ECE280 - Lab 5: Sampling and Aliasing

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I have adhered to the Duke Community Standard in completing this assignment.

Contents

1 Objectives

In this project, we will explore the phenomenon of aliasing that can occur when a continuous-time signal is sampled for digital processing by conducting both theoretical analyses and practical demonstrations—using visual and auditory examples—to illustrate how aliasing affects signal quality. Additionally, we will investigate how an anti-aliasing filter can help prevent or reduce these unwanted effects, enhancing the fidelity of the digitally processed signal.

1.**Analyzing Aliasing Effects** to gain hands-on experience with how aliasing impacts the quality of sampled continuous-time signals. We then use visual (graphing tools) and auditory tools to observe and understand the distortions that can arise when sampling does not adhere to appropriate frequency guidelines.

2. We also explore **anti-aliasing filtering** to investigate how applying an anti-aliasing filter before sampling can improve the representation of continuous-time signals. Throught he visual and auditory examples, we will illustrate how this filter helps to suppress higher frequency components that could cause aliasing.

3. We finally explore the importance of **anti-aliasing pre-filtering** by learning the theoretical basis for why anti-aliasing filters are essential in many digital processing applications.

2 Background

Sampling
Sampling is a fundamental process in digital signal processing, where a continuous-time signal (one that varies smoothly over time) is converted into a discrete-time signal by measuring its amplitude at fixed intervals, known as sampling intervals. These measurements, or "samples," capture snapshots of the original signal at specific moments in time. Sampling allows analog signals, like audio or radio waves, to be represented digitally, enabling their storage, manipulation, and transmission by digital systems, such as computers or communication networks. The rate at which samples are taken is called the **sampling rate** or **sampling frequency** and is measured in samples per second, or Hertz (Hz). According to the **Nyquist-Shannon Sampling Theorem**, to accurately capture all the information in a continuous signal, the sampling rate must be at least twice the highest frequency present in the signal. This minimum rate is called the **Nyquist rate**. For example, human hearing ranges up to about 20,000 Hz, so audio signals are typically sampled at 44,100 Hz (44.1 kHz) to ensure fidelity. In essence, sampling is the bridge between continuous analog signals and digital processing, making it essential in fields like telecommunications, music production, medical imaging, and more, where continuous signals must be stored, transmitted, or manipulated digitally. It can be observed for instance by convolving a continuous time signal with spaced impulses.

Aliasing
Aliasing occurs when the sampling rate is insufficient to accurately capture the variations in the continuous-time signal, resulting in distortions or misrepresentations of the original signal. Specifically, frequencies higher than half the sampling rate appear as lower frequencies in the sampled signal, creating unwanted artifacts and loss of signal fidelity. When the sampling rate is lower than the Nyquist rate, aliasing can occur. Aliasing causes high-frequency components of the original signal to appear as lower frequencies in the sampled signal, distorting the signal and making it impossible to accurately reconstruct the original waveform from its samples. This is why, in applications like audio processing, an anti-aliasing filter is often applied before sampling to remove frequencies higher than half the sampling rate, thus preventing these unwanted artifacts.

Steps for Transmitting Information via the Standard Telephone System

1. **Microphone Input**: When a person speaks, the microphone in the telephone handset converts the sound waves from their voice into an analog electrical signal (continuous) that varies in amplitude and frequency to match the chatacteristics of the person's voice.

2. **Analog-to-Digital Conversion (ADC)**: To send this analog signal over digital networks, the telephone system converts it into a digital signal through sampling and quantization. For telephony, the typical sampling rate is 8 kHz,which is sufficient for the frequency range of human speech. Each sample is quantized (assigned a digital value), typically at 8 bits per sample, using the G.711 codec. This process transforms the analog waveform into a series of digital values representing sound intensity at each sampled point.

3. **Compression and Transmission**:Once digitized, the signal is often compressed to reduce the amount of data and to optimize bandwidth use. This can involve encoding methods that reduce redundancies in the signal. The compressed data is then formatted into small packets for transmission over digital networks. The digital signal packets travel through various telecommunications infrastructure, such as copper wires, fiber optic cables, or wireless networks, to reach the receiver's telephone. In traditional landline telephony, the Public Switched Telephone Network (PSTN) handles routing. For mobile or internet-based calls, cellular networks or internet protocols direct the data to the receiver.

4. **Reception and Digital-to-Analog Conversion (DAC)**: At the receiving end, the process reverses. The digital packets are reassembled, decompressed, and passed through a Digital-to-Analog Converter (DAC). This converter reconstructs an analog signal from the digital data, producing an approximation of the original audio waveform.The receiving end thus decodes the digital data back into an analog signal using a DAC.

5. **Speaker Output:** The analog signal drives a small speaker in the telephone's handset. The speaker converts the electrical signal back into sound waves, reproducing the original voice so that the listener can hear the transmitted speech. The clarity and quality of the sound depend on the fidelity of the sampling, compression, and reconstruction processes.

Example of Digital Processing of a Continuous-Time Signal: Medical Imaging (MRI)
In Magnetic Resonance Imaging (MRI), the continuous signals produced by the body's response to magnetic fields are sampled and digitally processed. The MRI machine samples this data, applies signal processing algorithms, and reconstructs it into detailed digital images of internal structures. These digital images allow for non-invasive diagnosis and monitoring in healthcare.

3 Results and Discussion

3.1 Deliverables

- Deliverable 1: For each frequency, describe what you hear and explain why this is what you would expect.
  100 Hz: For the 100 Hz input frequency, a clean low-pitch sound is heard, and the analyzer has a peak at 100 Hz. This is because the input frequency is well below the Nyquist limit, hence it does not undergo aliasing.

  200 Hz:For the 200 Hz input frequency, a clean low-pitch sound is heard, and the analyzer has a peak at 200 Hz. This is because the input frequency is well below the Nyquist limit, hence it does not undergo aliasing.

  400 Hz: For the 400 Hz input frequency, a clean low-pitch sound is heard, and the analyzer has a peak at 400 Hz. This is because the input frequency is well below the Nyquist limit, hence it does not undergo aliasing.

  800 Hz:For the 800 Hz input frequency, a clean low-pitch sound is heard, and the analyzer has a peak at 800 Hz. This is because the input frequency is well below the Nyquist limit, hence it does not undergo aliasing.

  7900 Hz: A low frequency sound is heard and the analyzer shows a peak at a frequency much lower than 7900Hz. It is a sound that is almost imperceptible. This is because the sound undergoes aliasing as the input frequency is well above the Nyquist limit.

7800 Hz: A low frequency sound is heard and the analyzer shows a peak at a frequency much lower than 7900Hz. It is a sound that is almost imperceptible. This is because the sound undergoes aliasing as the input frequency is well above the Nyquist limit.

7600 Hz: A low frequency sound is heard and the analyzer shows a peak at a frequency much lower than 7900Hz. It is a sound that is almost imperceptible. This is because the sound undergoes aliasing as the input frequency is well above the Nyquist limit.

7200 Hz: A low frequency sound is heard and the analyzer shows a peak at a frequency much lower than 7900Hz. It is a sound that is almost imperceptible. This is because the sound undergoes aliasing as the input frequency is well above the Nyquist limit.

- Deliverable 2: Explain what is the maximum frequency that will pass through the system without aliasing.
  The maximum frequency that would pass through the system without aliasing can be determined by considering the effective sampling rate post downsampling. After downsampling, the effective sampling rate is 8kHz. However by the Nyquist Theorem, the maximum frequency that would go through the system without aliasing has to be half the effective sampling rate which is **4kHz**. With an input frequency of 4kHz, a peak is observed at he input frequency without any distortions to the signal. A gradual increase shifts the peak significantly to lower values.

- Deliverable 3: Describe your observations about the quality of the output signal, and why this is the case.
  The sound played or produced is a coarse sound. The words aren't distinct and there is so much background noise possibly due to the lack of an anti-aliasing filter. It is thus harsh to the ears as there is no clarity to the sound produced. The lack of clarity may be attributed to the fact that the background noise which is not filtered out but is rather aliased to a perceptible sound, introduces a degree of complexity that makes the original sound imperceptible.

- Deliverable 4: Explain what is the cutoff frequency of the filter, how this frequency is appropriate for the telephone system application, and how the cutoff frequency of an anti-aliasing filter should generally be chosen.
  The cutoff frequency appears to be above 3.4kHz and beneath **4.0kHz**, more accurately (4.0kHz) around the same frequency range as normal speech. By limiting the frequency range to that of speech, the filter allows all necessary components of human speech to pass while removing higher frequencies that are irrelevant(noise). Generally speaking, the cutoff frequency for anti-aliasing filters are set to be exactly or right below the Nyquist frequency of the downsampled rate. This ensures that only the frequencies within the permissible range are passed, while higher frequencies are filtered out.

- Deliverable 5: For each frequency, describe what you hear and explain why this is what you would expect.
  Frequencies below the Nyquist frequency (4kHz) are allowed to pass through the system without aliasing.
  100 Hz: A clear distinct, low-tone, sine wave is heard and a corresponding peak observed.
  200 Hz: A clear higher-pitched 200Hz sound is heard and a corresponding peak observed.
  400 Hz: An even higher-pitched sound is heard and a corresponding peak observed.
  800 Hz:An even higher-pitched sound is heard and a corresponding peak observed.
  For frequencies above 4kHz, we expect aliasing to occur, however the anti aliasing filter with a cutoff of 4kHz, blocks most of the higher frequencies, and stops them before the downsampling block. 7900

Hz: A very silent and almost attenuated sound is heard.
7800 Hz: A very silent and almost attenuated sound is heard. The sound produced is almost absent and close to imperceptible.
7600 Hz:There is little to no output sound since the sound is very weak.
7200 Hz:There is little to no output sound since the sound is very weak.

- Deliverable 6: Explain what is the maximum frequency that will pass through the system undistorted, and how this compares to your answer for Exercise 1.
  The maximum frequency which would pass through the system undistorted is 4kHz. This is the frequency at which the anti-aliasing filter begins to attenuate the signal, allowing only frequencies at or below 4kHz pass through without significant distortion. When the input signal is set to 4kHz, a clear peak is seen on the analyzer. After increasing the frequency by a small amount, the anti-aliasing filter starts filtering and removing higher frequencies, resulting in a weaker or absent output signal that could be slightly distorted.

- Deliverable 7: Describe your observations about the quality of the output signal, and how they differ from your observations from Exercise 1.
  With the anti-aliasing filter, the quality of the output sound is greatly improved. There is no distortion, and the background noise has been significantly removed. It is a clearer and more distinct sound.

- Deliverable 8: Explain why the performance of the telephone system would degrade significantly if anti-aliasing pre-filtering were not used and how this filtering prevents these negative effects.
  When signals are sampled digitally (like in telephone systems at 8kHz), frequencies above 4kHz cannot be accurately captured and rather they fold back into lower frequencies, causing distortions. Without anti-aliasing filtering, high frequency components would create distorted, muddy audio. Speech intelligibility would thus suffer due to the noise and interference with critical voice frequencies. The overall quality of phone calls would thus be significantly degraded. The anti-aliasing filter solves this problem by preserving only the frequencies needed for clear voice transmission, ensuring clear consistent distortion-free audio

- Deliverable 9: Include all your 8 graphs in your lab, and relate them to the results you obtained in the previous exercises.
  Image after conclusion. A visual distinction that can be made is that frequencies beneath the Nyquist frequency have a smoother curve since there is no aliasing whereas those with frequncies above the Nyquist frequncy appear to be formed by series of conjoined lines instead of a smooth curve.

3.2 Discussion Questions

- Explain the purpose of each block in the **Sampling_wAliasing** model in Exercise #1.

  Audio Device Reader: This block reads an analog signal from the microphone and converts it into a digital signal by sampling it at a specified rate. In the case, the sampling rate parameter is 48kHz. This high sampling rate ensures that any input is captured in its entirety regardless of high-frequency content, including content that may have a frequency very well above the downsampled rate of 8kHz.

  DownSample Block: This block reduces the sampling rate of the signal, simulating the process of adjusting the sampled signal to a lower rate for transmission. In this case, the downsampling factor is 6. Reducing the sampling rate to 8kHz simulates the standard sampling rate, which has a Nyquist rate of 4kHz. Since the initial signal may have higher frequencis, these high frequencies would alias into lower frequencies when downsampled, causing distortions in the output.

  Subsystem Block to restore Sampling frequency: The subsystem restores the original sampling frequency before sending the signal to the Audio Device Writer for playback. It does this by unsampling

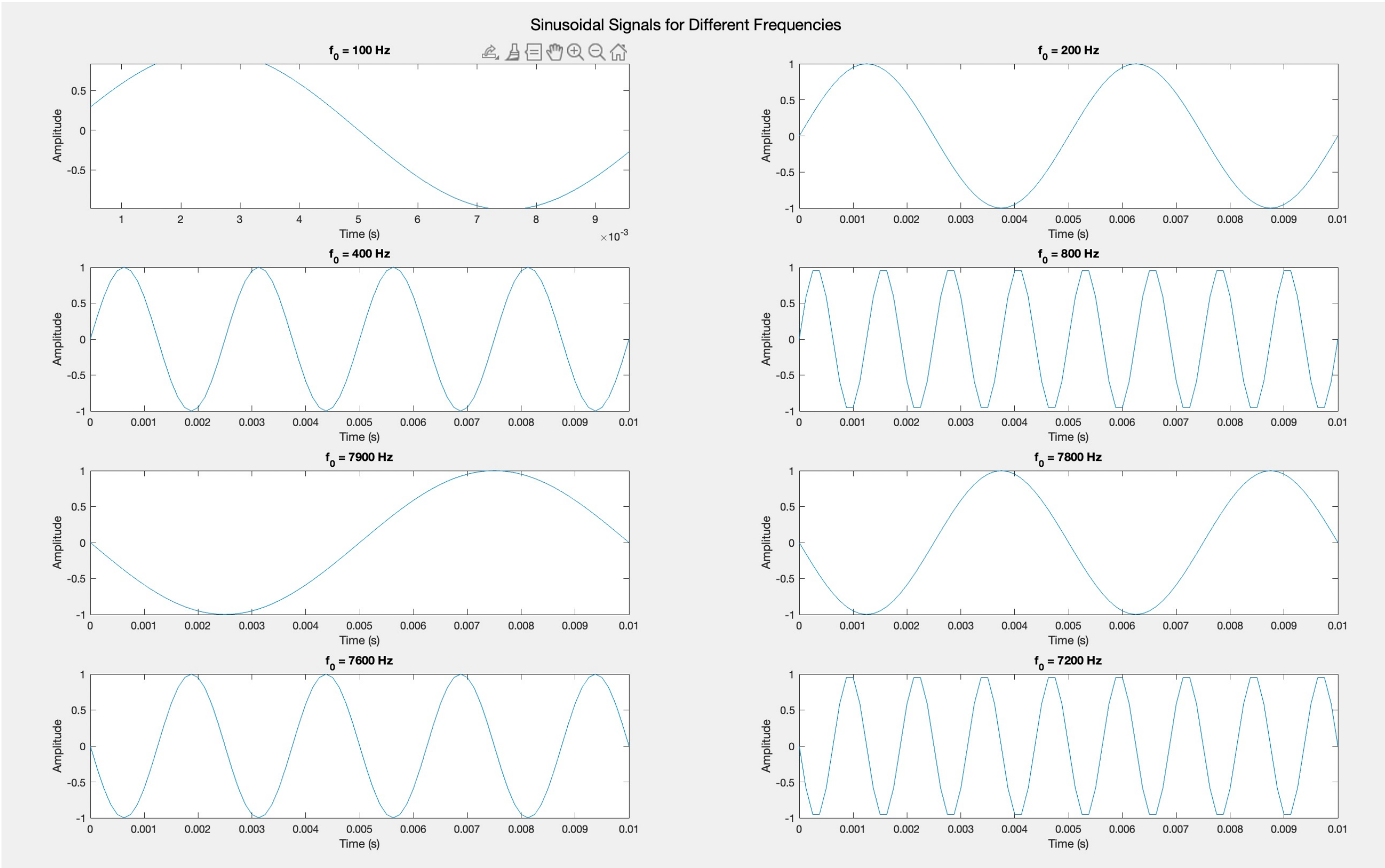


Figure 1:

the downsampled signal back to 48kHz. This is because the audio device writer expects a sampling rate of 48kHz, same as inout signal.

Audio Device Writer: This block converts the digital signal back into analog format for playback through headphones or speakers, allowing users to hear the output of the system.

Spectrum Analyzer: The spectrum analyzer displays the frequency spectrum of the output signal. It allows for a visual interpretation of aliasing effects in the frequency domain.

- Using the plots generated by your Matlab simulation, explain how the system without anti-aliasing works and why it behaves the way it does.
  In the setup, the analog input generated using a waveform generator, is first sampled at 48kHz, to capture all components, including the really high frequencies. The signal is then downsampled to a lower rate, typically 8kHz in this case. Since the system lacks an anti-aliasing filter, any frequency components above the Nyquist frequency of the 8kHZ rate will alias, causing frequencies beyond 4kHz to appear as lower frequency components. The plots show the frewuncy spectrums of a series of input signals sampled at 48kHz, up to 24kHz before downsampling. Post downsampling, the Nyquist frequency is 4kHz. Thus all input signals above 4kHz alias as seen in the plots. They appear as distortions in the graph when analyzed in the time domain, evident in the linear distortion of the presented graphs. Without an anti-aliasing filter, there is no mechanism to limit the input signal's bandwidth thus all frequencies pass through and are indistinguishable.

The code for lab 6 is shown below

```
function [x, t] = sineWave(A, f0, fs)
    % Define the time vector from 0 to 10ms with a spacing of 1/fs
    t = 0 : 1/fs : 0.01;

    % Calculate the sinusoidal signal
    x = A * sin(2 * pi * f0 * t);
end

% Parameters
fs = 8000;       % Sampling rate
A = 1;           % Amplitude

% Frequencies to analyze
frequencies = [100, 200, 400, 800, 7900, 7800, 7600, 7200];

% Plot each frequency
figure;
for i = 1:length(frequencies)
    f0 = frequencies(i);
    [x, t] = sineWave(A, f0, fs);

    % Plot the signal
    subplot(4, 2, i);
    plot(t, x);
    title(['f_0 = ', num2str(f0), ' Hz']);
    xlabel('Time (s)');
    ylabel('Amplitude');
end

% Adjust layout
sgtitle('Sinusoidal Signals for Different Frequencies');

end

    % MATLAB Code for Sampling without Anti-Aliasing
fs_original = 48000; % Original sampling frequency in Hz
fs_downsampled = 8000; % Downsampled frequency in Hz
f_input = [100, 200, 400, 800, 7900, 7800, 7600, 7200]; % Test frequencies

t = 0:1/fs_original:0.05; % Time vector for 50 ms
signal = sin(2 * pi * f_input' * t); % Generate sinusoidal signals

% Plot Original Signal Spectrum
figure;
subplot(2,1,1);
for i = 1:length(f_input)
    spectrum_original = fft(signal(i, :));
    freq = linspace(0, fs_original, length(spectrum_original));
    plot(freq, abs(spectrum_original)); hold on;
end
title('Original Signal Spectrum at 48 kHz Sampling');
xlabel('Frequency (Hz)'); ylabel('Magnitude');
```

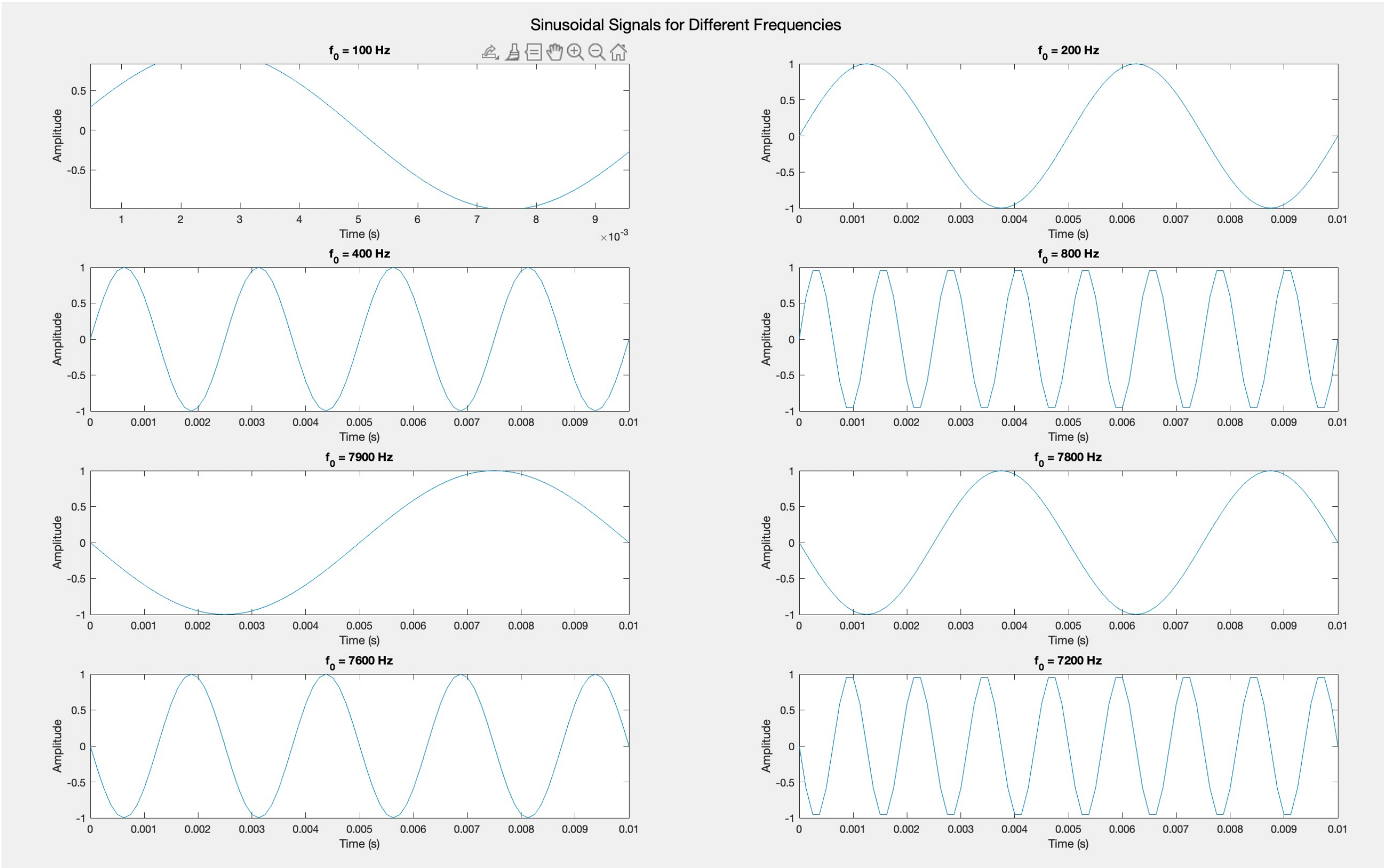
```
% Downsample and plot spectrum
downsampled_signal = downsample(signal', fs_original / fs_downsampled)';
subplot(2,1,2);
for i = 1:length(f_input)
    spectrum_downsampled = fft(downsampled_signal(i, :));
    freq_downsampled = linspace(0, fs_downsampled, length(spectrum_downsampled));
    plot(freq_downsampled, abs(spectrum_downsampled)); hold on;
end
title('Downsampled Signal Spectrum at 8 kHz Sampling');
xlabel('Frequency (Hz)'); ylabel('Magnitude');

% Save figures for report
saveas(gcf, 'Aliasing_Effects.png');
```

- Describe, in your own words, how the **Sampling_AntiAliasing** model from Exercise #2 differs from the **Sampling_wAliasing** model in Exercise #1. Why is the anti-aliasing filter necessary?
  The sampling process in the anti-aliasing model includes an anti-aliasing filter that removes high-frequency components from the input signal before it is downsampled. The filter acts as a low-pass filter, allowing only frequencies that are below a certain cutoff to pass through, effectively limiting the bandwidth of the signal. By contrast, the aliasing model has no filter to remove high-frequency content before downsampling. In the anti-aliasing model, the initial sampling rate is set high (e.g., 48 kHz), which captures the full range of frequencies in the input. The anti-aliasing filter then removes any high-frequency content before the signal is downsampled to the desired lower rate (e.g., 8 kHz).In the aliasing model, the signal is downsampled without filtering, causing frequencies above the new Nyquist frequency (4 kHz in this case) to alias and appear as false lower frequencies, which distort the signal. The anti-aliaising filter is necessary because it prevents aliasing. Without this filter:Frequencies above half the new sampling rate (the Nyquist frequency) are misrepresented as lower frequencies in the sampled signal, creating distortions and artifacts that interfere with the original audio content.The resulting signal quality degrades, and the output may include unwanted sounds or frequencies that were not part of the original signal.In a communication system like a telephone, anti-aliasing is crucial for maintaining clarity and fidelity of the transmitted signal. By filtering out frequencies above the Nyquist limit before downsampling, the anti aliasing model preserves the integrity of the signal and ensures that only the desired frequency range is represented in the sampled data.

- Assume that the input to your system is music (with a highest frequency of 22.05 kHz). Sketch the spectrum of the signal at the specified points in the lab manual:

4 Conclusions

This lab explored the impact of aliasing in digital signal processing, specifically focusing on how a continuous-time signal is affected when sampled without proper filtering. By examining and comparing two models—one without an anti-aliasing filter and one with an anti-aliasing filter—we observed aliasing effects and understood how they distort the signal when frequencies above the Nyquist rate are present. In the model without anti-aliasing, high-frequency components of the input signal alias into lower frequencies, causing distortions. The anti-aliasing model, however, includes a low-pass filter that removes these high frequencies before downsampling, effectively preventing aliasing and preserving signal integrity. The lab emphasizes the importance of anti-aliasing pre-filtering in digital processing applications, such as telecommunications, to ensure clarity and fidelity in transmitted audio signals. Through hands-on exercises with frequency adjustments, spectral analysis, and simulations, students gain practical insights into sampling theory and anti-aliasing techniques.



Sinusoidal Signals for Different Frequencies
f0 = 100 Hz
f0 = 200 Hz
f0 = 400 Hz
f0 = 800 Hz
f0 = 7900 Hz
f0 = 7800 Hz
f0 = 7600 Hz
f0 = 7200 Hz
Amplitude
Time (s)

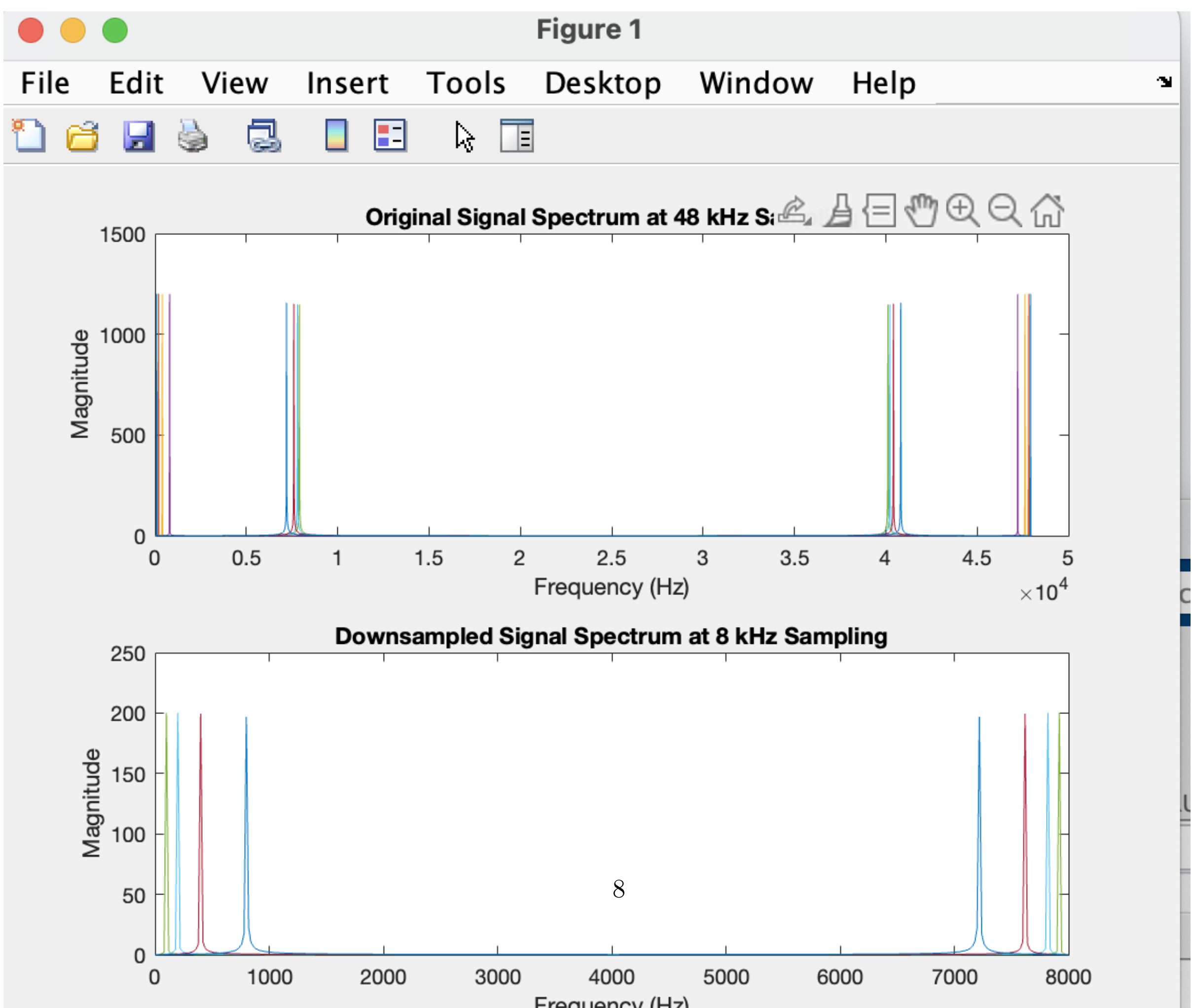


Figure 1
File Edit View Insert Tools Desktop Window Help
Original Signal Spectrum at 48 kHz Sa
Magnitude
Frequency (Hz)
×10^4
Downsampled Signal Spectrum at 8 kHz Sampling
Magnitude
Frequency (Hz)

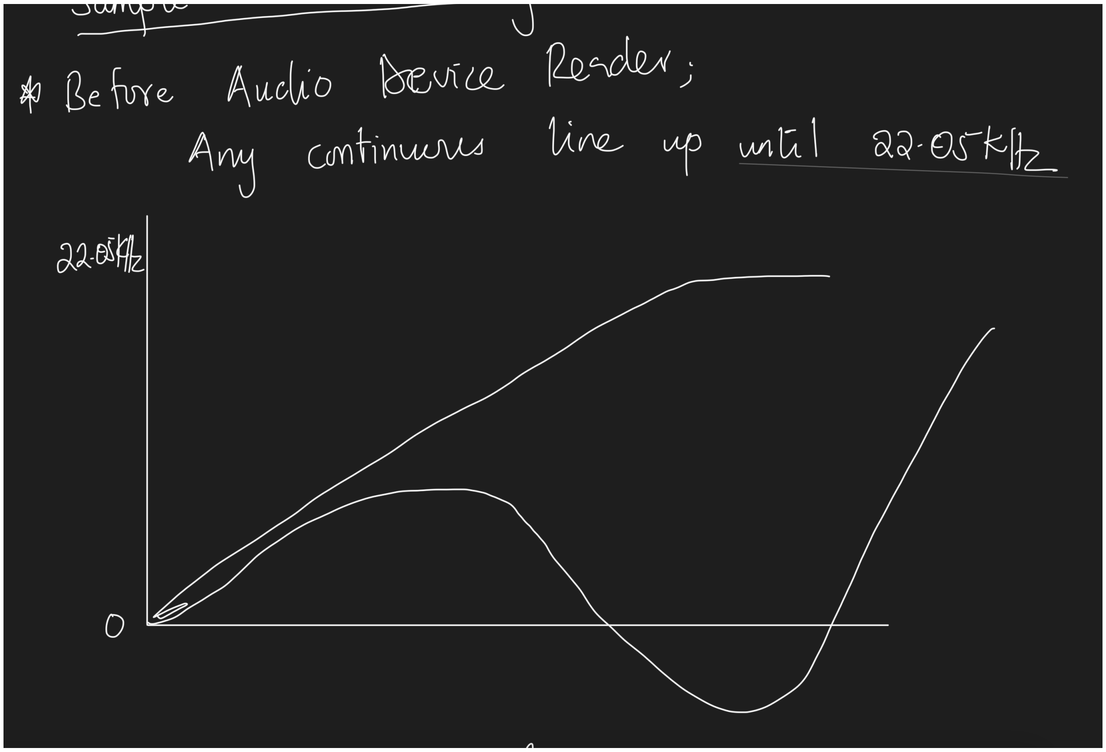


Figure 3:

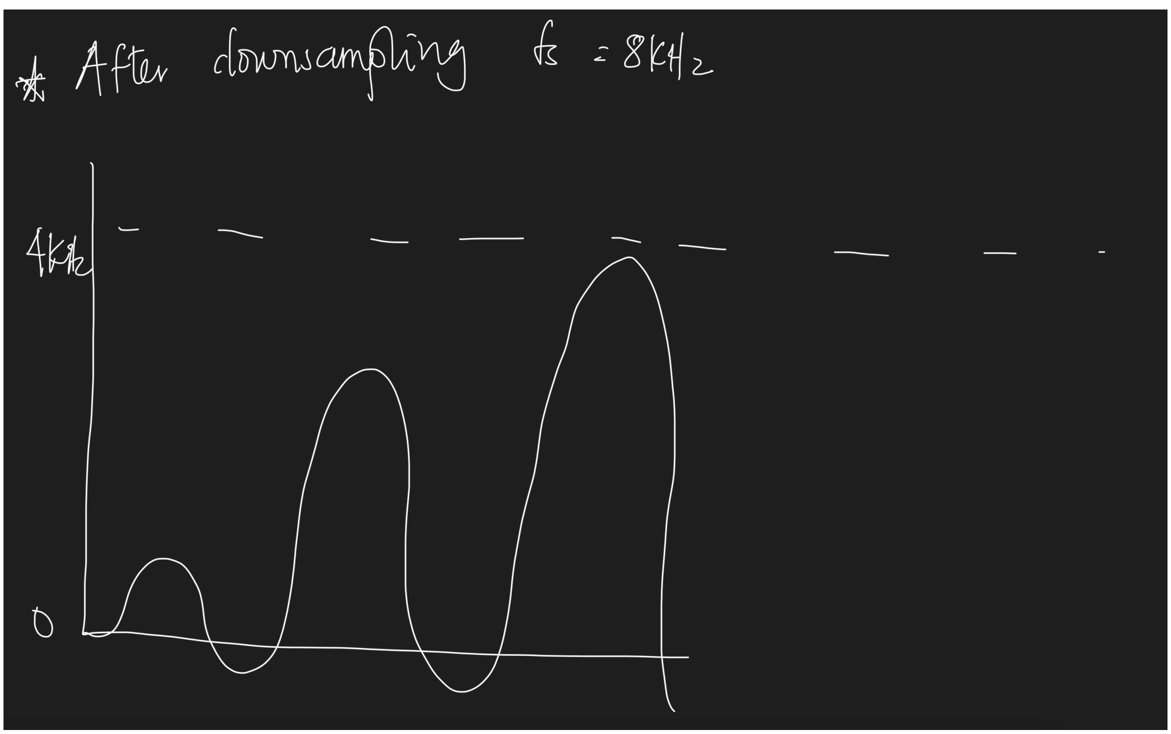


Figure 4:

Figure 5:

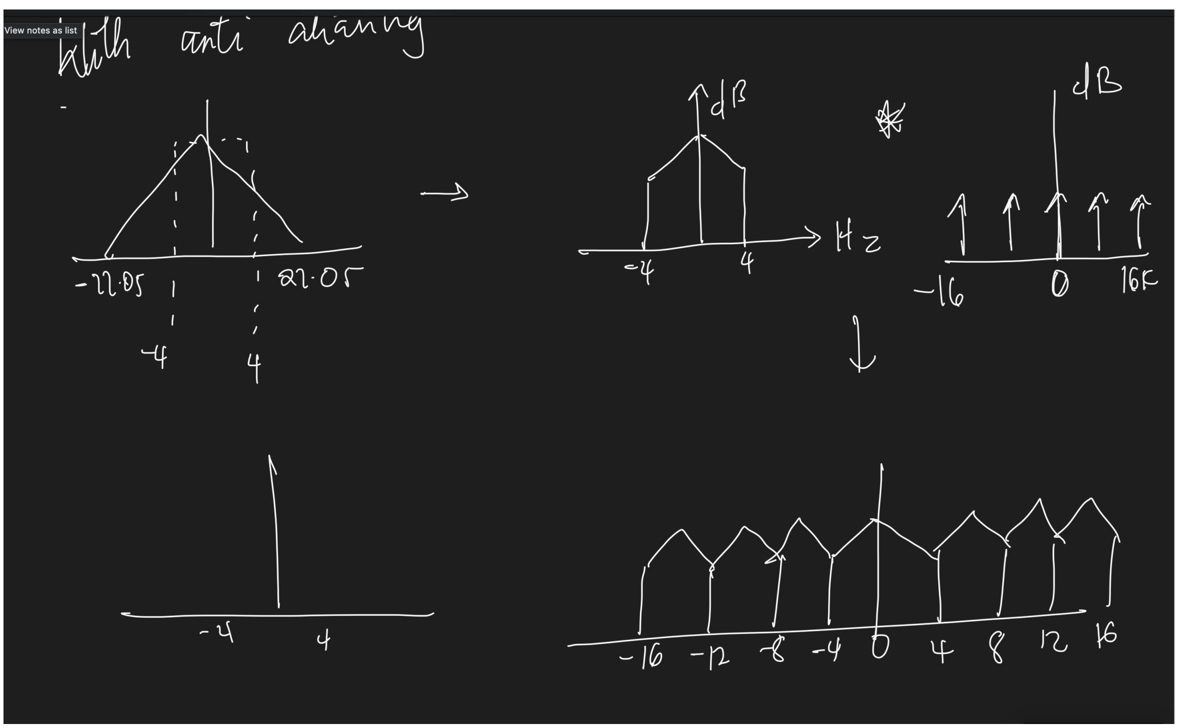


Figure 6:

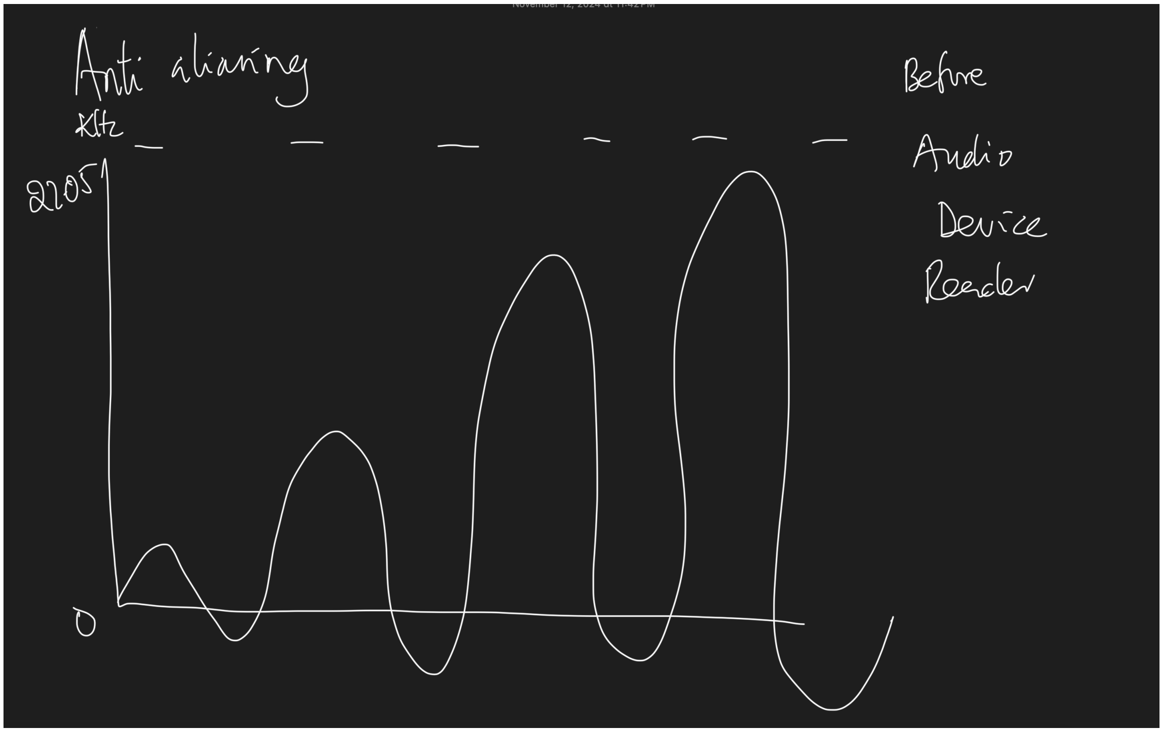


Figure 7:

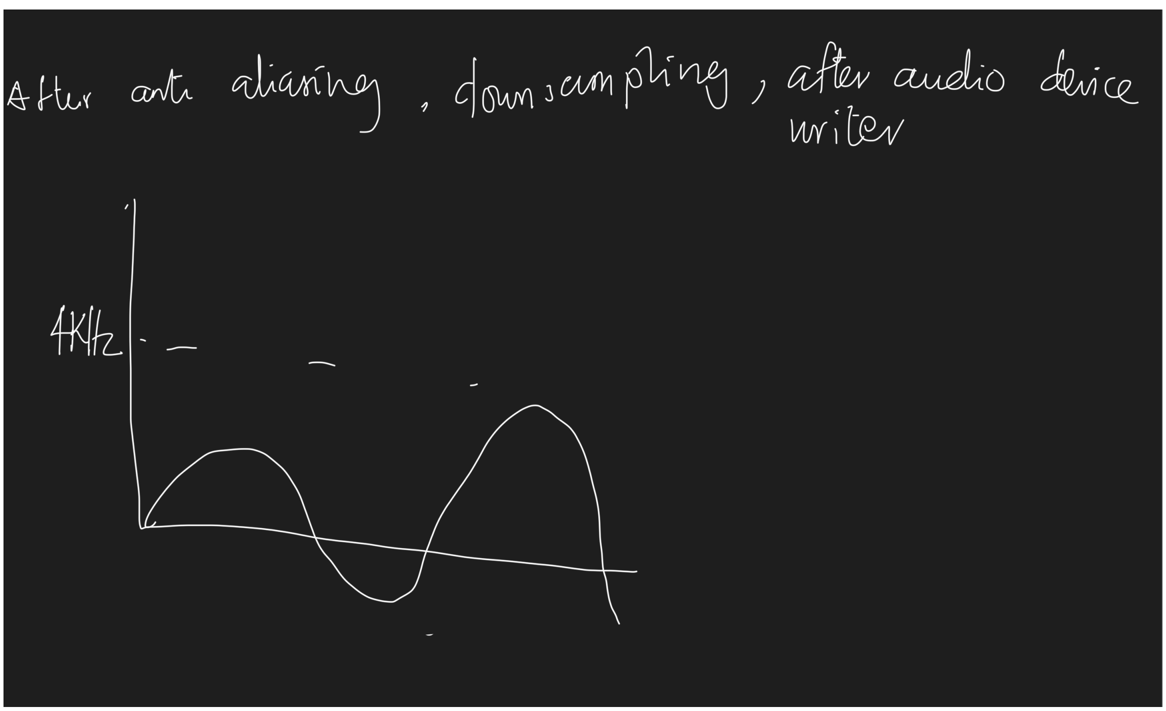


Figure 8:

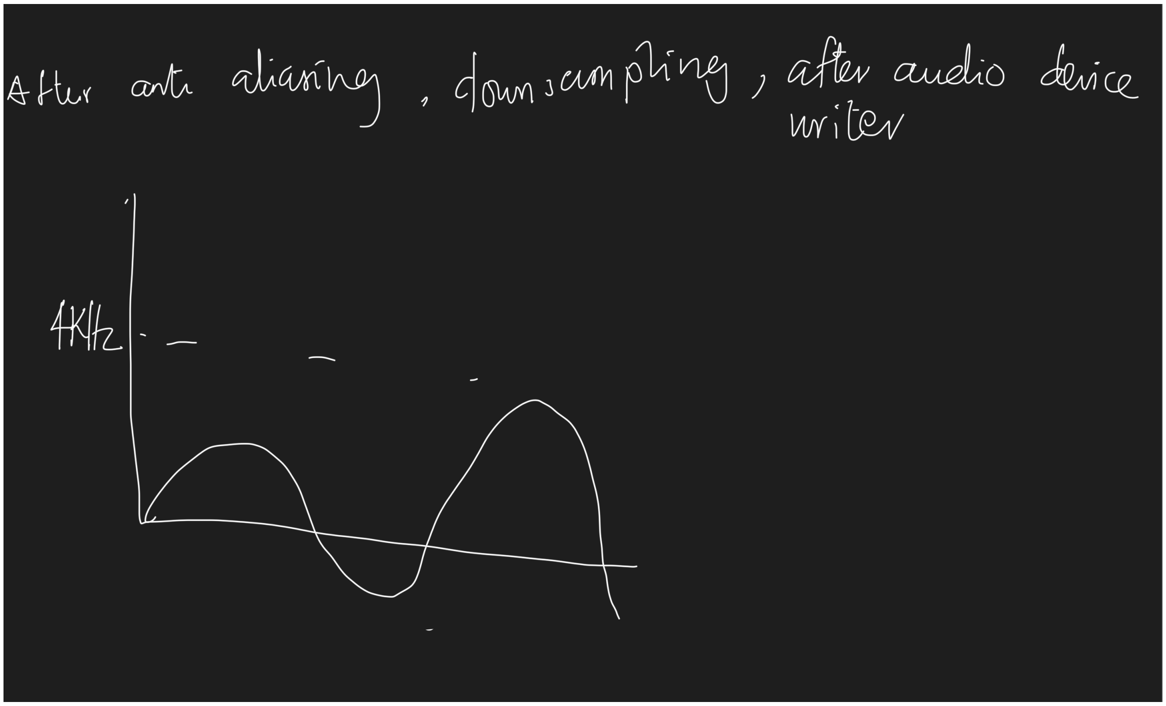


Figure 9:

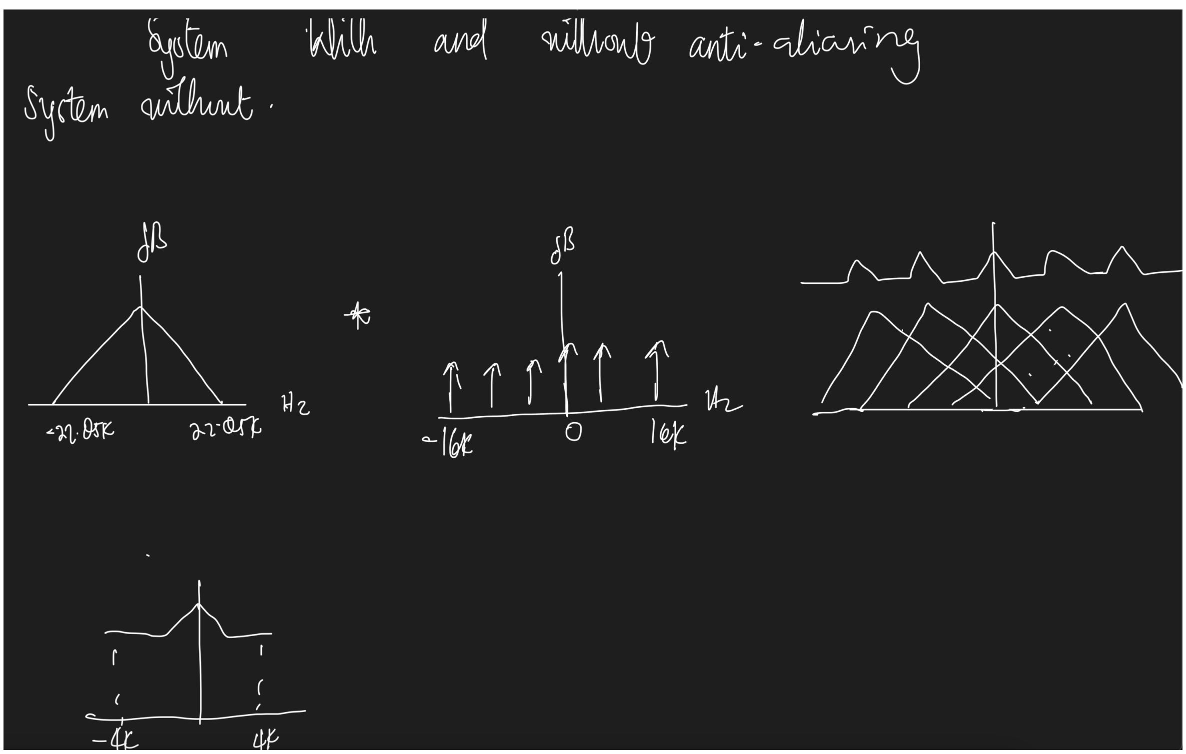


Figure 10: