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AM25M009

Assignment -1
AM5030

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1. $u+v+w=2$
$u+2v+3w=1$
$v+2w=0$

(i) Explain why singuler

singular matrix $\longrightarrow$ Linearly dependat
$\downarrow$
$\det(A)=0$

~~$u+v+w=2$~~ $\det\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 1(1)-2+1 = \underline{\underline{0}}$

Since $\det(A)=0$
matrix is said to be singular

(ii) what should 0 be swapped to have a solution?
converting to row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right] \quad R_2 \to R_2 - R_1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

This is the case of ~~Ba~~ $\underline{\text{no solution}}$
sin $1 \neq 0$

if 0 was swaped with ~~2~~ $\underline{\underline{1}}$

it would be $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ although ∞ mans solution, system becoms consistet

general equation when $w = t$
↳ any free variable

$w = t$

$v = -1-2t$

$u + (-1-2t) + t = 2$

$\cancel{-t = 2}$

$u = 3 + t$

if $\underline{t = 0}$

$w = 0$

$v = -1$

$u = 3$

$(u, v, w) = (3, -1, 0)$

(iii) To show that they lie on the same plane

then $\alpha(\underset{\sim}{v_1}) + \beta(\underset{\sim}{v_2}) = \underset{\sim}{v_3}$

when α and β are some combination that gives us $\underset{\sim}{v_3}$

To show linear dependency: ~~columns~~

$$\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$\alpha + \beta = 1$

$\alpha + 2\beta = 3$

$\beta = 2$

$\boxed{\beta = 2 \\ \alpha = -1}$ and $\alpha = 1-2 = -1$

↳ since α, β exist, then they linearly depend ∴ on same plane.

(iv) solution of RHS = 0

the row echelon form [Page ①] will be

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$w = t$

$v + 2t = 0$ $v = -2t$

$u - 2t + t = 0$

$u = t$

$(u, v, w) = (t, -2t, t)$

③

(i) $A = \begin{array}{cc} c_1 & c_2 \\ \end{array}$
$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

colum space
↳ set of all possible linear combinations of its colums

$\underset{\sim}{A}\underset{\sim}{x} = \underset{\sim}{b}$
all $\underset{\sim}{b}$ where solution occurs

Null space
↳ for $A\underset{\sim}{x} = 0$ all the places $\underset{\sim}{x}$ can be to map 0 vector

Col. space
c_2 and c_1 are clearly related and dependent

subspace $= \begin{bmatrix} c \\ 0 \end{bmatrix}$ wh $c \in \mathbb{R}^2$

Columspace in the ~~x~~ x-axis in $\mathbb{R}^2$

colspace = x-axis

Null space

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

y = free variable
$x - y = 0$

Nullspace $= \text{span}\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$

(ii)
$$\begin{array}{ccc} c_1 & c_2 & c_3 \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

Colspace clearly c_2 is dependent on c_1

$c_2 = 2c_1$

col. space in 2D $[\mathbb{R}^2]$ = ~~span~~ span$\{\underset{\sim}{c_1}, \underset{\sim}{c_3}\}$

Null space

$x_3 = 0$
$x_1 + 2x_2 = 0$
↳ $x_1 = -2x_2$

~~if x_2 =~~ if $x_2 = t$
$x_1 = -2t$
$x_3 = 0$

$$= \begin{bmatrix} -2t \\ t \\ 0 \end{bmatrix} = t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad t \in \mathbb{R}$$

iii) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Columnspace = Zero vector

Nullspace = R^3

↳ because $A\underset{\sim}{x} = 0$

↳ if A is zero vector then $\underset{\sim}{x} \in R^n$

(3) a. plane of vectors (b_1, b_2, b_3) with $b_1 = 0$

↳ $W = \{(b_1, b_2, b_3) : b_1 = 0\}$

1. closed under 0 vector?

Yes, since b_2, b_3 are free variables and b_1 is already 0

2. Closed under addition?

Yes, since any addition operand is also in W, with $b_1 = 0$

example $(0, \overset{b_2}{1}, \overset{b_3}{2}) + (0, 5, 6) = (0, 6, 8)$

↑ also in W

3. closed under multiplication?

Yes, since any multiplication result is also in W

example : $(\overset{b_1}{0}, \overset{b_2}{1}, \overset{b_3}{2}) * 12 = (0, 12, 24)$

↑ also in W

⟹ subset is a subspace

b. $W = \{(1, b_2, b_3)\}$

1. closed under 0 vector?

No $b_1 \neq 0$

subset is not subspace.

c) $W = \{(b_1, b_2, b_3),\ b_2 b_3 \in = 0\}$ ⑤

This could lead to 2 cases

$P_1 = (b_1, 0, b_3) \in W$

$P_2 = (b_1, b_2, 0) \in W$

1- Closed under 0 vector?

↳ Yes, 0 vector is contained

2- Closed under addition?

↳ P_1 and P_2 are individually closed under addition

but $P_1 \cup P_2$ is not

example $u = (1, 0, 5)$, $v = (5, 2, 0)$ → $u + v = (6, 2, 5)$

$b_2 b_3 = 10 \neq 0$

Not closed under addition

⇒ subset is not subspace

d. All possibilities of $(1, 1, 0)$ and $(2, 0, 1)$

→ all the combination of 2 vectors (passing through origin)

can either form line, plane, entire $\mathbb{R}^3$,

→ Since it is the case all the possible combination

of vectors already were exhausted in making subspace

→ Yes, subset is subspace.

e. $W = \{(b_1, b_2, b_3) : b_3 - b_2 + 3b_1 = 0\}$

or $W = \{(b_1, b_2, b_2 - 3b_1)\}$

1. Closed under 0 vector?

Yes, $(0, 0, 0 - 3 0) = (0, 0, 0)$

2. Closed under addition?

lets take ~~2~~ example

$u = (5, 6, 9)$ $\quad 6-15$

$v = (6, 7, 15)$ $\quad 13-33$

$u+v = (11, 13, 24)$

~~24~~ $13 - (11 \times 3) = -20 \neq 24$

$\Rightarrow$ Not closed under addition

subset is not subspace.

(a). subset is subspace ✓

(b) subset is not subspace ✗

(c) subset is not subspace ✗

(d) subset is subspace ✓

(e) subset is not subspace ✗

4. (a) closed under addition?

$u = (0, 1, 0 - - -)$
$v = (1, 0, 1 - - -)$ $+ = (1, 1, 1 - - -)$

No, zeros available

$\Rightarrow$ Not closed under addition

(b) $(x_1, x_2, x_3 \; - - - - 0, 0, 0 - - -)$

1. Closed under 0?

Yes ~~*~~ example $x_1 = 0 \; x_2 = 0 \; - - -$
$(0, 0, - - \; 0, 0, 0, - - 0)$

2. Closed under addition?

Yes $\Rightarrow$ after a distinct j^{th} index all 0's occur. and ~~an~~ adding any vector from the subspace result will remain in the subspace.

3. Closed under multiplication?

Yes $\Rightarrow$ example $5 \times (x_1, x_2, \ldots 0, 0)$

$= (5x_1, 5x_2 \ldots 0, 0) \in$ subspace

$\Rightarrow$ subspace of R^∞

ⓒ ~~$W = \{(x_1, x$~~

all sequence of $x_{j+1} \leq x_j$

1. closed under multiplication

example $u = (-5, -6, -7)$

$c = -1$

$c \cdot u = (5, 6, 7)$ when $x_{j+1} > x_j$

$\Rightarrow$ Not a subspace of R^∞

ⓓ x_j when $j \to \infty$

1. closed under 0

↳ Yes

2. closed under addition?

↳ Yes, addition of 2 convergent sequence is still convergent

3. Closed under multiplication?

↳ yes if $x_j \to L$

$j \to \infty$

$cx_j \to cL$ → still finite

ⓔ AP series

1. Closed under 0?

Yes, $(0, 0 \ldots 0)$ is also a AP with

Common difference $= 0$

2. Closed under Addition?

↳ Yes

common differe of 2 different AP

~~$AP_1 + AP_2$~~ ~~$(v_{j+1} - v_j)$~~ + ~~$(u_{j+1}$~~

$d_1 + d_2$

$$\Delta P_1 + \Delta P_2 = (u_{j+1} + v_{j+1}) - (u_j + v_j)$$

$$(u_{j+1} - u_j) + (v_{j+1} - v_j) = \boxed{d_1 + d_2}$$

3. Closed under multiplicatn

if common differen $= d$

then under mutiplication ~~CD~~ =

common difference $= cd = D$

(f) All G.P $(x_1, kx_1, + k^2 x_1, \ldots)$

i. Closed under multiplication

$u = (1, 2, 4 \ldots)$ $k = 2$

$v = (1, 3, 9 \ldots)$ $k = 3$

$u + v = (2, 5, 13 \ldots)$ no single common ratio

$\therefore$ u+v is note G.P

Summary.

(a) Not subspace of R^∞ X

(b) subspace of R^∞ ✓

(c) Not a subspace of R^∞ X

(d) Subspace of R^∞ ✓

(e) subspace of R^∞ ✓

(f) ~~so~~ Not a subspace of R^∞ X

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a (i) Zero subspace {(0,0)}

-> Dim = 0

-> contains zero vector

-> closed under multiplication and addition

(ii) Line

-> Passes through (0,0)

-> Dim = 1

-> closed under addition, multiplication

(iii) $\mathbb{R}^2$ itself

-> Dim = 2

-> contains all vector in plane.

-> closed under addition, multiplication

b (i) Zero subspace. {[0,0,0,0]}

(ii) Line

(iii) ~~$\mathbb{R}^3$~~ Plane

-> Dim = 2

-> span of any two Independent vectors

(iv) any $\mathbb{R}^3$

-> Span of 3 Independent vector.

(v) $\mathbb{R}^5$ itself