

Tecnológico de Costa Rica
Escuela de Ingeniería Electrónica



Adversarial Anomaly Detector: Use of Generative Adversarial Networks for the detection of tomato diseases

A thesis submitted in partial fulfillment of the requirements for the degree of Master of
Science in Electronics, Major in Digital Signal Processing

Luis Alonso Murillo Rojas

Cartago, November 19th, 2017

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Proyecto de Graduación
Tesis de Maestría
Tribunal Evaluador

Tesis de maestría defendida ante el presente Tribunal Evaluador como requisito para optar por el grado académico de maestría, del Instituto Tecnológico de Costa Rica.

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Estudiante: Luis Alonso Murillo Rojas

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Resumen

El resumen es la síntesis de lo que aparecerá en el tesis. Tiene que ser lo suficientemente consiso y claro para que alguien que lo lea sepa qué esperar del resto de la tesis si la leyera completamente. Puede concluir con palabras clave, que son los temas principales tratados en el documento. El resumen queda fuera de la numeración del resto de secciones.

No se acostumbra utilizar referencias bibliográficas, tablas, o figuras en el resumen.

Palabras clave: Tomato, Deep Learning, Anomaly detection, Autoencoders, Generative Adversarial Networks

Abstract

The same as before, but in English.

Keywords: word 1, word 2,

a Mariángel

Agradecimientos

El resultado de este trabajo no hubiese sido posible sin el apoyo de Thevenin, Norton, Einstein y mi querido amigo Ohm.

Luis Alonso Murillo Rojas

Cartago, November 30, 2019

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Lista de símbolos y abreviaciones

Abreviaciones

ASM	Modelos Activos de Forma
PCA	Análisis de componentes principales
WSN	Redes Inalámbricas de Sensores

Notación general

A	Matriz.
$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$	
\mathbb{C}	Conjunto de los números complejos.
$\text{Im}(z)$ o z_{Im}	Parte imaginaria del número complejo z
j	$j = \sqrt{-1}$
$\text{Re}(z)$ o z_{Re}	Parte real del número complejo z
$\mathcal{T}[\cdot]$	Transformación realizada por un sistema
$\underline{\mathbf{x}}$	Vector.
$\underline{\mathbf{x}} = [x_1 \ x_2 \ \cdots \ x_n]^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$	
y	Escalar.
z^*	Complejo conjugado de z

Chapter 1

Introduction

The disease and pest strategy controls are becoming more important in the latest years for crop management, thanks to the consequences that brings climate change. In [6] list a series of problems that climate change provokes, like the alteration of stages and rates of development of pathogens, modifying host resistance and changes in the physiology of host-pathogen interactions. These could cause changes in the geographical distribution and growth of plant species.

All this situation has the potential to cause catastrophic plant disease, which can cause food crops to be destroyed, which according to [20] has the consequence of aggravating deficit of food supply. In this sense, the tomato has a key role, due to its big impact in the diet of people. As stated in [5], the tomato represents the most economically important vegetable crop worldwide.

[17] states the importance of the use of non-invasive methods to deal with crop disease. Normally, to combat this problem, the agricultors make use of chemicals like pesticides which leads to negative impacts like environment consequences and health issues to the people that consume the vegetable.

In Costa Rica, the disease the causes the most damage is the *Phytophthora infestans* and the most important pest in the whiteflies [2].

The general goal of this project is the study of different unsupervised learning algorithms for anomaly detection purposes in tomato images. As part of the specific goals are the following:

- Define a capture protocol of tomato images to the creation of the dataset that shall be used in the training of the generative models.
- Study and selection of unsupervised learning algorithms.
- Evaluation of the generative models for the detection of anomalies in tomato images.

The literature review section will be cover all the related works for the detection of

anomalies in different contexts like in agriculture or health care; also will be explained the main concepts that are the basis of this project.

In the proposed solution section is described the solution strategy to evaluate different deep learning architecture that is able to detect anomalies.

The results section presents a discussion of the performance of the different architecture and which of them are the best option for the detection of diseases in tomato.

Finally, the conclusion has a final analysis of the project and establish the future work.

Chapter 2

State of the art

2.1 Related work

Most of the work related to the detection of diseases and pests in tomato make use of deep learning techniques, but specifically the supervised learning algorithms. In [1] is presented work of the use of CNN classifiers and object detectors like Faster R-CNN, Region-based Fully Convolutional Network or Single Shot Multibox Detector for the detections of disease in tomato in South Korea. One of the problems that this project faced was the false positives, in [10] is proposed a filter to reduce this issue.

In [3] propose the use of an adversarial autoencoder for anomaly detection. This corresponds to an unsupervised method that is just able to replicate healthy data. This method has the advantage that the network is able to find the main features that represent normal data.

In [4] presents a new architecture proposal where is combined the variational autoencoder and the generative adversarial networks, for the detection of anomalies in MR brain images. This new architecture is compared with other approaches like general VAE, spatial VAE, and AnoGAN. The authors claim that its VAEGAN architecture slightly outperforms most of the other architectures.

2.2 Literature review

2.2.1 Deep Feedforward Networks

The Deep Feedforward Networks, also known as multilayer perceptrons (MLPs), has a big importance in the field of Machine Learning. The problem that this kind of network tackle is the approximation of some function f^* . As mentioned in [12], a common example is in the implementation of a classifier where $y = f^*(x)$ maps the input x to some category represented in y . In order to achieve that the feedforward networks define a mapping of

the form $Y = f(X, \theta)$, and during the training, this network should be able to learn the set of parameters θ that best fits to approximate the function.

These networks are typically composed of many different functions, which represent the layers of the network. For example, we could have four functions that are connected in cascade, to the form $f(x) = f_{(4)}(f_{(3)}(f_{(2)}(f_{(1)}(x))))$. The length of these chains of functions can be seen as the depth of the network, where $f_{(1)}$ is the first layer and $f_{(4)}$ is the last layer of the output. Between the first and the output layers are the hidden layers. During the training, generally is provided explicitly the output values given some input, but is no specified what values should have the hidden layers. The learning algorithm should decide how to use those hidden layers to generate the desired output.

2.2.2 Convolutional Neural networks

The convolutional neural networks (CNN) [14] are an extension of the deep feedforward networks, that are specialized in the processing of grid-like data. As its name suggests, this kind of network makes use of a mathematical operation called convolution. The convolution is an operation on two functions of a real-valued argument and is defined as follows:

$$s(t) = \int x(a)w(t-a)da \quad (2.1)$$

In terms of CNN, the first argument of the convolution ($x(a)$) is referred as the input and the second argument ($w(t-a)$) as the kernel. The output of this operation is known as the feature map. The common architecture of a CNN, as shown in figure 2.1, is composed of a convolutional layer, then a pooling layer (which is in charge of subsampling the data) and in the end, there is typically a fully connected network.

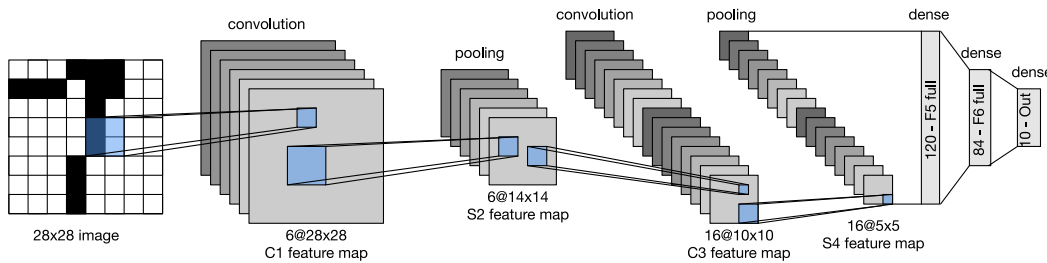


Figure 2.1: Convolutional Neural Network architecture

2.2.3 Generative models

In machine learning, there are mainly three types of learning algorithms: supervised learning algorithms, unsupervised learning algorithms, and semi-supervised learning algorithms. The supervised learning algorithms are the type of algorithms that make use of labeled data for its training process. In the last years, this kind of algorithm is presenting very important results in applications like object detection [15].

The unsupervised learning algorithms are the ones that do not need a structured dataset and can find a pattern by itself. One particular type of unsupervised learning algorithms is the generative models. The generative models are powerful algorithms that are capable of learning the probabilistic data distribution of some datasets, allowing them to generate new samples. Two of the most important generative models are the variational autoencoders (VAE) and the generative adversarial networks (GAN).

Autoencoders

In a similar way as the CNNs, the autoencoders are an extension of the deep feedforward network, with the difference that in this case, the goal is to replicate the input in the output. This type of networks are divided into two parts: the encoder that tries to generate a latent space h where is extracted some features of the input data $h = f(x)$; and the decoder that takes the latent space generated by the encoder as input and reconstruct the data $r = g(h)$. One of the main applications of the type of neural networks is the dimensionality reduction, image compression, image denoising or image generation. The last application example shall be cover in more detail in the next section with the variational autoencoders as part of the generative models.

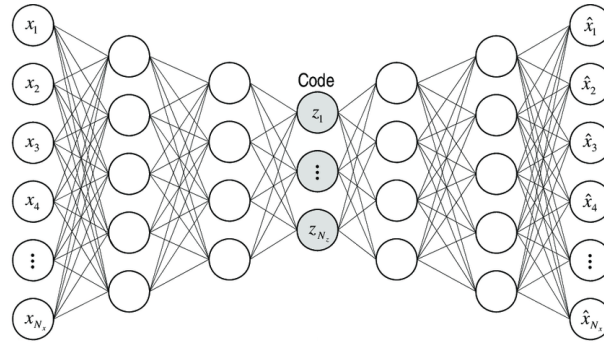


Figure 2.2: Autoencoder architecture

Variational Autoencoder

The objective of the variational autoencoders [13] is the generation of data samples from a learned latent space. This latent space is obtained from a large dataset. In order to achieve this generation process, this autoencoders must try to learn the probability distribution $P(x)$ of the data.

From a probabilistic perspective, the latent variables will get from a prior $P(z)$ and the generated data has a likelihood of $P(X|z)$ that is conditioned by the latent space. So the goal here is to model the data distribution as follows:

$$\mathbb{P}(\mathbf{X}) = \int_z \mathbb{P}(\mathbf{X}|z)\mathbb{P}(z)dz \quad (2.2)$$

However, this integral is computationally too costly, due to the fact of computing all the possibilities in the latent space. To avoid this, VAEs try to infer the distribution $\mathbb{P}(x)$ from data using $\mathbb{P}(z|\mathbf{X})$. Variational inference approximates the distribution $\mathbb{P}(z|\mathbf{X})$, using a simpler distribution, where a common choice is a Gaussian distribution. Then, with a parametric inference model $\mathbb{Q}(z|\mathbf{X})$ that maps the input data with the latent space; the difference between the distribution $\mathbb{P}(z|\mathbf{X})$ and $\mathbb{Q}(z|\mathbf{X})$ is calculated using the Kullback-Leibler divergence.

$$\begin{aligned} D_{KL}(\mathbb{Q}(z|\mathbf{X})\|\mathbb{P}(z|\mathbf{X})) &= \sum \mathbb{Q}(z|\mathbf{X}) \log \frac{\mathbb{Q}(z|\mathbf{X})}{\mathbb{P}(z|\mathbf{X})} \\ &= \mathbb{E} \left[\log \frac{\mathbb{Q}(z|\mathbf{X})}{\mathbb{P}(z|\mathbf{X})} \right] \\ &= \mathbb{E}[\log \mathbb{Q}(z|\mathbf{X}) - \log \mathbb{P}(z|\mathbf{X})] \end{aligned} \quad (2.3)$$

Using $\mathbb{P}(z|\mathbf{X}) = \frac{\mathbb{P}(\mathbf{X}|z)\mathbb{P}(z)}{\mathbb{P}(\mathbf{X})}$, the equation 2.3 can be rewritten as:

$$\begin{aligned} D_{KL}(\mathbb{Q}(z|\mathbf{X})\|\mathbb{P}(z|\mathbf{X})) &= \mathbb{E} \left[\log \mathbb{Q}(z|\mathbf{X}) - \log \frac{\mathbb{P}(\mathbf{X}|z)\mathbb{P}(z)}{\mathbb{P}(\mathbf{X})} \right] \\ &= \mathbb{E}[\log \mathbb{Q}(z|\mathbf{X}) - \log \mathbb{P}(\mathbf{X}|z) - \log \mathbb{P}(z) + \log \mathbb{P}(\mathbf{X})] \end{aligned} \quad (2.4)$$

$\mathbb{P}(x)$ does not depend on z , hence it can be taken out of the expectation:

$$\begin{aligned} \implies \log \mathbb{P}(\mathbf{X}) - D_{KL}(\mathbb{Q}(z|\mathbf{X})\|\mathbb{P}(z|\mathbf{X})) &= \mathbb{E}[\log \mathbb{P}(\mathbf{X}|z)] - \mathbb{E}[\log \mathbb{Q}(z|\mathbf{X}) - \log \mathbb{P}(z)] \\ &= \mathbb{E}[\log \mathbb{P}(\mathbf{X}|z)] - D_{KL}(\mathbb{Q}(z|\mathbf{X})\|\mathbb{P}(z)) \end{aligned} \quad (2.5)$$

Therefore, the loss function corresponds to:

$$\log \mathbb{P}(\mathbf{X}) \geq \mathbb{E}[\log \mathbb{P}(\mathbf{X}|z)] - D_{KL}(\mathbb{Q}(z|\mathbf{X})\|\mathbb{P}(z)) \quad (2.6)$$

The first term of the loss function can be seen as the reconstruction error and the second term corresponds to the KL error [9].

Spatial Variational Autoencoder

Consist of an improvement of the typical variational autoencoder. Whereas in the classic VAEs the latent space are vectors where its variables have a dimension of 1x1, in the

spatial VAE the idea is to extend these latent variables to have a bigger dimension and, in that way, be able to capture more spatial features of the input data.

In [21] is proposed a spatial variational autoencoder, where the latent variables are sampled from a matrix-variable normal (MVN) distribution. The authors claim that this architecture outperforms the original VAEs due to the capture of richer structural and spatial information from data.

Gaussian-Mixture Variational Autoencoder

This architecture corresponds to another variant of the VAEs models. In this case, the prior distribution $\mathbb{P}(z)$ is a Gaussian-mixture that allows the net to perform unsupervised clustering of the data. This autoencoder, proposed in [8], has the potential of group the data, and each group can represent or share a specific feature of the original data, besides to have a competitive performance in comparison with the regular VAEs.

Generative adversarial networks

The generative adversarial networks (GAN) were proposed by Ian Goodfellow [11] in 2014. The basic idea behind GANs is in the competence of two models: from one side is the generator G that tries to learn the data distribution of the data and for the other side, a discriminator D that decides if the input data is real or generated by G . The goal of the generator is to try to create images as real as possible that provokes the discriminator to make mistakes. This game is described as the minmax value function in equation 2.7.

$$\min_G \max_D V(D, G) = \mathbb{E}_{x \sim p_{\text{data}}(x)} [\log D(x)] + \mathbb{E}_{z \sim p_z(z)} [\log(1 - D(G(z)))] \quad (2.7)$$

A well trained GAN model is able to reach the Nash equilibrium, where the discriminator has an accuracy of around 0.5, which means that it is not able to discern between fake or real data; and the generator should reach value loss of approximately 0.7.

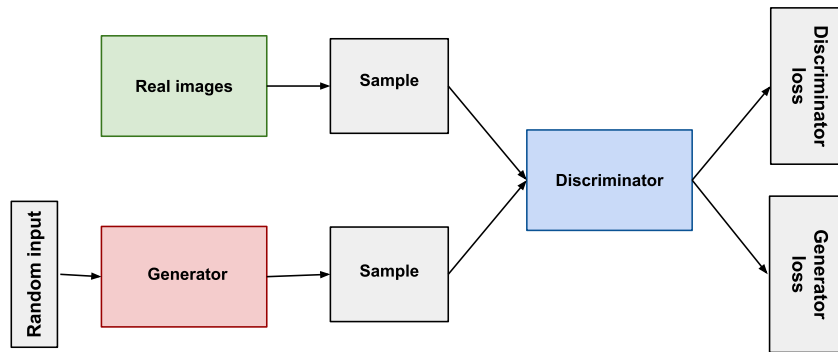


Figure 2.3: Generative Adversarial Network architecture

One of the major challenges that the GANs has is precisely the training process, where something is really hard to reach the Nash equilibrium. Sometimes, if the capacity of the

model is not enough, the model is susceptible to collapse. Another possible scenario is when the learning rate of the model is too aggressive, provoking that the model never converges.

Architectures based on GANs

In [18] presents a series of architectures based on GANs, along with some metrics to evaluate its performance.

- Convolution base GAN: The original GAN is implemented based on the multi-layer perceptron but it has been proven that CNN are better than the MLP in extracting features to the images. This kind of network is known as Deep Convolutional Generative Adversarial Network (DCGAN).
- Conditional GAN: Normally, the generator of the GAN receives as input some random noise, which sometimes makes the model prone to collapse. It is for this reason that in the conditional GAN a variable C is introduced as input to the generator and also to the discriminator, with the objective of add some constraints and, therefore, have more control in that latent space. The type of constraint will depend on the type of data that the GAN is dealing with.
- Autoencoders based GANs: This type of GAN architecture is presented in [16] where the idea is to make use of an adversarial training to the autoencoder performing variational inference by matching the aggregated posterior latent space of the autoencoder with an arbitrary prior distribution. This will allow overcoming one of the main challenges of the autoencoders, where sometimes is not able to learn correctly the data distribution.

2.2.4 Use of generative model in the anomaly detection

The generative models discussed so far have their utility in the detection of anomalies. The general idea is to train a model to be able to learn the data distribution of a normal dataset. In that way, the model should be able to reconstruct or generate a query image as similar as possible. If for some reason that is not the case, there is a high probability that the regions that the model was not able to generate, correspond to an anomaly. All the autoencoders described before can be used in this way, as well as the GANs.

For the specific case of the GANs, in [7] is presented different GAN based architectures, with the propose of anomaly detection:

AnoGAN

This GAN is first trained with just normal data and be able to learn the manifold of the data X . Then, with the generator trained, each time that some image has to be evaluated,

an iterative process is performed in order to find the latent variables that generate the more similar $G(x)$ to the query image. This iterative process has the disadvantage that is too time-consuming [19].

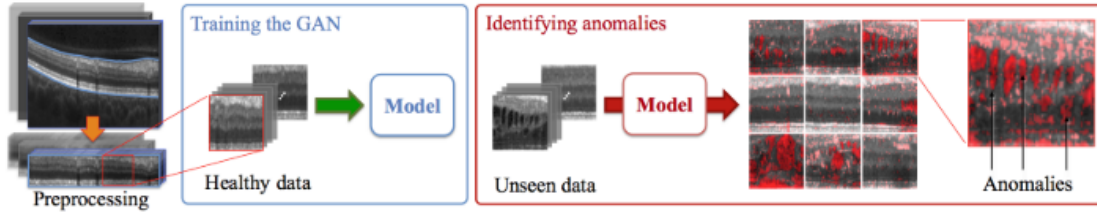


Figure 2.4: AnoGAN training with healthy images first and then reconstruction of unseen images[19]

GANomaly

This architecture is inspired by the AnoGan but tries to overcome the long detection time that it has. In order to do that, make use of an encoder that is able to learn the latent space variables that the generator receives during the GAN training. This has the advantage of having a faster GAN training and reduces the times to generate a similar image to the query image. The generator also has an encoder at the end of its structure that helps in the training in the learning of the manifold of the input data x .

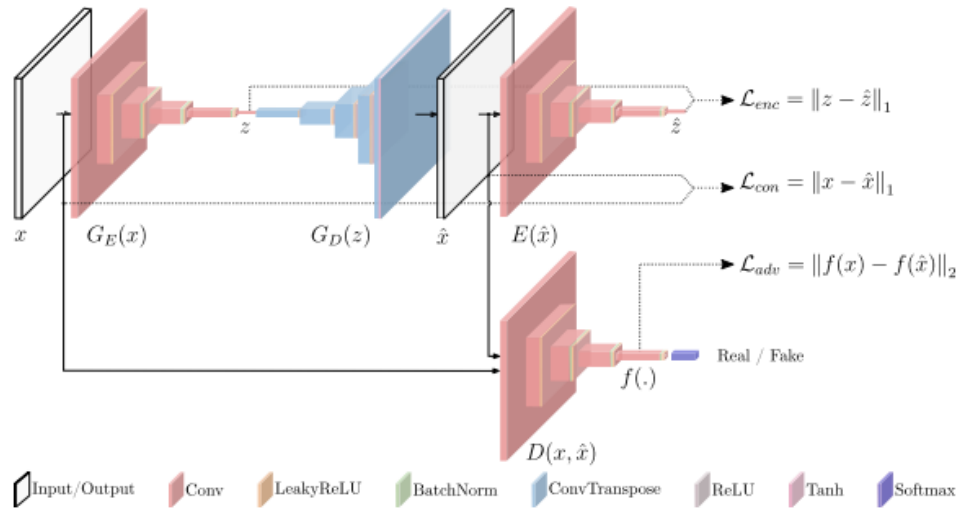


Figure 2.5: GANomaly architecture and loss functions [7]

Chapter 3

Proposed solution

Chapter 4

Results and analysis

Chapter 5

Conclusions

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Appendix A

Demostración del teorema de Nyquist

El título anterior es solo un ejemplo ilustrativo. Éste teorema no ameritaría un apéndice pues es parte normal del currículum de Electrónica, pero apéndices usualmente involucran aspectos de esta índole, que se salen de la línea de la tesis, pero que es conveniente incluir por completitud.

Los anexos contienen toda información adicional que se considere pertinente agregar, como manuales de usuario, demostraciones matemáticas que se salen de la línea principal de la tesis, pero que pueden considerarse parte de los resultados del trabajo.