

Mathematics problems.

① ②

1.1

$$\frac{y^{58}}{y^4 \cdot y^{12}} = \frac{y^4 \cdot y^{12} \cdot y^{42}}{y^4 \cdot y^{12}} = y^{42}$$

1.2

$$8^2 \cdot 2^x = 2^9$$

$$2^6 \cdot 2^x = 2^9$$

$$x = 3$$

1.3

$$\frac{x}{y} = 3 \quad x^{-2}y^2 = ?$$

$$(xy^{-1})^2 = (3)^2$$

$$(x^2y^{-2}) = (9)^1$$

$$x^{-2}y^2 = \frac{1}{9}$$

1.4

$$\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \frac{\sqrt{2^{13}}}{\sqrt{2^3 \cdot 2^3 \cdot 2^3}} = \sqrt{\frac{2^{13}}{2^9}} = \sqrt{2^4} = 2^2 = 4$$

1.5

a) true

b) true

c) true

d) true.

1.6

$$\frac{x^2 - 25}{x - 5} = 3$$

$$\frac{(x-5)(x+5)}{(x-5)} = 3$$

$$x+5=3$$

$$x = -2$$

2.1.

$$0K = -460F$$

$$1000K = 1340F$$

$$K = a + bF$$

$$0 = a + b(-460) \quad (1)$$

$$1000 = a + b(1340) \quad (2)$$

$$(1)-(2) \quad -1000 = 0 - 1800b$$

$$b = \frac{1000}{1800} = \frac{5}{9}$$

$$(1) \quad 0 = a + \left(\frac{5}{9} \cdot -460\right)$$

$$a = 255,55$$

$$K = 255,55 + \frac{5}{9}F$$

$$F = \frac{9}{5}K - 460$$

$$F - \frac{9}{5}F = -460$$

$$-\frac{4}{5}F = -460$$

$$F = 460 \cdot \frac{5}{4} = 575$$

2.2. $f(x) = 2x + 3$

$$f(y) = 17$$

$$f(y) = 2x + 3 = 17$$

$$2x = 14$$

$$x = 7$$

2.3.

$$3^{2x^2-4x+3} = 27$$

$$3^{2x^2-4x+3} = 3^3$$

$$2x^2-4x+3=3$$

$$2x^2-4=0$$

$$2x \cdot (x-2) = 0$$

$$\downarrow$$

$$x=0$$

$$\downarrow$$

$$x=2$$

$$S = \{0, 2\}$$

2.4.

$$1,5 = (1+0,01)^m$$

$$\ln(1,5) = \ln(1,01)^m$$

$$\ln(1,5) = m \ln(1,01)$$

$$m = \frac{\ln(1,5)}{\ln(1,01)} = 40,75 \text{ years.}$$

$$2.5 \quad \ln\left(\frac{e^2}{e^3}\right) = \ln e^2 - \ln e^3$$

$$= 2 - 3 = -1$$

$$3.1. \sum_{i=0}^{\infty} \left(\frac{1}{6^i} + 0.25^i \right) = \left(\frac{1}{6} \right)^i + \left(\frac{1}{4} \right)^i$$

$$i=0 \Rightarrow 2$$

$$i=1 \quad \frac{1}{6} + \frac{1}{4} = \frac{11}{24} + \frac{6}{24} = \frac{17}{24} = \frac{5}{12}$$

$$i=2 \quad \left(\frac{1}{6} \right)^2 + \left(\frac{1}{4} \right)^2 = \frac{1}{36} + \frac{1}{16} = \frac{13}{144}$$

$$i=3 \quad 0.0202$$

$$i=4 = 0.0046$$

$$i=5 = 0.0011$$

$$\text{Sum at } i=5$$

$$= 2.532$$

$$3.2 \quad \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{0}{0}$$

$$\lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)} = 6$$

$$3.3. \quad f(x) = x^3 - 4 \quad (-1, -5)$$

$$f(x)' = 3x$$

$$f'(-1) = 3 \cdot (-1)^2 = 3$$

$$f'(-5) = 3 \cdot (-5)^2 = 75$$

$$3.4. \left(\frac{x^2+3}{x+2} \right)' = \frac{(x^2+3)' \cdot (x+2) - (x+2)' \cdot (x^2+3)}{(x+2)^2} \quad (3)$$

$$= \frac{2x \cdot (x+2) - 1 \cdot (x^2+3)}{(x+2)^2}$$

$$= \frac{2x^2 + 4x - x^2 - 3}{(x+2)^2}$$

$$= \frac{x^2 + 4x - 3}{(x+2)^2}$$

$$3.5. f(x) = (x^7 + 4x^2)''$$

$$= (7x^6 + 8x)'$$

$$= 42x^5 + 8$$

$$3.6. f(x) = \frac{x^4 + 4^x}{\ln x}$$

$$= \frac{(x^4 + 4^x)' \cdot (\ln x) - (\ln x)' \cdot (x^4 + 4^x)}{(\ln x)^2}$$

$$= \frac{(4x^3 + 4^x \ln 4) \ln x - \frac{1}{x} \cdot (x^4 + 4^x)}{(\ln x)^2}$$

$$3.7. f(x) = 3x^3 - 9x$$

$$f'' = 18x$$

$$f'(x) = 9x^2 - 9$$

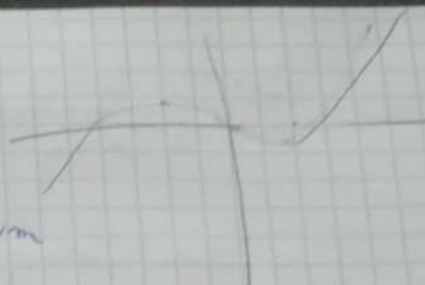
$$9(x^2 - 1)$$

$$x_1 = 1$$

$$x_2 = -1$$

$$f'(1) = 18$$

$$f''(-1) = -18$$



$x_1 = 1 \Rightarrow$ local minimum

$x_2 = -1 =$ local maximum

$$3.8 \quad f(x, y) = x^2 + 2y^3$$

$$f(x, y) = 2^2 + 2 \cdot 3^3 = 4 + 2 \cdot 27 = 58.$$

$$3.9. \quad f(x, y) = \ln(2x - y) > 0$$

$$2x - y > 0$$

$$2x > y$$

$$x > \frac{y}{2}$$

$$3.10. \quad f(x, y) = x^5 e^y + x^2 y^3$$

$$\frac{\partial f(x, y)}{\partial x} = e^y 5x^4 + 2xy^3$$

$$\frac{\partial f(x, y)}{\partial y} = e^y x^5 + 3y^2 x^2$$

3.11.

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$$f(x,y) = \sqrt{xy} - 0,7x - 0,7y$$

$$(xy)^{0,5} - 0,7x - 0,7y$$

$$f(x,y)' = 0,5 x^{-0,5} y^{0,5} - 0,7 = 0$$

3.12.

$$x^2 y^2$$

$$x+y=10$$

$$x+y-10=0$$

$$L = x^2 y^2 - \lambda(x+y-10)$$

$$\frac{\partial L}{\partial x} = 2xy^2 - \lambda$$

$$2xy^2 = \lambda$$

$$\frac{\partial L}{\partial y} = 2yx^2 - \lambda$$

$$2yx^2 = \lambda$$

$$2xy^2 = 2yx^2$$

$$y = x$$

$$x+x=10$$

$$2x=10$$

$$x=5$$

$$y=5.$$

4.1. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \\ 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

2 3	2+6 3+3 2+6
4 1	4+2 16+1 4+2
1 2	1+4 4+2 1+4

$$\begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 8 \\ 5 & 6 & 5 \end{bmatrix}$$

4.2. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

	2 3 1	2 3 1
1 4 1	2+16+1	3+4+2
2 1 2	4+4+2	6+1+4

$$= \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$$

4.3. $\begin{bmatrix} 3,3 & 5,1 \\ 6,1 & 1,23 \\ 45,76 & 0 \end{bmatrix}^T = \begin{bmatrix} 3,3 & 6,1 & 45,76 \\ 5,1 & 1,23 & 0 \end{bmatrix}$

4.4. det.

~~$$\begin{bmatrix} 2 & 3 & 0 & 2 & 3 \\ 4 & 5 & 2 & 4 & 5 \\ 2 & 5 & 3 & 2 & 5 \end{bmatrix}$$~~

$$\begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$$

$$= 30 + 12 + 0 - 0 - 20 - 36$$

$$42 - 56 = -14$$

$$S.1. \Omega = (H, H) (H, T) (T, H) (T, T).$$

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$$S.2. \quad 30 \cdot 28 \cdot 28 = 24360.$$

$$S.3. \quad \begin{array}{c} \textcircled{1} \\ 2 \\ \textcircled{3} \\ 4 \\ \textcircled{5} \\ 6 \end{array} \quad \begin{array}{c} \textcircled{1} \\ 2 \\ \textcircled{3} \\ 4 \\ \textcircled{5} \\ 6 \end{array} \quad \{1, 3, 5\}$$

$$\Omega = 36. \quad (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,3), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5) \\ (3,6), (4,1), (4,3), (4,5), (5,1), (5,2), (5,3), (5,4) \\ (5,5), (5,6), (6,1), (6,3), (6,5) = 27$$

$$P = \frac{27}{36} = \frac{3}{4}.$$