal{C}}$ the set of routes $\mathcal{R}(u, v)$ for any (u, v) in \mathcal{C} . $\mathcal{R}_{\mathcal{C}}$ is called an **assigned graph**, and contains all the information needed in forthcoming problems (couples of vertices, routes and delay of the arcs).

TODO: Si on s'en sert, ajouter ici que le routage est cohérent.

2.2 Slotted time Model

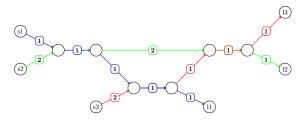
Consider now a positive integer P called the **period**. In our problem the messages we send in the network will be periodic of period P and thus we consider slices of time of P discrete slots. Assume we send a message at the source of the route r, at the time slot m in the first period, then a message will be sent at time m at each new period. We define the first time slot at which a message reaches a vertex v in this route by $t(v,r) = m + \lambda(v,r)$ mod P.

A message usually cannot be transported in a single time slot. We denote by τ the number of consecutive slots necessary to transmit a message. Let us call [t(v,r)] the set of time slots used by r at a vertex v in a period P, that is $[t(v,r)] = \{t(v,r) + i \mod P \mid 0 \le i \le \tau\}$.

A P-periodic affectation of $\mathcal{R}_{\mathcal{C}}$ consists in a set $\mathcal{M} = (m_0, \dots, m_{c-1})$ of c integers that we call **offsets**, with c the cardinal of the assignment \mathcal{C} . The number m_i represents the index of the first slot used in a period by the route $r_i \in \mathcal{R}_{\mathcal{C}}$ at its source. A P-periodic affectation must have no **collision** between two routes in $\mathcal{R}_{\mathcal{C}}$, that is $\forall (r_i, r_j) \in \mathcal{R}_{\mathcal{C}}^2, i \neq j$, we have

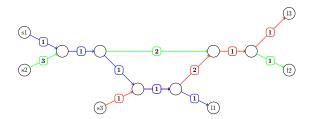
$$[t(u,r_i)] \cap [t(u,r_i)] = \emptyset.$$

Notice that the notion of P-periodic affectation is not monotone with regard to P. Indeed, consider messages of size 1, we can build a graph, with c routes $r_i \in \mathcal{R}_{\mathcal{C}}$, i < c which all intersect two by two and such that if r_i and r_j have v as first common vertex we have $\lambda(v, r_i) - \lambda(v, r_j) = 1$. Therefore there is a 2-periodic affectation by setting all m_i to 0.



$$\lambda(v, r_i) - \lambda(v, r_j) = 1$$

On the other hand if we set all $\lambda(v, r_i) - \lambda(v, r_j) = P$, there is no P-periodic affectation if P < c.



$$\lambda(v, r_i) - \lambda(v, r_i) = 2$$

Here for P=2, there is no P-periodic affectation.

Therefore if we choose and odd value for P and c = P + 1, if there is no P-periodic affectation but modulo 2 all $\lambda(v, r_i) - \lambda(v, r_j)$ are equal to one thus we have a 2-periodic affectation.

2.3 Problems

The main theoretical problem we have to deal with in this context is the following.

Problem Periodic Routes Assignment (PRA)

Input: a graph G = (V, A), a set C of pair of vertices, a routing function R and an integer P.

Question: does there exist a P-periodic affectation of C in (G, \mathbb{R}) ?

We will prove in Sec. 3 that the Problem PRA is NP-complete, even in restricted settings. In fact, even approximating the smallest value of P for which there is a P-periodic assignment is hard.

In the context of cloud-RAN applications, we consider here the digraph G = (V, A) modeling the target network and two disjoint subsets of vertices S and L, where S is the set of BBU and L is the set of RRH. We denote by n the size of S and L. We are given a period P, a routing function R and a bijection $\rho: L \to S$ which assigns a BBU to each RRH. Let $C_{\rho} = \{(l, \rho(l))\}_{l \in L} \cup \{(s, \rho^{-1}(s))\}_{s \in S}$. Let consider a P-periodic affectation of C_{ρ} which associates m_l to $(l, \rho(l))$ and $m_{\rho(l)}$ to $(\rho(l), l)$.

This affectation represents the following process: first a message is sent in l, through the route r_l , at time m_l . This message is received by $\rho(l)$ at time $t(\rho(l), r_l)$. It is then sent back to l at time \bar{m}_l , which is in the same period if $m_{\rho(l)} > t(\rho(l), r_l)$ otherwise it is in the next. The time between the arrival of the message and the time it is sent back is called the **waiting time** and is defined by $w_l = m_{\rho(l)} - t(\rho(l), r_l)$ if $m_{\rho(l)} > t(\rho(l), r_l)$ and $w_l = m_{\rho(l)} + P - t(\rho(l), r_l)$ otherwise. When a BBU receives a message, it must compute the answer before sending it back to the BBU. This time is taken into account in the last arc leading to the BBU and thus we need not to consider it explicitly in our model.

Thus, the whole process time for a vertex l is equal to

$$PT(l) = \lambda(r_l) + w_l + \lambda(r_{\rho(l)}).$$

The maximum process time of the P-periodic affectation \mathcal{M} is defined by $MT(\mathcal{M}) = \max_{l \in L} PT(l)$. The problem we want to solve is the following.

Problem Periodic Assignment for Low Latency(PALL)

Input: a digraph G, a matching ρ from L to S two disjoint set of vertices of G, a routing function \mathcal{R} , a period P, an integer T_{max} .

Question: does there exist a P-periodic affectation \mathcal{M} of \mathcal{C}_{ρ} in (G, \mathcal{R}) such that $MT(\mathcal{M}) \leq T_{max}$?

3 Solving PRA

3.1 NP-Hardness

TODO: Definition of the load. La preuve se réfère à la précédente qui n'a pas été retenue, il faut du coup tout expliquer.

For each arc $(u, v) \in A$, we denote by $\rho(u, v)$ the subset of routes of \mathcal{R} containing (u, v). We define the load of an arc (u, v) as the number of routes that use this arc thus: $load(u, v) = |\rho(u, v)|$.

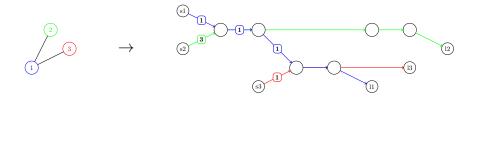
Theorem 1. Problem PRA cannot be approximated within a factor $n^{1-o(1)}$ unless P = NP even when the load is two and n is the number of pairs in the assignment.

Proof. We reduce PRA to graph coloring. Let G be a graph instance of the k-coloring problem. We define H in the following way: for each vertex v in G, there is a route r_v in H. Two routes r_v and r_u share an edge if and only if (u,v) is an edge in G and this edge is only in this two routes. We put a weight inbetween shared edges in a route so that there is a delay k between two such edges.

Let ϕ be an edge coloring with k colors of G. We can build a k periodic affectation by assigning to the source $s_{u,v}$ of each route an offset of $\phi(s_{u,v})$. Indeed, if two routes r_1 and r_2 share the same edge, say (v_1, v_2) then they represent two edges e_1 and e_2 of G incident to the vertex v. Therefore $\lambda(v_1, r_1) = \phi(e_1) \mod k$ because the delays of the edges before v_1 sum to d(d+1) or 2(d(d+1)) which are equal to 0 modulo k since k=d or k=d+1. Thus $\lambda(v_1,r_1)-\lambda(v_1,r_2) \mod k=\phi(e_1)-\phi(e_2) \mod k$ and $\lambda(v_1,r_1)-\lambda(v_1,r_2) \mod k>0$ since $\phi(e_1)\neq\phi(e_2)$.

Now consider a k-periodic affectation of ρ . For each (u, v) in G, we define $\phi(u, v)$ to be the offset of the route beginning at $s_{u,v}$. For the same reasons as in the last paragraph, ϕ is an edge coloring with k colors.

Therefore if we can approximate the value of PRA within a factor f, we could approximate the minimal number of colors needed to color a graph within a fator f, by doing the previous reduction for all possible k. The proof follows from the hardness of approximability of finding a minimal coloring [?].



Η

3.2 MIN-PRA

G

Exemple de cas simple

4 Proposed Solutions, solving PALL

In this section, we consider a particular case of the model, in which for each (u,v), the route is the same in both directions. This means that $\mathcal{R}(u,v)$ uses the same arcs as $\mathcal{R}(v,u)$ in the opposite orientation.

4.1 Intro

PALL NP-Hard car PRA NP-Hard

Résultats valables sur Topologie 1 avec nos paramètres TODO: J'ai viré star affectation, car je pense qu'il n'y a rien à dire là dessus.

4.2 No waiting times

4.2.1 Shortest-longest

Algo

Period

4.2.2 Greedy Algorithm with higher bound

Algorithm 1 Greedy with Higher bound Period(GHP)

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Input: \mathcal{R}_{\mathcal{C}}, period P

Output: A P-periodic affectation in p \leq P, or FAILURE

P1[P] slots of size \tau in first way period.

P2[P] slots of size \tau in back way period.

for all route i in \mathcal{R}_{\mathcal{C}} do

for all slot j in P1 do

if P1[j] is free AND P2[j + \lambda(r_i)] is free then

o_i \leftarrow j

end if

if No intervals are found for i then

return FAILURE

end if

end for
end for
```

Period This algorithm gives us a solution without waiting times in a maximum period $3.\tau.c$, if we have c routes.

Suppose that we have a period of $3.\tau.c$ slots, divided in τ macro-slots. Let us call forward period the period in the central node when the messages goes to RRH to BBU, and backward period the period when the messages comes back in the other way. Put a message in the first slot of size τ in the forward period, such that the corresponding area in the backward period is free. This message takes at most 2 slots of time τ in the backward period.

TODO: Dessin qui illustre ça

When k < c messages are put in the forward period, and we want to add another message, there is 3.c - k free slots of size τ in the forward period. Those 3.c - k gives us 3.c - k possible slots in the backward periods. The k messages uses at most 2k slots of size τ used in the backward period. Since k < c, 2k < 3.c - k, thus using the pigeonhole principle, there is at

least one free slot in the backward period for the new message.

4.2.3 Exhaustive generation

Décrire l'algo, expliquer les coupes

4.2.4 Results

Resultats des simulations : Shortest-longest optimal pour ces parametres.

4.3 Allowing waiting times

4.3.1 Intro

Importance des waiting times quand la période est donnée (Résultats D'éxepriences et preuve avec l'exemple)

4.3.2 LSG

${\bf Algorithm}$

Analysis Parler de LSO et expliquer pourquoi LSG mieux avec nos params

4.3.3 Results

Random

Distributions

5 Conclusion