ENRICHMENT ACTIVITY

Philip borrowed money from Rodnie. He will pay the principal plus the interest by paying Php2,215 each month for 2 years. How much money did he borrow if the interest is 6% compounded quarterly?

MathPACKS

SHS
General Math

Quarter 2 Week 4 – A
(M11GM-IIc-d-1)
The learner finds the present value of general annuities.

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GENERALIZATIONS

Present Value (P) of General Ordinary Annuity

$$P = R\,\frac{1-(1+j)^{-n}}{j}$$

where **R** is the regular payment;
j is the equivalent interest rate per payment interval converted from the interest rate per period; and
n is the number of payments.

Steps in Finding the Present Value of a General Ordinary Annuity:

1. Convert the given interest to its equivalent rate for its corresponding payment interval (monthly, quarterly, semi-annually or annually).
2. Use the formula in finding the present value of ordinary annuity.

EXAMPLE

Mr. Sanchez would like to buy a refrigerator payable for 18 months starting at the end of the month. How much is the cost of the refrigerator if his monthly payment is Php1,145 and the interest is 9% compounded semi-annually?

SOLUTION

Given: R = Php 1,14 $i^{(2)} = 0.09$ n = 18 payments m = 2
Find: cost (present value P)

Step 1:

Convert 9% compounded semi-annually to its equivalent rate for monthly payment interval.

$$F_1 = F_2$$

$$P\left(1+\frac{i^{(12)}}{12}\right)^{(12)t} = P\left(1+\frac{i^{(2)}}{2}\right)^{(2)t}$$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{0.09}{2}\right)^{2}$$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = (1.045)^2$$

$$1+\frac{i^{(12)}}{12} = [(1.045)^2]^{\frac{1}{12}}$$

$$\frac{i^{(12)}}{12} = (1.045)^{\frac{1}{6}} - 1$$

$$\frac{i^{(12)}}{12} = 0.00736123 = j$$

The interest rate per monthly payment interval is 0.007363123 or 0.7363123%.

Step 2:

Apply the formula in finding the present value of an ordinary annuity using the computed equivalent rate.

$$P = 1{,}145\left[\frac{1-(1+0.007363123)^{-18}}{0.007363123}\right]$$

$$P = 1{,}145\left[\frac{1-(1.007363123)^{-18}}{0.007363123}\right]$$

$$P = 1{,}145\left(\frac{1-0.8762966044}{0.007363123}\right)$$

$$P = 1{,}145\left(\frac{0.1237033956}{0.007363123}\right)$$

$$P = 1{,}145\,(16.80039781)$$

$$P = \text{Php } 19{,}236.50$$

The cost of the refrigerator is Php 19,236.50.

Note: In solving for an equivalent rate, six ore more decimal places or the exact value will be used.

EXERCISES

Solve the following problems.

1. A motorcycle can be bought on a monthly installment of Php3,600 for 2 years. If the rate of interest is 12% compounded annually, what is the present value of a motorcycle?
2. Eunice purchased a laptop to be paid Php3,050 every month for 9 months with an interest rate of 9% compounded quarterly. How much is the cost of the laptop Eunice purchased?

EXERCISES

Solve the following problems.

1. Kyla sold her laptop to Fely in a condition that Fely should give her a downpayment of Php8,000 and Fely will pay her Php1,000 every month for 1 year. If the money earns 0.5% compounded monthly, what is the market value of Fely's offer?
2. Grace wants to sell her old car. There are two interested buyers. Buyer 1 offers her to pay in cash worth Php300,000 while Buyer 2 will pay her Php26,000 every quarter for 3 years. Whose offer has a higher market value if money can earn 3% compounded annually?

ENRICHMENT ACTIVITY

Mrs. Rodriguez, a government employee will retire on her 60th birthday. In claiming her retirement benefits, she has two options.

Option 1:

She can get her five year pension in advance. The lump sum is equivalent to 60 months of her Basic Monthly Pension (BMP) payable at the time of her retirement. After 5 years, she will start receiving her BMP.

Option 2:

She will receive a cash payment equivalent to 18 months of her BMP payable upon her retirement, and a monthly pension payable immediately after the retirement date.

If the BMP of Mrs. Rodriguez is Php19,522.18 and assume that the money earns 5% compounded annually, which option has a better market value within 7 years?

MathPACKS

SHS
General Math

Quarter 1 Week 4 – B
(M11GM-IId-2)
The learner calculates the fair market value of a cash flow stream that includes an annuity.

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GENERALIZATIONS

The **Fair Market** Value (FMV) of a cash flow (payment stream) on a particular date refers to single amount that is equivalent to the value of the payment stream at that date. This particular date is called the **focal date**.

To determine the Fair Market Value (FMV), add the downpayment and the present value of the payments or just add the present value of all payments given.

Cash Flow

-refers to payments received (cash inflows) or payments or deposits made (cash outflows). Cash inflows can be represented by positive numbers and cash outflows can be represented by negative numbers.

EXAMPLE

Mr. Reyes received two offers on a lot that he wants to sell. Mr. Abad has offered Php50,000 and a Php1,000,000 lump sum payment 5 years from now. Mr. Delos Santos has offered Php50,000 plus Php45000 every quarter for five years. Compare the fair market value of the two offers if money can earn 5% compounded annually. Which offer has the higher market value?

Mr. Abad's offer:

Php 50000					1 million
0	1	2	3	4	5

Mr. Delos Santos' offer:

Php 50000	Php 45000	Php 45000	Php 45000	…	Php 45000
0	1	2	3	…	20

SOLUTION

Mr. Abad's offer:

The present value of Php1,000,000 offered 5 years from now is

$P = F\,(1+j)^{-n}$

$P = 1{,}000{,}000\ \ (1+0.05)^{-5}$

$P = 1{,}000{,}000\ \ (1.05)^{-5}$

$P = 1{,}000{,}000\ \ (0.7835261665)$

$P = \text{Php}\ 783{,}526.20$

FMV=downpayment+ present value

FMV= Php 50,000 + Php 783,526.20

FMV= Php 833,526.20

Mr. Delos Santos' offer:

We have to compute the present value of a general annuity with quarterly payments but with annual compounding interest at 5%. But first, we need to convert the 5% compounded annually to its equivalent interest rate for each quarterly payment interval.

$$F_1 = F_2$$

$$P\left(1+\frac{i^{(4)}}{4}\right)^{(4)5} = P\left(1+\frac{i^{(1)}}{1}\right)^{(1)5}$$

$$\left(1+\frac{i^{(4)}}{4}\right)^{20} = \left(1+\frac{0.05}{1}\right)^{5}$$

$$\left(1+\frac{i^{(4)}}{4}\right)^{20} = (1.05)^5$$

$$1+\frac{i^{(4)}}{4} = [(1.05)^5]^{\frac{1}{20}}$$

$$1+\frac{i^{(4)}}{4} = 1.012272234$$

$$\frac{i^{(4)}}{4} = 1.012272234 - 1$$

$$\frac{i^{(12)}}{12} = 0.012272234 = j$$

Applying the formula in finding the present value of a general annuity, we have:

$$P = 45{,}000\left[\frac{1-(1+0.012272234)^{-20}}{0.012272234}\right]$$

$$P = 45{,}000\left[\frac{1-(1.012272234)^{-20}}{0.012272234}\right]$$

$$P = 45{,}000(17.63931709)$$

$$P = \text{Php}\ 793{,}769.30$$

FMV=downpayment+ present value

FMV= Php 50,000 + Php 793,769.30

FMV= Php 843,769.30

Comparing the offer of the two, Mr. Delos Santos' offer has a higher market value.

EXERCISE 1

Solve the following problems.

1. A car is to be purchased in monthly payments of Php15,000 for 5 years starting at the end of 3 months. How much is the present value of the car if the interest rate is 12% converted monthly?
2. Alexa wants to purchase a smartphone to be given as a gift to her mother. However, she does not have enough money to purchase the product. So, she bought a smartphone at Php1,200 payable for 9 months to be paid starting at the end of 2 months. How much is the smartphone if the interest rate is 12% convertible monthly?

EXAMPLE 2

What is the period of deferral if the monthly payment of Php5,000 for 7 years will start 5 months from now?

SOLUTION

The first payment is at time 5. The period of deferral is from time 0 to 4, which is equivalent 4 periods or 4 months.

EXERCISE 2

Find the period of deferral in the following deferred annuity problems.

1. monthly payments of Php10,000 for 9 years will start 4 months from now
2. monthly payments of Php4,500 for 18 months will start 3 months from now
3. quarterly payments of Php6,000 for 5 years will start 6 months from now
4. annual payments of Php3,000 for 20 years will start 2 years from now
5. semi-annual payments of Php5,000 for 3 years will start 1 year from now

ENRICHMENT ACTIVITY

Mrs. Camino deposited her money in a fund such that it would allow her to withdraw Php10,000 monthly for 3 years starting at the end of 6 months. How much is the amount she deposited if the interest rate is 3% convertible monthly?

MathPACKS

SHS
General Math

Quarter 1 Week 4 – C
(M11GM-IId-3)
The learner calculates the present value and period of deferral of a deferred annuity.

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GENERALIZATIONS

Deferred Annuity

-an annuity that does not begin until a given time interval has passed

Period of Deferral

-time between the purchase of an annuity and the start of payments for the deferred annuity

Present Value of a Deferred Annuity

$$P = R\,\frac{1-(1+j)^{-(k+n)}}{j} - R\frac{1-(1+j)^{-k}}{j}$$

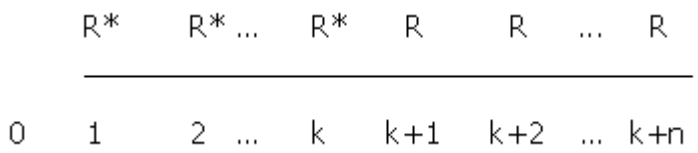
Where **R** is the regular payment;

j is the interest rate per period;

n is the number of payments;

k is the number of conversion periods in the deferral

Time Diagram for a Deferred Annuity



The period of deferral is **k** because the regular payments of **R** start at time **k+1**. The notation **R*** represent **k** "artificial payments" each equal to R, but are not actually paid during the period of deferral.

EXAMPLE 1

A private company offers Emma a deferred payment option for her cash loan. She is given an option to pay Php4,000 every month for 2 years. Her first payment is due 3 months from now. How much is the amount of the loan if the interest rate is 9% compounded monthly?

Given: R= Php 4,000 m=12 $i^{(12)}$= 0.09 t=2 years

Find: P

SOLUTION

The annuity is deferred for 2 months and it will go on for 2 years. The first payment is due at the end of 3 months, or at the end of the 3rd conversion period. Thus, there are 2 artificial payments.

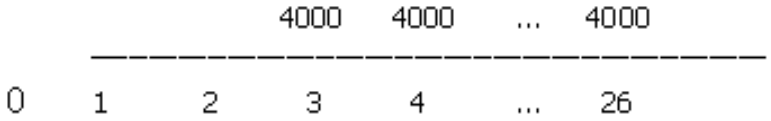
Number of artificial payments: **k**=2

Number of actual payments: **n**=(m)(t)=(12)(2)=24

Interest rate per period: $j = \frac{i^{(12)}}{m} = \frac{0.09}{12} = 0.0075$

If you assume that there are payments in the period of deferral, there would be a total of $k+n$ =2+24= 26 payments.

Time Diagram for a Deferred Annuity



Using the formula, the present value of the deferred annuity can be solved as

$$P = R\,\frac{1-(1+j)^{-(k+n)}}{j} - R\frac{1-(1+j)^{-k}}{j}$$

$$P = \left\{(4{,}000)\left[\frac{1-(1+0.0075)^{-26}}{0.0075}\right]\right\} - \left\{(4{,}000)\left[\frac{1-(1+0.0075)^{-2}}{0.0075}\right]\right\}$$

$$P = \left\{(4{,}000)\left[\frac{1-(1.0075)^{-26}}{0.0075}\right]\right\} - \left\{(4{,}000)\left[\frac{1-(1.0075)^{-2}}{0.0075}\right]\right\}$$

$$P = \left[(4{,}000)\left(\frac{1-0.8234335821}{0.0075}\right)\right] - \left[(4{,}000)\left(\frac{1-0.9851670782}{0.0075}\right)\right]$$

$$P = \left[(4{,}000)\left(\frac{0.1765664179}{0.0075}\right)\right] - \left[(4{,}000)\left(\frac{0.0148329218}{0.0075}\right)\right]$$

$$P = [(4{,}000)(23.54218905)] - [(4{,}000)(1.977722907)]$$

$$P = 94{,}168.75621 - 7{,}910.891628$$

$$P = \text{Php } 86{,}257.90837$$

$$P \approx \text{Php } 86\,257.90$$

Therefore, the amount of loan is Php 86,257.90.