ECE 280L Fall 2024

Laboratory 6:
Sampling and Aliasing

Contents

1 Introduction

In this project, you will investigate the effects of aliasing that can arise when digitally processing a continuous-time signal. You will make theoretical, visual, and aural observations that illustrate these effects and demonstrate how an anti-aliasing filter can be used to avoid or minimize these unwanted effects.

2 Objectives

1. To examine (visually and aurally) the effects of aliasing in the sampling of continuous-time signals
2. To examine (visually and aurally) the effects of an anti-aliasing filter when sampling continuous-time signals
3. To be able to explain why anti-aliasing pre-filtering is often necessary for digital processing of continuous-time signals.

3 Background

In order to digitally process any continuous-time signal, that signal must be converted into a digital signal through sampling, to create discrete time samples, and quantization, to create discrete amplitudes. Both quantization and sampling can introduce errors, or distortion, into the signal being processed. These errors become evident when, after processing, the digital signal is converted back to an analog signal through a process known as reconstruction.

There are many applications in which a continuous-time signal is processed digitally. One interesting application is the telephone system. During a telephone conversation, the digital telephone network transports streams of sampled voice signals between the phone exchanges to which the two connected handsets are attached. Voice signal transmission between the phone exchange and telephone handsets is normally done in analog form. At the central phone exchanges, the required analog-to-digital conversion (sampling and quantization) and digital-to-analog conversion (reconstruction) are performed.

Interestingly, the design of the telephone system was based on the fact that a large fraction of human voice energy is typically below about 3.5 kHz. Assuming 4 kHz just to be safe, the Nyquist sampling rate is 8000 samples-per-second (i.e., $f_s = 2$ x 4 kHz), which is the standard sampling rate used in the telephone system. At each sampling instant, an 8-bit sample is taken, which results in a total bit rate of 64 kbps (i.e., 8 bits x 8 kHz) for each one-way voice stream.

This design may work well for a typical conversation between two people, but what do you think would happen if something other than speech (e.g., music) was transmitted over the phone system? If the signal contained frequencies above 4 kHz, you would get aliasing and the resulting distortion might make it very difficult to communicate. Thus, an additional step of processing is needed to ensure that aliasing is avoided. In each telephone exchange, the incoming analog voice signal is first low-pass filtered to reduce the frequency components above 3.5 kHz. Following this, the signal is sampled as discussed above, and transported to its destination telephone.

In this lab, you will first explore what would happen if this pre-filtering operation were not in place and, therefore, if aliasing might occur. Then, the pre-filtering will be put in place, and you will make comparisons between the two systems.

4 Equipment Needed

- PC with Audio System Toolbox, MATLAB, and Simulink
- Microphone, headphones, and speakers
- Music files

5 Pre-Laboratory Assignment

Based on the Background information provided, answer the following questions on Gradescope:

1. You have a machine that samples incoming electrical signals at a frequency of 2kHz. If you send the signal $x(t) = \cos(2000\pi t)$ to this machine, which plot in Figure 1 would best represent the machine's interpretation of the signal?

2. To reduce the power consumption of your machine from Question 1, you lower the sampling frequency to 1kHz and send the same signal $x(t) = \cos(2000\pi t)$. Which plot in Figure 1 would now best represent the machine's interpretation of the signal?

3. Based on your observations from Questions 1 and 2, what is the Nyquist sampling frequency for the signal $x(t) = \cos(2000\pi t)$?

4. A different machine allows you to manually adjust its sampling frequency to any value between 1Hz and 200MHz. To optimize power consumption and minimize SSD storage usage, you are advised to set its sampling frequency to the Nyquist frequency for an incoming signal with a frequency of 64kHz. What should you set the machine's sampling frequency to?

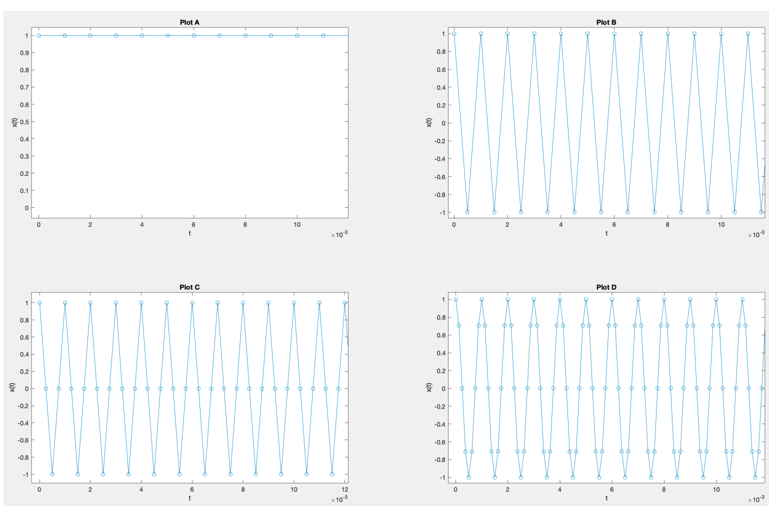


Figure 1. Plots for Questions 1 and 2

6 Instructions

We will begin by considering the effects of aliasing on a sinusoid which is sampled and transmitted through the phone system. (You can use your imagination to assume that the person on one end of the phone conversation hums various sinusoids into the handset. Later in the lab, you will use actual speech to study the system.) We are interested in what emerges at the other telephone.

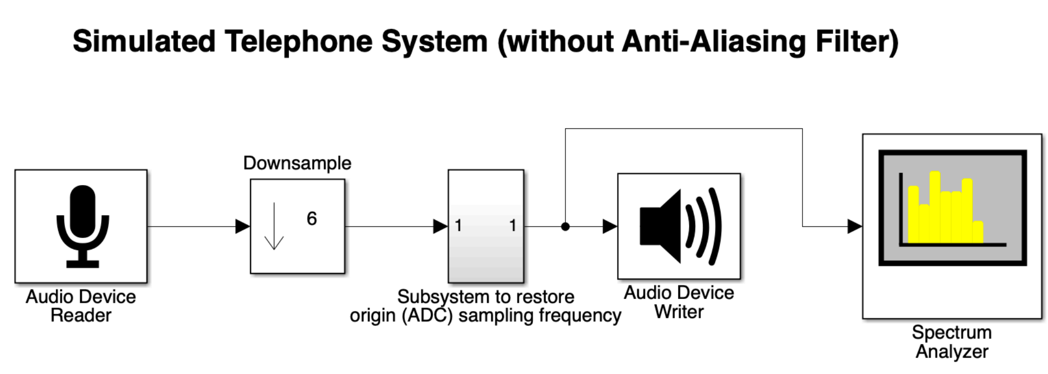
6.1 Exercise #1: Exploring Aliasing Effects (no pre-filtering)

This system samples the incoming signal (via the **Audio Device Reader**) at a 48 kHz rate. This rate satisfies the Nyquist criterion for any input signal we will use. Thus, there will be no aliasing at this point.

We will consider this sampled signal to effectively be our analog signal. This is followed by the **Downsampling** block which samples our "analog" signal by taking 1 out of every 6 samples. Thus, we have effectively sampled the original signal by $48/6 = 8$ kHz, which is our desired sampling rate. The reason we must do this two-step process is because the computer's sound card automatically applies an anti-aliasing filter (which you will explore further in Exercise #2) to the incoming signal as part of the **Audio Device Reader**.

Since we are interested in hearing what happens when aliasing occurs, we force aliasing to occur. The downside to doing this is that the **Audio Device Writer** needs a signal with the same sampling rate as the **Audio Device Reader** (i.e., 48 kHz). Thus, a subsystem has been added to restore the original sampling frequency (essentially by upsampling the signal) before the signal is sent to the **Audio Device Writer**. The details of this process are beyond the scope of ECE 280, so you can treat the subsystem in the model as a black box.

1. Load the SIMULINK model, `Sampling_wAliasing_AudioToolbox_a.slx`, which has been provided for you. The model is displayed below:



Discussion (1/8): What is the purpose of each block in the model?

2. For this exercise, you are going to use the function generator to produce sinusoidal signals that will be the input to the phone system (via LINE IN). Set the function generator so that a sinusoid with a frequency of 100 Hz and an amplitude of 1.0-Vpp is produced. Verify your signal by viewing it on the oscilloscope.
   NOTE: Make sure to set the amplitude to 1.0Vpp. Anything higher than that could potentially fry the computer.

3. Connect the output of the function generator (which should be generating a 100 Hz sine wave) to the microphone in jack on the front of the computer. Plug in your headphones to the headphones jack on the front of the computer.

4. Double-Click on the **Spectrum Analyzer**. You should now be able to both hear and see the output signal.

   Discussion (2/8): What do you expect to hear/see? Is this actually the case?

5. Now, vary the input signal so that the frequency of the sinusoid is:

   (a) 100 Hz
   (b) 200 Hz
   (c) 400 Hz
   (d) 800 Hz
   (e) 7900 Hz
   (f) 7800 Hz
   (g) 7600 Hz
   (h) 7200 Hz

   Discussion (3/8): What do you hear/see at the output for each frequency?
   Deliverable (1/9): For each frequency, describe what you hear and explain why this is what you would expect.

6. Decide what is the maximum frequency that will pass through the system without aliasing. Verify your answer by changing the frequency of your sinusoidal input signal and viewing the spectral analyzer.

   Checkpoint (1/3): Discuss with your TA what is the maximum frequency that will pass through the system without aliasing.
   Deliverable (2/9): In your lab report, explain what is the maximum frequency that will pass through the system without aliasing.

7. Change the audio device reader block to a **From Multimedia File** block and set the Samples per Audio Channel to 60. Set the File Name to an audio file (Audio File is available on Canvas).

   Discussion (4/8): What do you observe about the quality of the output signal? Why is this the case?
   Deliverable (3/9): Include your observations in your lab report.

6.2 Exercise #2: Explore the effect of an anti-aliasing filter

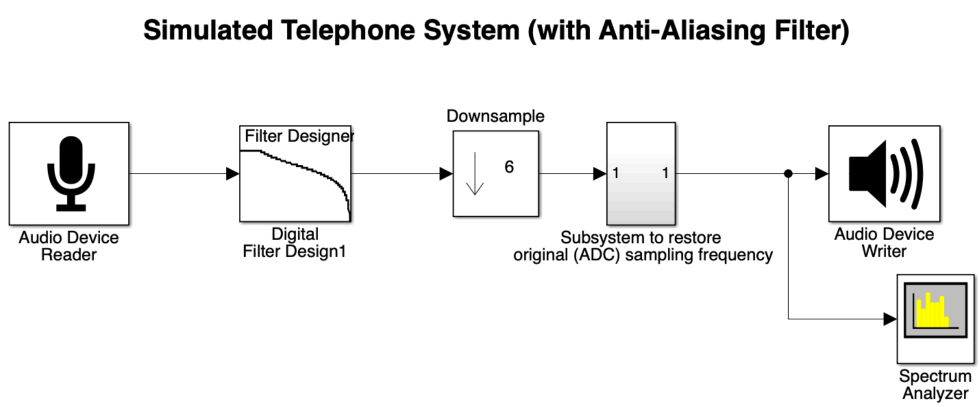
For this exercise, you are going to essentially repeat Exercise #1 using a different system model that incorporates anti-aliasing pre-filtering.

The most important difference between this model and the one you built in Exercise #1 is the addition of the anti-aliasing filter. For anti-aliasing to work, the filter must be applied to the analog signal before it is sampled.

However, since the **Audio Device Reader** performs sampling and we have no way to insert a block prior to the **Audio Device Reader**, we must use a trick to simulate filtering the analog signal. As in Exercise #1, we do this by using a higher sampling rate in the **Audio Device Reader** (48 kHz in this case) than our ultimate desired sampling rate (8 kHz). This rate satisfies the Nyquist criterion for any input signal we will use. Thus, there will be no aliasing at this point. We will consider this sampled signal to effectively be our analog signal.

Now, we apply the anti-aliasing filter to limit the bandwidth of our "analog" signal. This is followed by the **Downsampling** block which reduces the sampling rate to 8 kHz, which is our desired sampling rate. Again, we must include a subsystem to restore the original sampling frequency (of 48 kHz) before sending the signal to the **Audio Device Writer**. Finally, the sampled signal, with anti-aliasing pre-filtering, is reconstructed via the **Audio Device Writer**, as before.

1. Load the SIMULINK model, `Sampling_AntiAliasing_AudioToolbox_a.slx`, which has been provided for you. The model is displayed below:



2. Double click on the blocks of this model and analyze how this model works.

   Discussion (5/8): How does this model work? And how does it compare to the model from Exercise #1?

3. Double click on the **Digital Filter Design** block, and analyze its parameters.

   Deliverable (4/9): In your lab report, explain what is the cutoff frequency of the filter, how this frequency is appropriate for the telephone system application, and how the cutoff frequency of an anti-aliasing filter should generally be chosen.

4. Run the model and use the spectrum analyzer to analyze the frequency domain spectrum.

5. Using the function generator, create sinusoidal signals with frequencies:

   (a) 100 Hz
   (b) 200 Hz
   (c) 400 Hz
   (d) 800 Hz
   (e) 7900 Hz
   (f) 7800 Hz
   (g) 7600 Hz
   (h) 7200 Hz

   Discussion (6/8): What do you hear/see at the output for each frequency?
   Deliverable (5/9): For each frequency, describe what you hear and explain why this is what you would expect.

6. Decide what is the maximum frequency that will pass through the system undistorted. Verify your answer by changing the frequency of your sinusoidal input signal and viewing the spectral analyzer.

   Checkpoint (2/3): Discuss with your TA what is the maximum frequency that will pass through the system undistorted.
   Deliverable (6/9): In your lab report, explain what is the maximum frequency that will pass through the system undistorted, and how this compares to your answer for the system in Exercise #1.

7. Change the audio device reader block to a **From Multimedia File** block and set the Samples per Audio Channel to 60. Set the File Name to an audio file (Audio File is available on Canvas).

   Discussion (7/8): What do you observe about the quality of the output signal? Why is this the case?
   Deliverable (7/9): Include your observations in your lab report, nad explain how they differ from what you observed in Exercise #1

8. Now, think about the applications of anti-aliasing in telephone systems.

   Discussion (8/8): Based on what you have observed, discuss why the performance of the telephone system would degrade significantly if anti-aliasing pre-filtering were not used.
   Deliverable (8/9): In your lab report, explain why the performance of the telephone system would degrade significantly if anti-aliasing pre-filtering were not used and how this filtering prevents these negative effects.

6.3 Exercise #3: MATLAB Simulation

For this exercise, you will write a MATLAB function that produces samples of the sinusoidal signal $x(t) = A\sin(2\pi f_0 t)$. In this function, A is the amplitude of the signal, and $f0$ is the frequency of the signal. The samples are to be taken at periodic intervals of the sampling period, $T_s = 1/f_s$, where f_s is the sampling rate.

1. Begin by by creating a function named sineWave that take as inputs A, f0, and fs, and returns two vectors t, and x. This can be done as:

   function [x,t] = sineWave(A, f0, fs)

2. In your function, set x to be $A\sin(2\pi f0t)$, and t to be a time vector from 0 to 10ms with a space of 1/fs between points.

3. Use the function you wrote to plot the signals you analyzed in Exercise #1. (i.e., let f_s = 8000 Hz, A = 1, and f_0 = 100, 200, 400, 800, 7900, 7800, 7600, and 7200 Hz).

   Checkpoint (3/3): Show your 8 graphs to your TA.
   Deliverable (9/9): Include all your 8 graphs in your lab, and relate them to the results you obtained in the previous exercises.

7 Deliverables Summary

For your lab report **Results and Discussion** section, you should answer all the 9 deliverables listed in the Instructions:

1. **(3 Points)** #1: For each frequency, describe what you hear and explain why this is what you would expect.
2. **(3 Points)** #1: Explain what is the maximum frequency that will pass through the system without aliasing.
3. **(3 Points)** #1: Describe your observations about the quality of the output signal, and why this is the case.
4. **(3 Points)** #2: Explain what is the cutoff frequency of the filter, how this frequency is appropriate for the telephone system application, and how the cutoff frequency of an anti-aliasing filter should generally be chosen.
5. **(3 Points)** #2: For each frequency, describe what you hear and explain why this is what you would expect.
6. **(3 Points)** #2: Explain what is the maximum frequency that will pass through the system undistorted, and how this compares to your answer for Exercise #1.
7. **(4 Points)** #2: Describe your observations about the quality of the output signal, and how they differ from your observations from Exercise #1.
8. **(4 Points)** #2: Explain why the performance of the telephone system would degrade significantly if anti-aliasing pre-filtering were not used and how this filtering prevents these negative effects.
9. **(4 Points)** #3: Include all your 8 graphs in your lab, and relate them to the results you obtained in the previous exercises.

8 Discussion Questions

In your lab report you should also answer the following questions:

- **(10 Points)** Explain the purpose of each block in the `Sampling_wAliasing` model in Exercise #1. Be sure to explain the value assigned to each important parameter (e.g., the sampling rate of the Audio Device Reader). You may ignore the details of the Subsystem block.
- **(20 Points)** Using the plots generated by your MATLAB simulation, explain how the system (without anti-aliasing) works and why it behaves the way it does. Please append your MATLAB code and figures to the end of your report.
- **(10 Points)** Describe, in your own words, how the `Sampling_AntiAliasing` model (from Exercise #2) differs from the `Sampling_wAliasing` model in Exercise #1. (You may ignore the Subsystem block.) Why is the anti-aliasing filter necessary?
- Assume that the input to your system is music (with a highest frequency of 22.05 kHz). Sketch the spectrum of the signal at the following points in your models:
  - **(5 Points)** `Sampling_wAliasing` Model:
    * Before Audio Device Reader (i.e., original input spectrum)
    * After Downsampling (i.e., when sampled using $f_s = 8$ kHz)
    * After Audio Device Writer (assume Ideal Bandlimited Reconstruction)
  - **(5 Points)** `Sampling_AntiAliasing` Model:
    * Before Audio Device Reader (i.e., original input spectrum)
    * After anti-aliasing filter
    * After Downsampling (i.e., when sampled using $f_s = 8$ kHz)
    * After Audio Device Writer (assume Ideal Bandlimited Reconstruction)

9 Lab Report

You may work in teams on the lab but you must INDIVIDUALLY submit lab reports. Your lab report will be graded as follows:

9.1 Objectives (5)

Summarize the stated objectives in your own words.

9.2 Background (10)

- Briefly describe sampling and aliasing and describe the steps taken to transmit information via the standard telephone system.
- Briefly discuss one example (other than the telephone system described in the Background section) of an application where a continuous-time signal is processed digitally.

9.3 Results and Discussion (80)

- Include all the **9 Deliverables**.
- Answer all questions in the **Discussion Questions** section.

9.4 Conclusions (5)

Summarize what you have learned through this laboratory exercise.

Include the statement "I have adhered to the Duke Community Standard in completing this assignment", along with your signature, on the cover page and properly cite any sources you reference in your written report. Each group should turn in ONE report.

10 References

11 Updates

- October 2024: Added a prelab; made multiple adjustments based on recommendations of Jenny Green (Pratt '25), Eduardo Bortolomiol (Pratt '26), and Adam Davidson; eliminated extension and distributed points to required sections.
- June 2024: Converted to a standalone document
- October 2021: Converted to LaTeX.
- Fall 2020: Updated to MATLAB 2020a.