Contention Management for 5G

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Abstract

This article treats about Contention Management for 5G.

1 Introduction

- Context and problematic
- Related works
- Article contribution

2 Model

2.1 Definitions

We consider a directed graph $G = (V, A)$ modelling a network. Each arc (u, v) in A is labeled by an integer $dl(u, v) \geq 1$ that we call the delay and which represents the number of time slots taken by a signal to go from u to v using this arc.

A **route** r in G is a sequence of consecutive arcs $a_0, \ldots, a_{k-1}$, with $a_i = (u_i, u_{i+1}) \in A$. We will often refer to the first element of the route as a source and the last as a target.

The **latency** of a vertex u_i in r, with $i \geq 1$, is defined by

$$\lambda(u_i, r) = \sum_{0 \leq j < i} dl(a_j)$$

We also define $\lambda(u_0, r) = 0$. The latency of the route r is defined by $\lambda(r) = \lambda(u_k, r)$.

A **routing function** $\mathcal{R}$ in G associates to each pair of vertices (u, v) a route from u to v. Let $\mathcal{C}$ be an **assignment** in G, i.e., a set of couples of different vertices of G. We denote by $\mathcal{R}_\mathcal{C}$ the set of routes $\mathcal{R}(u, v)$ for any (u, v) in $\mathcal{C}$. We call $\mathcal{R}_\mathcal{C}$ a **routage graph**, it contains all the informations needed in the forthcoming problems (assignment, routes and delays of the arcs).

TODO: Si on s'en sert, ajouter ici que le routage est cohérent.

2.2 Slotted time Model

Consider now a positive integer P called the **period**. In our problem, we send in the network messages with period P. The time will thus be cut into slices of P discrete slots. Assume we send a message at the source of the route r, at the time slot m in the first period, then a message will be sent at time slot m at each new period. We define the first time slot at which the message reaches a vertex v in this route by $t(v, r) = m + \lambda(v, r) \mod P$.

A message usually cannot be transported in a single time slot. We denote by τ the number of consecutive slots necessary to transmit a message. Let us call $[t(v, r)]_{P,\tau}$ the set of time slots used by r at a vertex v in a period P, that is $[t(v, r)]_{P,\tau} = \{t(v, r) + i \mod P \mid 0 \leq i < \tau\}$. Usually P and τ will be clear from the context and we will denote $[t(v, r)]_{P,\tau}$ by $[t(v, r)]$.

A (P, τ)**-periodic affectation** of a routage graph $\mathcal{R}_\mathcal{C}$ is a sequence $\mathcal{M} = (m_0, \ldots, m_{c-1})$ of n integers that we call **offsets**, with n the number of routes in $\mathcal{R}_\mathcal{C}$. The number m_i represents the index of the first slot used in a period by the route $r_i \in \mathcal{R}_\mathcal{C}$ at its source. A (P, τ)-periodic affectation must have no **collision** between two routes in $\mathcal{R}_\mathcal{C}$, that is $\forall (r_i, r_j) \in {\mathcal{R}_\mathcal{C}}^2, i \neq j$, we have

$$[t(u, r_i)] \cap [t(u, r_j)] = \emptyset.$$

As an example of a $(2, 1)$-periodic affectation, let consider a routage graph with routes $\{r_i\}_{i=1,\ldots,n}$, such that all pairs of routes intersect at a different edge. We set $\tau = 1$ and the delays are chosen so that if r_i and r_j have v as first common vertex then we have $\lambda(v, r_i) - \lambda(v, r_j) = 1$.There is a $(2, 1)$-periodic affectation by setting all m_i to 0.

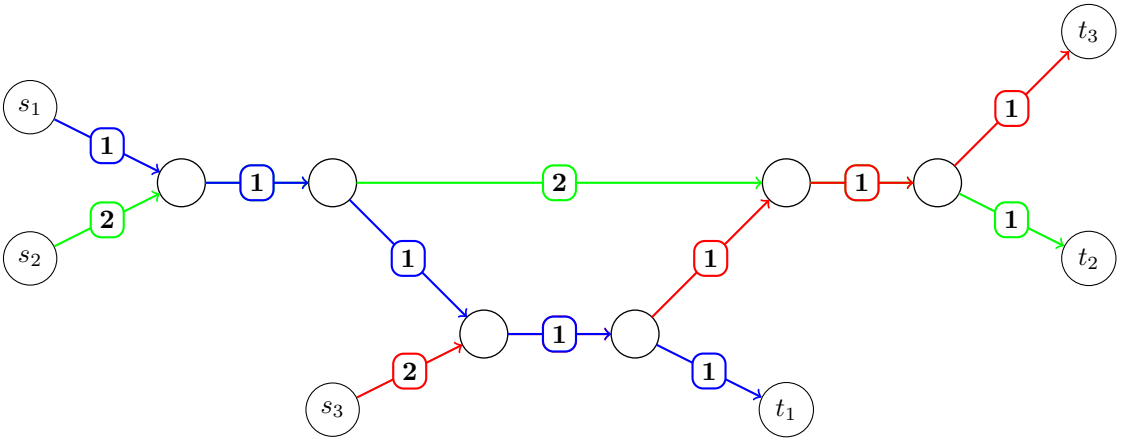


Figure 1: A routage graph with $(0, \ldots, 0)$ as a $(2, 1)$-periodic affectation

2.3 Problems

We want to ensure that there is an affectation which allows to send periodic messages from sources to target without collisions. The problem we need to

solve is thus the following:

Periodic Routes Assignment (PRA)

Input: a routage graph $\mathcal{R}_{\mathcal{C}}$, an integer τ and an integer P.

Question: does there exist a (P, τ)-periodic affectation of $\mathcal{R}_{\mathcal{C}}$?

We will prove in Sec. 3 that the problem PRA is NP-complete, even in restricted settings. Even approximating the smallest value of P for which there is a (P, τ)-periodic affectation is hard.

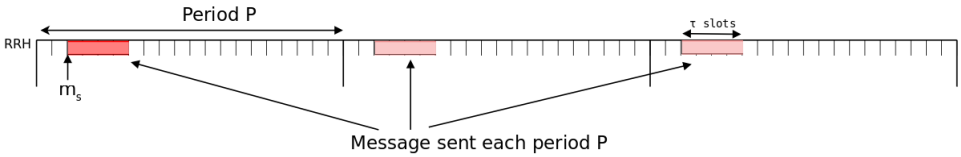
An unusual property of affectation is that given a routage graph, we may have a (P, τ)-periodic affectation but no (P', τ)-periodic affectation with $P' > P$: the existence of an affectation is not monotone with regards to P.

Lemma 1. *For any odd P, there is a routage graph such that there is $(2, 1)$-periodic affectation but no $(P, 1)$-periodic affectation.*

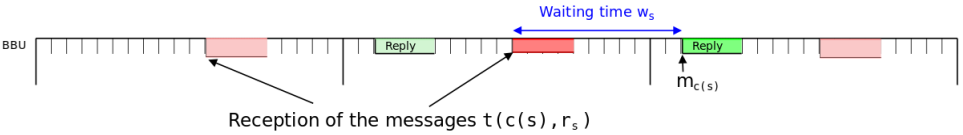
Proof. Consider the routage graph $\mathcal{R}_{\mathcal{C}}$ given in the previous subsection. We change the delays so that for v, the first vertex which belongs to r_i and r_j, we have $\lambda(v, r_i) - \lambda(v, r_j) = P$, where P is an odd number smaller than n, the number of routes in $\mathcal{R}_{\mathcal{C}}$. In such a graph, there is no (P, τ)-periodic affectation, because of $P < n$.
If we consider a period of 2, for all $i \neq j$, $\lambda(v, r_i) - \lambda(v, r_j) \mod 2 = 1$. Therefore $(0, \ldots, 0)$ is a $(2, 1)$-periodic affectation of $\mathcal{R}_{\mathcal{C}}$.

□

In the context of cloud-RAN applications, we consider here the digraph $G = (V, A)$ modeling the target network and two disjoint subsets of vertices S and T of equal cardinality, where S is the set of RRHs and T is the set of BBUs. A **symmetric** assignment $\mathcal{C}$, is an involutive function from S to T, which maps each element $s \in S$ to $\mathcal{C}(s)$ and $\mathcal{C}(s)$ to s. It can also be seen as the set of pairs $(s, \mathcal{C}(s))$ and $(\mathcal{C}(s), s)$.
The Routing function $\mathcal{R}$ associates to each couple $(s, \mathcal{C}(s))$ a route called r_s and to each couples $(\mathcal{C}(s), s)$ a route called $r_{\mathcal{C}(s)}$.
We are given a period P, a routing function $\mathcal{R}$, and we consider a (P, τ)-periodic affectation of $\mathcal{R}_{\mathcal{C}}$ which associates m_s to the route r_s which begins by s, and $m_{\mathcal{C}(s)}$ to the routes $r_{\mathcal{C}(s)}$ which begins by $\mathcal{C}(s)$. This affectation represents the following process: first a message is sent at s, through the route r_s, at time m_s.



This message is received by $\mathcal{C}(s)$ at time $t(\mathcal{C}(s), r_s)$. It is then sent back to s in the same period at time $m_{\mathcal{C}(s)}$ if $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$, otherwise at time $m_{\mathcal{C}(s)}$ in the next period. The time between the arrival of the message and the time it is sent back is called the **waiting time** and is defined by $w_s = m_{\mathcal{C}(s)} - t(\mathcal{C}(s), r_s)$ if $m_{\mathcal{C}(s)} > t(\mathcal{C}(s), r_s)$ and $w_s = m_{\mathcal{C}(s)} + P - t(\mathcal{C}(s), r_s)$ otherwise.



When a BBU receives a message, it must compute the answer before sending it back to the RRH. This time can be encoded in the last arc leading to the BBU and thus we need not to consider it explicitly in our model.

Thus, the whole process time for a message sent at vertex s is equal to

$$PT(s) = \lambda(r_s) + w_s + \lambda(r_{\mathcal{C}(s)}).$$

The **maximum process time** of the (P, τ)-periodic affectation $\mathcal{M}$ is defined by $MT(\mathcal{M}) = \max_{s \in S} PT(s)$. The problem we want to solve is the following.

Periodic Assignment for Low Latency(PALL)

Input: A routage graph $\mathcal{R}_\mathcal{C}$ with $\mathcal{C}$ a symmetric assignment, a period P, an integer τ, an integer T_{max}.

Question: does there exist a (P, τ)-periodic affectation $\mathcal{M}$ of $\mathcal{R}_\mathcal{C}$ such that $MT(\mathcal{M}) \leq T_{max}$?

TODO: Si on veut, on peut parler du temps moyen aussi ici, seulement si on fait quelque chose dessus dans la suite

In this article, we works on given routages, but we could imagine a problem, in which, considering a given graph $G = (V, A)$ and a period P, the question is to find, if it exists, a routage graph $\mathcal{R}_\mathcal{C}$, in which there is (P, τ)-periodic affectation.

3 Solving PRA

3.1 NP-Hardness

In this section we assume that the size of a message τ is equal to one. We will prove the hardness of PRA and PALL for this parameter, which implies the hardness of the general problems. Consider an instance of the problem PRA, i.e., a routage graph $\mathcal{R}_\mathcal{C}$, a message size τ and a period P.
The **conflict depth** of a route is the number of other routes which share an edge with it. The conflict depth of a routage graph $\mathcal{R}_\mathcal{C}$ is the maximum of the conflict depth of the routes in $\mathcal{R}_\mathcal{C}$.
The **load** of a routage graph is the maximal number of routes sharing the same arc. It is clear that a (P, τ)-periodic affectation must satisfy that P is larger or equal to the load times τ.

We give two alternate proofs that PRA is NP-complete. The first one works for conflict depth 2 and is minimal in this regards since we later prove that for conflict depth one, it is easy to solve PRA. The second one reduces the problem to graph coloring and implies inapproximability when one tries to minimize the

parameter P.

Proposition 1. *Problem PRA is* NP*-complete, when the routing is of conflict depth two.*

Proof. The problem PRA is in NP since given an offset for each route in an affectation, it is easy to check in linear time in the number of edges whether there are collisions.

Let $H = (V, E)$ be a graph and let d be its maximal degree. We consider the problem to determine whether H is edge-colorable with d or $d + 1$ colors. The edge coloring problem is NP-hard [1] and we reduce it to PRA to prove its NP-hardness. We define from H an instance of PRA as follows. For each v in V, the graph G has two vertices v_1, v_2, and for each $(u, v) \in E$,the graph g has two vertices $s_{u,v}, t_{u,v}$.

For each edge $(u, v) \in E$, there is a route $s_{u,v}, u_1, u_2, v_1, v_2, t_{u,v}$ in $\mathcal{R}$. The set of arcs of G is the union between all the arcs of the previous routes. The affectation $\mathcal{C}$ is the set of pair of vertices $(s_{u,v}, t_{u,v})$.

All these arcs are of weight 0.

Observe that the existence of a d-coloring of H is equivalent to the existence of a $(d, 1)$-periodic affectation of $\mathcal{R}_\mathcal{C}$. Indeed, a d-coloring of H can be seen as a labeling of its edges by the integers in $\{0, \ldots, d-1\}$ and we have a bijection between d-colorings of H and offsets of the routes of $\mathcal{R}_\mathcal{C}$. By construction, the constraint of having no collision between the routes is equivalent to the fact that no two adjacent edges have the same color. Therefore we have reduced edge coloring to PRA which concludes the proof. □

TODO: Faire un dessin d'illustration ?

Remark that we have used zero weight in the proof. If we ask the weights to be strictly positive, which makes sense in our model since they represent the latency of physical links, it is easy to adapt the proof. We just have to set them so that in any route the delay at u_1 is equal to d and thus equal to 0 modulo d. We now lift this hardness result to the problem PALL.

Corollary 1. *Problem PALL is* NP*-complete for routing of conflict depth two.*

Proof. We consider $(\mathcal{R}_\mathcal{C}, P, \tau)$ an instance of PRA. We assume that no vertex appears both in the first and second position in a pair of $\mathcal{C}$. Remark that this condition is satisfied in the previous proof, which makes the problem PRA restricted to this kind of instances NP-complete. Let us define $T_{max} = 2 \times \max_{r \in \mathcal{R}} \lambda(r) + P$. We consider $\mathcal{C}'$ and $\mathcal{R}'_{\mathcal{C}'}$ symmetrized versions of $\mathcal{C}$ and $\mathcal{R}_\mathcal{C}$ where for every route there is a symmetric route with new arcs and the same weights. The instance $(\mathcal{R}'_{\mathcal{C}'}, P, \tau, T_{max})$ is in PALL if and only if $(\mathcal{R}_\mathcal{C}, P, \tau)$ is in PRA. Indeed the waiting time of each route is by definition less than P and thus the maximal process time is always less than T_{max}. Moreover a (P, τ)-affectation of $\mathcal{R}_\mathcal{C}$ can be extended into a (P, τ)-affectation of $\mathcal{R}'_{\mathcal{C}'}$ in the following way. For each route $r_{u,v}$, the time at which the message arrives is $t(v, r_{u,v})$, then we choose as offset for $r_{v,u}$, $-t(v, r_{u,v}) \mod P$. The symmetry

ensures that each new route $r_{v,u}$ in $\mathcal{R}'_{\mathcal{C}'}$ uses the same times slot as $r_{u,v}$ and thus avoid collisions. □

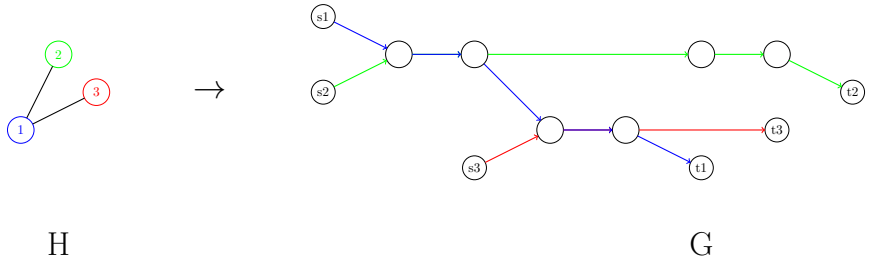
Let MIN-PRA be the problem, given a routage graph and an assignment, to find the minimal period P such that there is a P-periodic affectation.

Theorem 2. *The problem MIN-PRA cannot be approximated in polynomial time within a factor $n^{1-o(1)}$, with n the number of routes, unless* $\mathsf{P} = \mathsf{NP}$ *even when the load is two.*

Proof. We reduce graph coloring to PRA. Let H be a graph instance of the k-coloring problem. We define $\mathcal{R}$ in the following way: for each vertex v in H, there is a route r_v in $\mathcal{R}$. Two routes r_v and r_u share an arc if and only if (u, v) is an edge in H; this arc is the only one shared by these two routes. All arcs are of delay 0.

Observe that the existence of a k-coloring of H is equivalent to the existence of a $(k, 1)$-periodic affectation in G, by converting an offset of a route into a color of a vertex and reciprocally. Therefore if we can approximate the minimum value of P within some factor, we could approximate the minimal number of colors needed to color a graph within the same factor. The proof follows from the hardness of approximability of finding a minimal coloring [2]. □

In particular, this reduction shows that even with small maximal load, the minimal period can be large.



H G

3.2 Tractable cases of PRA

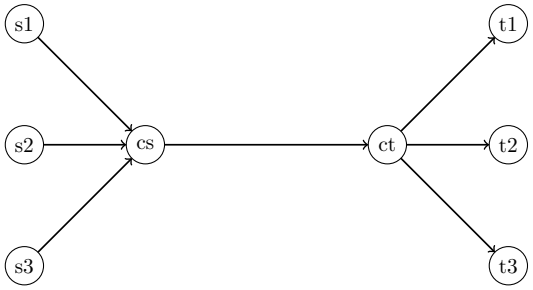
In this section, we present a few polynomial cases, in particular when the conflict depth is one.

The problem PALL seems to be hard to solve, even for very simple classes of graphs, as seen in the next section.

4 The Star Topology

In this section, we consider graphs with a very simple topology that we call the **star topology**. First, for each arc (u, v), there is also an arc (v, u) with the same weight. Moreover, there is a special arc, the central arc, which is shared by all routes. All routes consist from an arc from its source to the central source node, denoted by c_s, then the central arc to c_t, the central target node, and

an arc to its target. In addition to those central nodes, there are two sets of vertices, S and T, of cardinality n and $\mathcal{C}$ a symmetric assignment from S to T. The routes are the directed paths of the form s_i, c_s, c_t, t_i and t_i, c_t, c_s, s_i.



We assume the weight of the central arc to be 0 since it appears in every route, its value does not matter when considering affectations. Collisions between messages can only appear at node c_s on the way forward and at node c_t on the way back.
In such a topology, there is only two possible collision points between the routes : c_s and c_t. To avoid collisions, we only have to check if there is no collisions in a period on those two points. Let us take a period P for each central nodes, that are called the **forward period** and the **backward period** for, respectively c_s and c_t. A message issued by the source s_1 at time m_{s_1} will reach c_s at time $m_{s_1} + t(c_s, r_{s_1})$ in the forward period, and c_t at time $m_{\mathcal{C}(s_1)} + t(c_t, r_{\mathcal{C}(s_1)})$ in the backward period. A (P, τ)-periodic affectation consist in choosing $\forall i \in n, m_i$, and $m_{\mathcal{C}(s_i)}$ such that, for any couples of routes $(r_{s_j}, r_{s_k}), j, k \in n$, we have $[t(c_s, r_j)] \cap [t(c_s, r_k)] = \emptyset$. and for any couples of routes $(r_{\mathcal{C}(s_j)}, r_{\mathcal{C}(s_k)}), j, k \in n$, we have $[t(c_s, r_{\mathcal{C}(s_j)})] \cap [t(c_s, r_{\mathcal{C}(s_k)})] = \emptyset$

4.1 Solving PALL without waiting times

In this subsection, we deal with a simpler version of the problem PALL. We ask for a (P, τ)-periodic affectation with all waiting times equal to 0. In that case T_{max} is not relevant anymore. Since $w_i = 0$, choosing m_i, the offset of the route from s_i to t_i, also sets the offset of the route from t_i to s_i to $m_i + \lambda(r_i)$. Moreover, in this context the weight of the arcs from s_i to c_s have no impact, therefore we assume they are equal to 0.
For those reasons, in this case, we only have to consider the forward and backward periods. Indeed, setting the offset m_{s_i} in the forward period automatically set the offsets $m_{\mathcal{C}(s_i)}$ in the backward period. The goal is now to put messages in the forward periods such that there is no collisions in the both periods.
In this section, the offsets m_{s_i} are in reality $[t(c_s, r_{s_i})]$, but to find the real values, there is just to change the values of the offset considering the delay of the arcs $(s_i, c_s), \forall i$.

4.1.1 Shortest-longest policy

Algo We present a simple policy, which works when the period is large with regards to the delay of the routes. Send the messages in order from the shortest route to the longest route, on after each others in the forward period, without any slots between two messages, that is $m_{s_i} = \tau.i$, if the routes r_i are ordered by $\lambda(r_i)$ from the shortest to the longest one.

The message using the shortest route is sent before the others, so it comes back to the central node before the following ones. Then, the second message is sent before the others, and also will be back before the third etc... Because the second message is sent τ slots after the first message ($m_{s_2} = m_{s_1} + \tau$, it comes back after the end of the message one: $m_{s_1} + \tau + 2.\lambda(r_{s_1}) \leq m_{s_2} + 2.\lambda(r_{s_2})$ because of $\lambda(r_{s_2}) \geq \lambda(r_{s_1})$.

Period To ensure that there is no collisions in the backward period, we must ensure that the last message will not collide with the first message of the next period. We have to find the minimum period P such that the last message have totally came back before P slots after the first message.

The this policy ensure us a period of

$$P = \tau * n + 2(\lambda(r_{s_{n-1}}) - \lambda(r_{s_0}))$$

if routes $\{r_0, ..., r_{s_{n-1}}\}$ are ordered from the shortest to the longest.

Proof. We have to calculate the time taken by the last message to reach c_t. $2.\lambda(r_{s_0})$ is the first slot in which the first message is coming back in the backward period, and $2\lambda(r_{s_{n-1}}) + n.\tau$ is the time at which the last message has totally passed the switch. Therefore, the period only depends of the difference between the longest and the shortest route: the larger this value is, the larger the period is. □

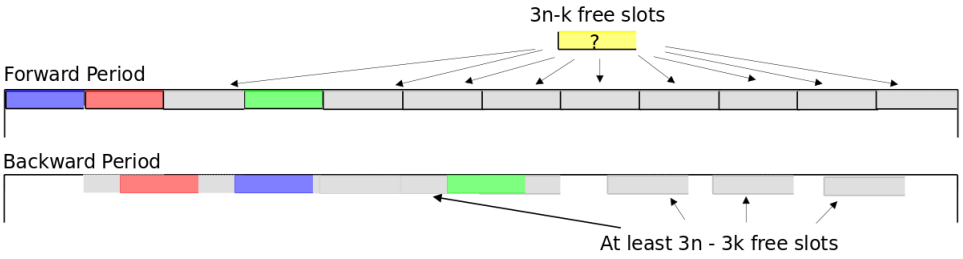
4.1.2 Greedy Algorithm

We introduce a greedy algorithm to build a (P, τ)-periodic affectation, which provably works when the period is large enough with regards to the number of routes and size of messages. In other words, we can always find a solution when the network is far from saturated.

Proposition 2. *There is an algorithm, which finds a (P, τ)-periodic affectation in time $O(n^2)$ when $P \geq 3n\tau$.*

Proof. We consider the forward period and cut it into consecutive macro slots of time of size τ. The algorithm works in the following way: select the first route for which the offset as not been chosen. Try every offset which put the message in a free macro slot in the forward period. It also fixes the position of a message in the backward period, and we choose the first offset which does not create a collision in the backward period. Assume we choose the offset of the route r_{k+1}, we have at least $3n - k$ free macro-slots, since $P \geq 3n\tau$. Each of these $3n - k$ possible offset values translates into $3n - k$ positions of messages in the

backward period. All these positions are separated by at least τ slots. There are already k messages of size τ in the backward period. One such message can intersect at most $2k$ potential positions, therefore amongst the possible $3n-k$ positions, there are $3n-k-2k$ which are without collision. Since $k < n$, one of the choice is without collision, which proves that the algorithm terminates and find a (P, τ)-periodic affectation. □



TODO: Avec une structure de données plus maligne on doit pouvoir faire n, non ?

Algorithm 1 Greedy affectation

Input: $\mathcal{R}_\mathcal{C}$, period P
Output: A P-periodic affectation in p $\leq P$, or FAILURE

```
P1[P] slots of size τ in forward period.
P2[P] slots of size τ in backward period.
for all source i in S do
  for all slot j in P1 do
    if P1[j] is free AND P2[j + λ(r_i)] is free then
      m_{s_i} ← j
    end if
    if No intervals are found for i then
      return FAILURE
    end if
  end for
end for
```

4.1.3 Exhaustive generation

Algorithm 2 Exhaustive Generation

Input: A routage graph $\mathcal{R}_\mathcal{C}$, period P, packet size τ
Output: (P, τ)-periodic affectation of $\mathcal{R}_\mathcal{C}$

```
Forward-budget ← P - n * τ
Backward-budget ← P - n * τ
Free-Intervals ← list of free intervals, init to [0; P[
for all route i in R_C do
  for all j in Free-Intervals do
    if Message of the route i does not collides with scheduled routes then
      m_{s_i} ← the first slot of Free-Intervals[j]
      Split the Free-Intervals considering the new packet
      Forward-budget ← Forward-budget - lost size
      Backward-budget ← Backward-budget - lost size
      call Exhaustive Generation on remaining routes
    end if
  end for
end for
```

Lost size, is the size lost when an interval is too few to put another packet. Ex : if the interval [250;3250[receives a packet of size 2500, there is 500 slots lost, because there is no way to uses the slots for another packet.

This algorithm enumerates all the solutions by traversing a tree. The leaves of the tree are the $(P\tau)$-periodic affectations, and the nodes are partial solutions. A partial solution is a choice of a starting time for a subset of the routes. For each route, the algorithm tries every possible starting offset until it finds one available (that allows the message to come back without collisions). When a route have been scheduled, it take another route that have not been scheduled yet. This creates a new sub-tree in which the algorithm will try every possible starting offset too, and create a sub-tree on the following route too. If no offset are available for a route, this means that the partial solution is not good, so the algorithm backtracks over the tree that is, it goes back to the father of the current sub-tree. Once a leaf is obtained, the algorithm stops and return the scheduling.
To reduce the number of useless calculation, in each time we add another packet, we consider the lost size in both periods, thus, we know when a period has no many great enough free area to put a packet.

4.1.4 Results

Resultats des simulations : Shortest-longest optimal pour ces parametres.

4.2 Allowing waiting times

4.2.1 Intro

Importance des waiting times quand la période est donnée (Résultats D'éxepriences et preuve avec l'exemple)

4.2.2 LSG

To find a solution allowing waiting times, the following heuristic is suggested:

On leaves switch, the messages are scheduled so that they are following each others, from the one using the longest route, to the one using the shortest route.

On sources switch, the politic is the following one, after setting the clock to 0, do:

1. To be eligible, a job needs to be able to come back on the switch at the current clock (if clock = 0, take the first message able to come back).
2. Between the eligible jobs, schedule first the one which has the longest route gives it the starting time $o_i = clock$.
3. Update the clock at the time in which the scheduled task is over:

$$clock = clock + message\ length$$

.

This algorithm defines the offsets of the first P-periodic affectation by sending the messages from the longest route to the shortest route, then it uses a greedy algorithm to schedule the messages in the second P-periodic affectation. We denote this algorithm by LSG (Longest Shortest + Greedy).

Algorithm 3 LSG

Input: Matched graph N, period P, packet size T
Output: 2way Trip affectation in P

```
clock ← 0
for all route i in ρ from the longest to the shortest do
    m_i ← clock
    clock ← clock + T
end for
clock ← 0
take i such that r_i is the first route to come back in sources switch
o_i ← clock;
clock ← a_i + T
while there is a route which has no w_i do
    take i such that r_i is the eligible route.
    o_i ← clock - a_i;
    clock ← clock + T
end while
```

Algorithm

Analysis Parler de LSO et expliquer pourquoi LSG mieux avec nos params

4.2.3 Results

Random

Distributions

5 Conclusion

Open questions. Can we improve the results if:

- The routes are smaller than the size of a message.
- The routes are smaller than the period.
- The largest difference between two routes is smallest than one of these parameters

On a general graph:

- The routing is coherent
- The graph is symmetric

References

[1] Ian Holyer. The np-completeness of edge-coloring. *SIAM Journal on computing*, 10(4):718–720, 1981.

[2] David Zuckerman. Linear degree extractors and the inapproximability of max clique and chromatic number. In *Proceedings of the thirty-eighth annual ACM symposium on Theory of computing*, pages 681–690. ACM, 2006.