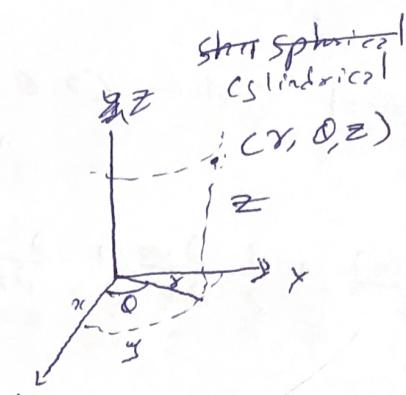


$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

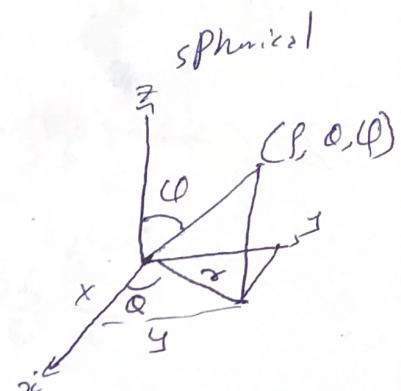
$$z = z$$



$$\cos \theta = \frac{x}{r} = x = r \cos \theta$$

$$r \sin \theta = y$$

$$z = z$$



$$x = \rho \sin \theta \cos \phi$$

$$y = \rho \sin \theta \sin \phi$$

$$z = \rho \cos \theta$$

1. Cartesian Coordinates

i) Vector form

$$\rho \left(\frac{\partial u}{\partial t} + (\mathbf{u} \cdot \nabla) u \right) = -\nabla p + \mu (\nabla^2 u) + \rho g$$

$$+ \nabla \cdot \mathbf{u} = 0$$

ii) Tensor form

$$\frac{\partial u_i}{\partial t} = 0$$

$$\rho \left[\left(\frac{\partial u_i}{\partial t} \right) + u_j \frac{\partial u_i}{\partial x_j} \right] = - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \rho g_i$$

iii) Cartesian form

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y$$

$$\rho \left(\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} + w \frac{\partial \omega}{\partial z} \right) = - \frac{\partial P}{\partial z} + M \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right) + \rho g_z$$

2. Cylindrical System (r, θ, z) (u_r, u_θ, u_z)

continuity:

$$\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

eqn:

$$r \text{-eqn: } \rho \left[\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \right] = - \frac{\partial P}{\partial r} + M \left[\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2u_\theta}{r^2} \frac{\partial u_\theta}{\partial \theta} \right] + \rho g_r$$

$$\theta \text{-eqn: } \rho \left[\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} \right] = - \frac{1}{r} \frac{\partial P}{\partial \theta} + M \left[\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \right] + \rho g_\theta$$

$$z \text{-eqn: } \rho \left[\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + u_\theta \frac{\partial u_z}{\partial \theta} + \frac{u_z}{r} \frac{\partial u_z}{\partial z} + u_z \frac{\partial u_z}{\partial r} \right] = - \frac{\partial P}{\partial z} + M \left[\nabla^2 u_z \right] + \rho g_z$$

where

$$\nabla^2 u_r = \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} \left(\frac{\partial u_r}{\partial r} \right)$$

$$\nabla^2 u_\theta = \frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{1}{r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \left(\frac{\partial u_\theta}{\partial \theta} \right)$$

$$\nabla^2 u_z = \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \left(\frac{\partial u_z}{\partial r} \right)$$

3. Spherical

$$(r, \theta, \phi) \quad u = (u_r, u_\theta, u_\phi)$$

continuity

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta u_\theta)}{\partial \theta} + \frac{1}{r \sin^2 \theta} \frac{\partial u_\phi}{\partial \phi} = 0$$

$$P \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_r}{\partial \phi} - \frac{u_\theta^2 + u_\phi^2}{r} \right) \\ = - \frac{\partial P}{\partial r} + \mu (\nabla^2 u) r + p_{gr}$$

$$\left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi} + \frac{u_\theta u_\phi}{r} \frac{\partial u_\theta}{\partial \phi} + \frac{-u_\theta^2 \cot \theta}{r} \right) = \\ - \frac{1}{r} \frac{\partial P}{\partial \theta} + \mu (\nabla^2 u) \theta + p_{g\theta}$$

$$\frac{\partial u_\phi}{\partial t} + u_r \frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\phi}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_r u_\theta + u_\theta u_\phi}{r} \cot \theta = \\ - \frac{1}{r \sin \theta} \frac{\partial P}{\partial \phi} + \mu (\nabla^2 u) \phi + p_{g\phi}$$

curv.

$$(\nabla^2 u)_r = \cancel{\frac{\partial^2 u_r}{\partial r^2}} + \frac{2}{r} \frac{\partial u_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\cot \theta \partial u_r}{r^2 \partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \\ - \frac{2 u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{2 \cot \theta u_\theta}{r^2 \sin^2 \theta} - \frac{2}{r^2 \sin^2 \theta} \frac{\partial u_\phi}{\partial \phi}$$

$$(\nabla^2 u)_\theta = \frac{\partial^2 u_\theta}{\partial r^2} + \frac{2}{r} \frac{\partial u_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\cot \theta \partial u_\theta}{r^2 \partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\theta}{\partial \phi^2} - \frac{u_\theta}{r^2 \sin^2 \theta} \\ + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{2}{r^2} \frac{\cos \theta}{\sin^2 \theta} \frac{\partial u_\phi}{\partial \phi}$$

$$(\nabla^2 u)_\phi = \frac{\partial^2 u_\phi}{\partial r^2} + \frac{2}{r} \frac{\partial u_\phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u_\phi}{\partial \theta^2} + \frac{\cot \theta \partial u_\phi}{r^2 \partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_\phi}{\partial \phi^2}$$

⑧ $-\frac{u_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin^2 \theta} \frac{\partial u_r}{\partial \phi} + \frac{2}{r^2} \frac{\cos \theta}{\sin^2 \theta} \frac{\partial u_\theta}{\partial \phi}$