Question 1 – Exoplanet Characterization

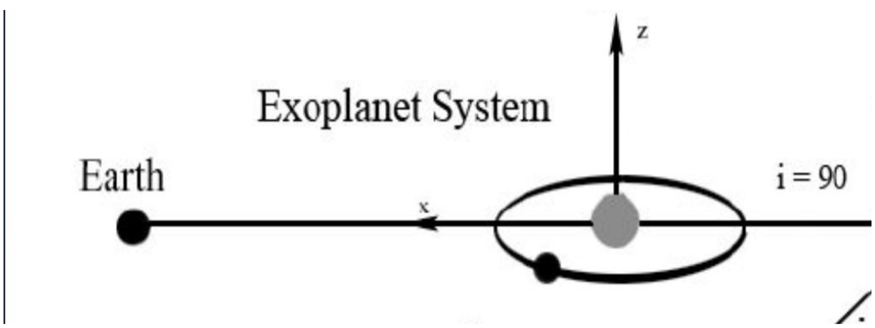
In this question, you will estimate the mass and radius of a planet from its radial velocity and transit data.

A mysterious new (and fake!) planet, GJ 8999 b, has been detected orbiting the M dwarf GJ 8999. GJ 8999 is a *very* small star, with a mass of $0.2M_\odot$ and a radius of $0.2R_\odot$. (If you haven't seen those symbols before, $M_\odot$ and $R_\odot$ are the mass and radius of the Sun, respectively.)

The cunning astronomer you are, you have been measuring transit and radial velocity data of this star to figure out the planet's mass and radius of this planet, so you can publish a paper on the system! Let's characterize this planet now.

a) What is the inclination of GJ 8999 b?

As we are measuring transit of the star (GJ 8999), the planet must be "edge-on". So, it is safe to assume that the inclination of the planet (GJ 8999 b) must be 90°



b) New transit data from the Transiting Exoplanet Survey Satellite (TESS) has come in, and it very much looks like we have some exoplanet transits! A plot of the flux from the full 28-day observation period of TESS is shown here, as well as a plot that is zoomed into a single transit.

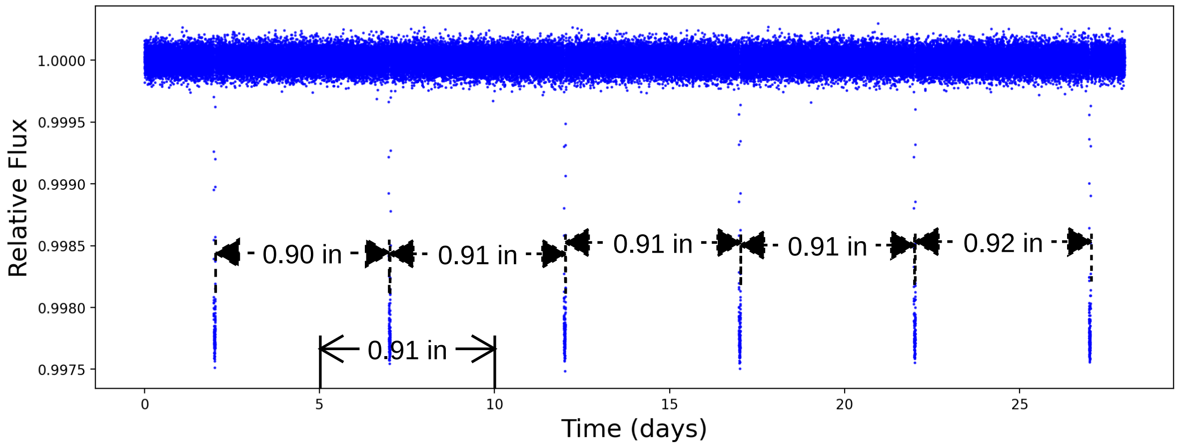


Figure 1: A plot of the flux of GJ 8999 over time over a 28-day period.

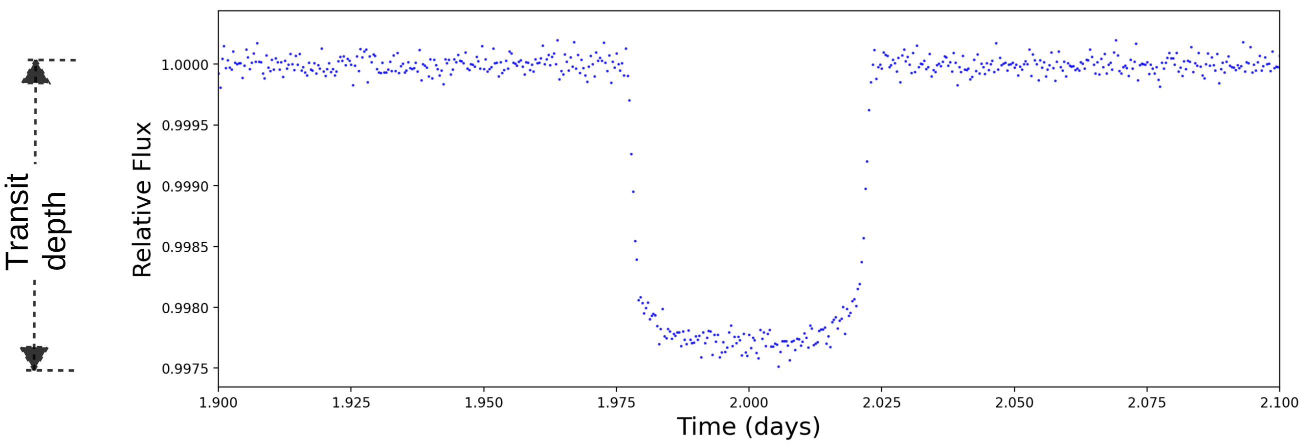


Figure 2: A plot of the flux of GJ 8999 over time, zoomed into a single exoplanet transit.

What is the period of this exoplanet?

Using software (shown above), a 0.91 inch distance in the plot corresponds to roughly 5 days.

Taking the average of all the time periods in terms of distance between the spikes in the graph,
Average=(0.90+0.91+0.91+0.91+0.92)/5 = 0.91 in

Thus, the time period is also roughly <u>5 days.</u>

c) What is the radius of this planet?

We know,

$$
\begin{aligned}
Z &= \left(\frac{R_p}{R_*}\right)^2 \\
\implies R_p &= \sqrt{Z} \times R_* \\
\implies R_p &= \sqrt{1 - 0.9975} R_* \qquad \text{(putting the values)} \\
&= \sqrt{0.0025} R_* \\
&= 0.05 R_* \\
&= 0.05 \times 0.2 R_\circ \qquad \text{(Given, mass of the star is 0.2 times the mass of the Sun)} \\
&= 0.01 R_\circ
\end{aligned}
$$

The transit depth Z has been taken from the graph (marked there).

Thus, the radius of this planet is 1% of the Sun's radius. (~) 6963.4 km

Also, (to be used later), radius with respect to Earth= 1.09 $R_\oplus$

d) Luckily for us, we have gotten some radial velocity data to figure out this planet's mass, too. This data, taken over a period of 30 days, measures the star's Doppler shift as it moves back and forth due to the planet's gravity.

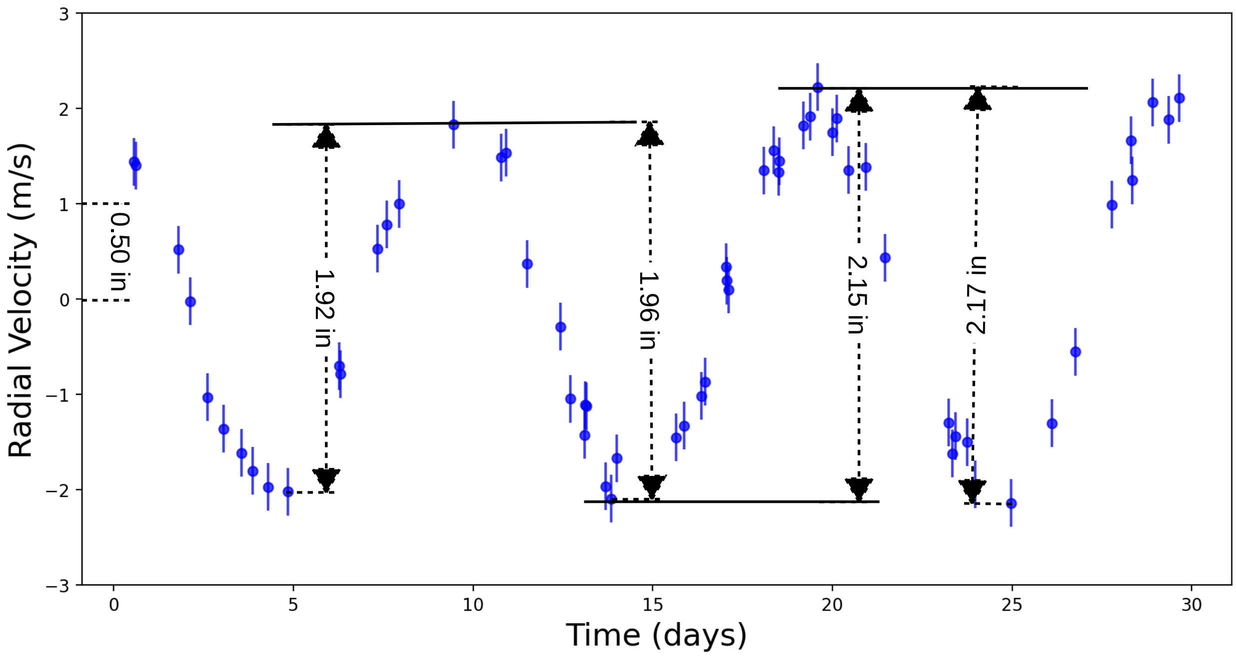


Figure 3: A plot of the radial velocity of GJ 8999 over time.

What is the semi-amplitude K of this planetary signal?

The semi-amplitude K is defined as half of the distance between crest and trough of this graph. Taking the average of the distances from the graph, we have,

d_avg=(1.92+1.96+2.15+2.17)/4 ~ 2.05 in

Now, according to the graph 1 m/s of radial velocity corresponds to 0.50 inch.

Then, 2.05 in correspond to (2.05/0.5) m/s = 4.10 m/s

Therefore, semi-amplitude, K = 4.10/2 m/s= 2.05 m/s

e) What is the mass of this planet?

We have

$$K = M_p \sin i \left(\frac{2\pi G}{PM_*^2}\right)^{1/3}$$

$$\implies M_p \sin i = K \left(\frac{PM_*^2}{2\pi G}\right)^{1/3}$$

K=2.05 m/s (solved in the previous question, Semi-amplitude)
P=5 days = 432,000 s , (Time period)
M*= 0.2 times the mass of the Sun~ 3.978×10^29 kg, (Stellar mass)
π=3.1416
G=6.674 × 10^{-11} N·m²/kg² (Universal Gravitational Constant)

Putting all these values in the equation, we get

Minimum mass of the planet= 1.1199 × 10^25 kg =1.875 $M_\oplus$

However, as we had assumed that the inclination is 90° , we can take this value as approximately the mass of the planet.

f) So, now that we've found the mass and radius of our planet, let's try to figure out what it's made of!

The following plot shows (very rough) 'mass-radius curves' of rocky exoplanets of different compositions. A planet lying on a given curve has a mass and radius consistent with being made of the corresponding composition.

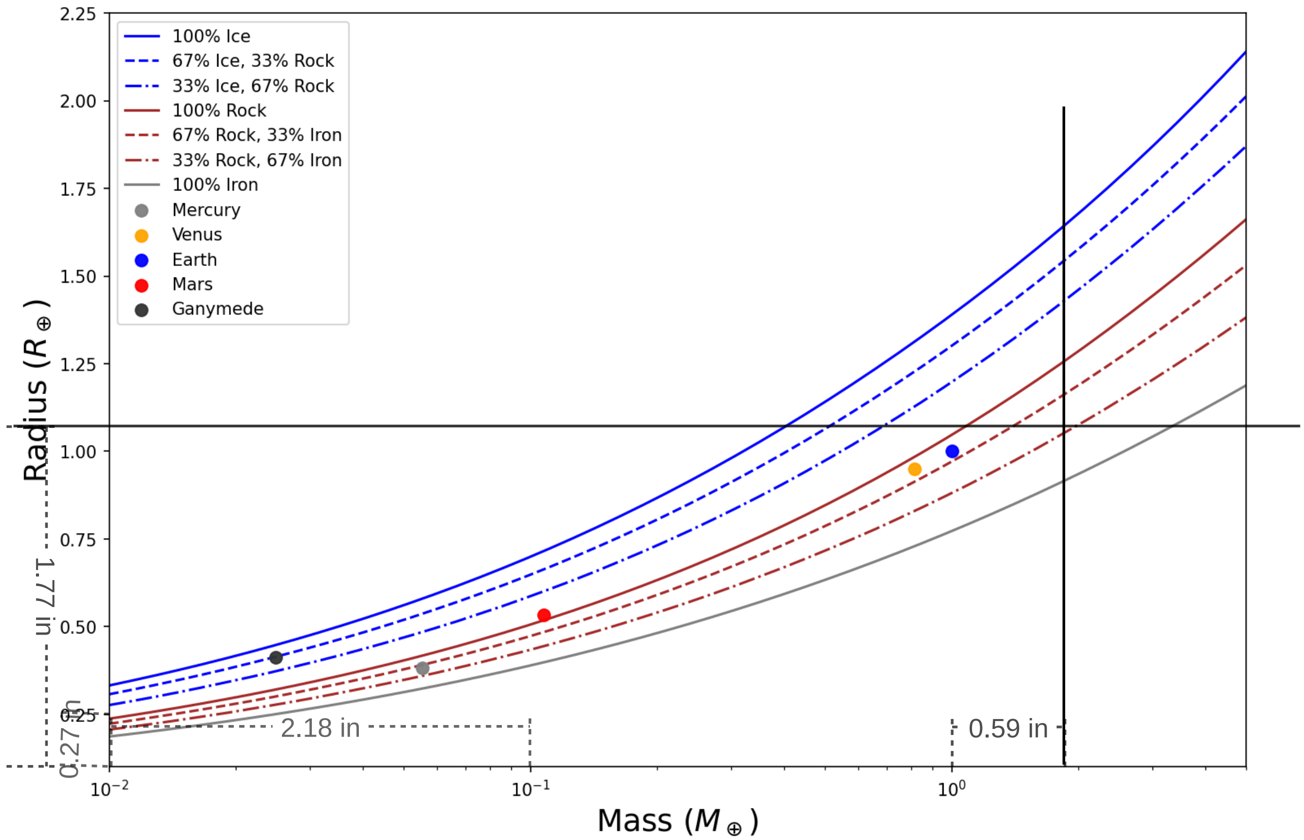


Figure 4: A plot showing the mass-radius curves for different exoplanet compositions.

The five rocky planets (plus Ganymede) are all shown on the plot as well. For example, Earth lies very near the '67% rock, 33% iron' curve, and Earth's composition IS indeed about 67% rock and 33% iron.

With this in mind, what is the composition of GJ 8999 b?

Using the data we obtained in the previous questions, and after converting the data to measure values in the table itself (convert into inches with correct proportionality),

we draw the two lines that satisfy the conditions for Mass of the planet and radius of the planet with respect to Earth. They meet, as shown, very near the 33% rock and 67% iron curve.

This should be an approximate composition of the planet.

--- Devankur Kashyap