Punto 2

$$R^i_j = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix}, \quad T^i = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$$

$$g^{ij} = g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) $$S^i_j = \frac{1}{2}\left(R_{ij} + R_{ji}\right) = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix} =$$

Simétrica

$$\frac{1}{2}\begin{pmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{pmatrix} = \begin{pmatrix} 1/2 & 3/2 & 5/2 \\ 3/2 & 5/2 & 7/2 \\ 5/2 & 7/2 & 9/2 \end{pmatrix}$$

$$A^i_j = \frac{1}{2}\left(R_{ij} - R_{ji}\right) = 1/2\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{pmatrix}$$

Antisimétrica

b) · $R_{kj} = g_{ik} R^i_j = g_{ki} R^i_j$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1 & 3/2 \\ -2 & -5/2 & -3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

· $R^{ki} = g^{jk} R^i_j = g^{kj} R^i_j$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1 & 3/2 \\ -2 & -5/2 & -3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

Se concluye que la métrica funciona para subir o bajar índices sin repercutir en el valor del tensor en cuestión, mostrándolo como covariante o contravariante

$T_j = g_{ij} T^i = g_{ji} T^i$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix} \rightsquigarrow$$

mediante índices vemos que se comporta como forma, por lo tanto es adecuado trasponerlo.

$$= \begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix}$$

c). $R^i_j T_i = \cancel{T_i R^i_j}$

$$\begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} = \begin{pmatrix} 7/3 & 8/3 & 3 \end{pmatrix}$$

· $R^i_j T^j =$

$$\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 1 & 9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7/3 \\ 16/3 \\ 25/3 \end{pmatrix}$$

· $R^i_j T_i T^j$

$$= \begin{pmatrix} 7/3 & 8/3 & 3 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = 7/9 + 16/9 + 3$$

$$\boxed{= \frac{50}{9}}$$

d) $R^i_j S^j_i =$

$R^i_1 S^1_i + R^i_2 S^2_i + R^i_3 S^3_i$

$$R^i_j = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1/2 & 3/2 & 5/2 \\ 3/2 & 5/2 & 7/2 \\ 5/2 & 7/2 & 9/2 \end{pmatrix}$$

$1/4 + 3/2 + 15/4 + 3 + 25/4 + 21/2 + 35/4 + 14 + 81/4$

$= \dfrac{273}{4}$

- $R^i_j A^j_i$

$$\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{pmatrix}$$

$1/2 + 3/2 - 1 + 3/2 - 7/2 - 2 = -3$

- $A^j_i T^i$

$$\begin{pmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} -4/3 \\ -1/3 \\ 2/3 \end{pmatrix}$$

- $A^j_i T^i T_j$

$$\begin{pmatrix} -4/3 \\ -1/3 \\ 2/3 \end{pmatrix} \begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix} = -4/9 + 2/9 + 2/3 = \boxed{4/9}$$

Escaneado con CamScanner

e) $\cdot\ R^i_j - 2\delta^i_j R^l_l$

$$\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} - \begin{pmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix} = \begin{pmatrix} -29/2 & 1 & 3/2 \\ 2 & \frac{-25}{2} & 3 \\ 7/2 & 4 & -21/2 \end{pmatrix}$$

$\cdot\ (R^i_j - 2\delta^i_j R^l_l)T_i = T_i(R^i_j - 2\delta^i_j R^l_l)$

$$\begin{pmatrix} \frac{1}{3} & -2/3 & 1 \end{pmatrix} \begin{pmatrix} -29/2 & 1 & 3/2 \\ 2 & -25/2 & 3 \\ 7/2 & 4 & -21/2 \end{pmatrix}$$

$$= \begin{pmatrix} -8/3 & 30/3 & -12 \end{pmatrix}$$

$\cdot\ \underbrace{(R^i_j - 2\delta^i_j R^l_l)T_i}_{j} T^j$

$$\begin{pmatrix} -8/3 & 30/3 & -12 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{40}{9} \end{pmatrix}$$

8) Suponga un sistema de coordenadas ortogonales generalizadas (q^1, q^2, q^3):

$$q^1 = x + y \; ; \; q^2 = x - y \; , \; q^3 = 2z$$

a) Compruebe que el sistema es ortogonal

$$q^1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad q^2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad q^3 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$q^1 \cdot q^2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \cancel{1 + (-1)} + 0 = 0$$ ✓ Ortogonales

$$q^1 \cdot q^3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = 0 + 0 + 0 = 0$$ ✓ Ortogonales

$$q^2 \cdot q^3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = 0 + 0 + 0 = 0$$ ✓ Ortogonales

Bajo el producto punto usual, los vectores q^1, q^2 y q^3 son ortogonales

b) $x^{i'} = \dfrac{\partial \tilde{x}^{i'}}{\partial x^{\nu}} x^{\nu}$

Es el resultado de derivar parcialmente las coordenadas q_1, q_2, q_3

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Nueva base

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{\partial \tilde{x}^{i}}{\partial x^{\nu}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Vector canónico.

$$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \frac{\partial \tilde{x}^{i}}{\partial x^{\nu}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \frac{\partial \tilde{x}^{i}}{\partial x^{\nu}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

c) Tensor métrico → producto punto de los vectores

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad U_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad U_2 = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

Elemento de volumen

$$dV = dx\,dy\,dz \qquad dx^i = (dx, dy, dz)$$

$$dx^i = \frac{\partial x^i}{\partial \tilde{x}^j} d\tilde{x}^j \qquad d\tilde{x}^j = (dq^1, dq^2, dq^3)$$

$$dx^i = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} dq^1 \\ dq^2 \\ dq^3 \end{pmatrix} = \begin{pmatrix} dq^1 + dq^2 \\ dq^1 - dq^2 \\ 2dq^3 \end{pmatrix}$$

$$dx = dq^1 + dq^2$$
$$dy = dq^1 - dq^2$$
$$dz = 2dq^3$$

$$dV = (dq^1 + dq^2)(dq^1 - dq^2)(2dq^3)$$

$$dV = ((dq^1)^2 - (dq^2)^2)\, 2dq^3$$

$$\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \frac{\partial \tilde{x}}{\partial x^j} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \frac{\partial \tilde{x}^i}{\partial x^j} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

d) $A = 2\hat{j} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ $B = \hat{i} + 2\hat{j} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ $C = \hat{i} + 7\hat{j} + 3\hat{k} = \begin{pmatrix} 1 \\ 7 \\ 3 \end{pmatrix}$

$$A\,|q^i\rangle = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \checkmark$$

$$B\,|q^i\rangle = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \checkmark$$

$$C\,|q^i\rangle = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 6 \end{bmatrix} \checkmark$$

e) $A \times B = (2\ \ -2\ \ 0) \times (3\ \ -1\ \ 0)$

$= (0 - 0\ ,\ 0 - 0\ ,\ -2 - (-6))$

$= (0\ ,0\ ,4)$

$A \cdot C = (2\ \ -2\ \ 0) \cdot (8\ \ -6\ \ 6)$

$16 + 12 + 0 = 28$

$(A \times B)\ C = (0, 0, 4) \cdot (8\ \ -6\ \ 6)$

$0 + 0 + 24 = 24$

f) $R^i_j = \begin{bmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{bmatrix}$

$$R^{i'}_{j'} = \frac{\partial \tilde{x}^{i'}}{\partial x^k} \frac{\partial x^L}{\partial x^{j'}} \cdot R^k_L$$

$$R^{i'}_{j'} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{i' \times K} \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}}_{L \times j'} \underbrace{\begin{bmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{bmatrix}}_{K \times L}$$

Por orden indicial.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

NEXT

$$R^{i'}_{j'} = \frac{1}{4}\begin{bmatrix} 12 & -2 & 9 \\ -6 & 0 & -3 \\ 30 & 2 & 18 \end{bmatrix}.$$

• $T^{i} = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} \longrightarrow T^{i'} = \dfrac{\partial \tilde{x}^{i'}}{\partial x_j} T^{j}$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1/3 \\ 2 \end{bmatrix} \checkmark$$

• $g^{i'j'} = \dfrac{\partial \tilde{x}^{i'}}{\partial x^{k}} \dfrac{\partial \tilde{x}^{j'}}{\partial x^{l}} g^{kl}$ $\qquad g_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{i' \times k} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{j' \times L \,=\, L \times j'} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{k \times L}$$

Por efectos de notación

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$g^{i'j'} = \begin{bmatrix} 0 & -1/2 & 0 \\ -1/2 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

6) Considere el tensor de Maxwell

$$F_{\mu\alpha} = \begin{pmatrix} 0 & E^x & E^y & E^z \\ -E^x & 0 & -cB^z & cB^y \\ -E^y & cB^z & 0 & -cB^x \\ -E^z & cB^y & cB^x & 0 \end{pmatrix}, \quad \text{con } \eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) Observador 1 (O) → Solo detecta campo en x

$$F_{\mu\alpha} = \begin{pmatrix} 0 & E^x & 0 & 0 \\ -E^x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \text{Se anulan las demás componentes}$$

$\beta = v\hat{\imath}$

Observador 2 (O') → Velocidad respecto al 1

$F'_{\mu\alpha} = $ ¿? → Transformación de Lorentz

¿Por qué no de Galileo? Cambia las ecuaciones

$F'_{\mu\alpha} = \Lambda^{\mu}_{\ \mu'} \Lambda^{\nu}_{\ \nu'} F_{\mu\nu}$ → Solo tengo componentes F_{01} y F_{10}

$F'_{\mu\alpha} = \Lambda^{0}_{\ \mu'} \Lambda^{1}_{\ \nu'} F_{01} + \Lambda^{1}_{\ \mu'} \Lambda^{0}_{\ \nu'} F_{10}$ $\quad F_{01} = E_x \quad F_{01} = -F_{10}$

$F_{10} = -E_x$

$F'_{\mu\alpha} = \Lambda^{0}_{\ \mu'} \Lambda^{1}_{\ \nu'} F_{01} - \Lambda^{1}_{\ \mu'} \Lambda^{0}_{\ \nu'} F_{01}$

$F'_{\mu\alpha} = (\Lambda^{0}_{\ \mu'} \Lambda^{1}_{\ \nu'} - \Lambda^{1}_{\ \mu'} \Lambda^{0}_{\ \nu'}) F_{01}$

Tomando en cuenta que: $\Lambda^{0}_{\ 0'} = \gamma$, $\Lambda^{i}_{\ 0'} = \gamma v^i$, $\Lambda^{i}_{\ j'} = \delta^{i}_{j'} + v^i v_{j'} \dfrac{\gamma - 1}{\underbrace{v^k v_k}_{|v|^2}}$

Como $v^i = (v, 0, 0)$ → solo está en i

$$\Lambda^{\mu}_{\ \mu'} = \begin{pmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Lambda^{1}_{\ 1'} = \delta^{1}_{1'} + v^1 v_{1'} \frac{\gamma - 1}{|v|^2} = \gamma$$

Entonces:

$F_{0'0'} = (\Lambda^{0}_{\ 0'} \Lambda^{1}_{\ 0'} - \Lambda^{1}_{\ 0'} \Lambda^{0}_{\ 0'}) F_{01} = (-\gamma^2 v + \gamma^2 v) F_{01} = 0$

$F_{1'1'} = (\Lambda^{0}_{\ 1'} \Lambda^{1}_{\ 0'} - \Lambda^{1}_{\ 0'} \Lambda^{0}_{\ 1'}) F_{01} = (-\gamma^2 v^2 + \gamma^2 v^2) F_{01} = 0$

$F_{1'0'} = (\Lambda^{0}_{\ 1'} \Lambda^{1}_{\ 0'} - \Lambda^{1}_{\ 1'} \Lambda^{0}_{\ 0'})(-E^x) = (\gamma^2 v^2 - \gamma^2)(E^x)$

→ Análogamente $F_{0'1'} = (\gamma^2 v^2 - \gamma^2)(-E^x)$

Las demás componentes son 0 y se harán necesariamente 0

$$F_{\mu'\alpha'} = \begin{pmatrix} 0 & -\gamma^2(v^2-1)E^x & 0 & 0 \\ \gamma^2(v^2-1)E^x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

De esta manera $\vec{E}' = (E^{x'}, E^{y'}, E^{z'}) = ((v^2-1)\gamma^2 E^x, 0, 0)$

$\vec{B}' = (B^{x'}, B^{y'}, B^{z'}) = (0, 0, 0)$

Por ende el observador 2 (O') solo detectará campo magnético en dirección $\hat{x}$ incrementado en un factor $(v^2-1)\gamma^2$ //

b) Tomando en cuenta las ecuaciones de Maxwell para la ley de Gauss y la ley de Ampere-Maxwell

$$\vec{\nabla}\times\vec{B}-\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J}\ ,\ \vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$

$J^\mu=(c\rho,\vec{J})$, $\vec{J}=(J^1,J^2,J^3)$

$x^\nu=(ct,x,y,z)$

$\vec{E}=(E^x,E^y,E^z)$

$\vec{B}=(B^x,B^y,B^z)$

Demostrar $\rightarrow$ $\dfrac{\partial F^{\mu\nu}}{\partial x^\nu}=F^{\mu\nu}{}_{,\nu}=c\mu_0 J^\mu$

Necesitamos subir índices (versión contravariante):

$F^{\mu\nu}=\eta^{\mu\alpha}\eta^{\nu\beta}F_{\alpha\beta}=\eta^{\mu\alpha}F_{\alpha\beta}\eta^{\beta\nu}$ $\longrightarrow$ Para que se cumpla producto matricial

Partiendo de:

Operamos con el tensor métrico

$$F_{\alpha\beta}=\begin{pmatrix}0&-E^x&-E^y&-E^z\\E^x&0&cB^z&-cB^y\\E^y&-cB^z&0&cB^x\\E^z&cB^y&-cB^x&0\end{pmatrix}\longrightarrow\begin{pmatrix}0&E^x&E^y&E^z\\-E^x&0&cB^z&-cB^y\\-E^y&-cB^z&0&cB^x\\-E^z&cB^y&-cB^x&0\end{pmatrix}$$

Ley de Gauss $\longrightarrow$ "Derivadas parciales"

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu}=F^{\mu\nu}{}_{,\nu}=F^{\mu0}{}_{,0}+F^{\mu1}{}_{,1}+F^{\mu2}{}_{,2}+F^{\mu3}{}_{,3}$$

$x^\nu=(x^0,x^1,x^2,x^3)$
$(0,x,y,z)$

$$=\cancel{F^{00}{}_{,0}}+F^{01}{}_{,1}+F^{02}{}_{,2}+F^{03}{}_{,3}=0+\underbrace{\frac{\partial E^x}{\partial x}+\frac{\partial E^y}{\partial y}+\frac{\partial E^z}{\partial z}}_{\vec{\nabla}\cdot\vec{E}\ \rightarrow \text{Divergencia}}$$

$$=\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0}=\frac{c\rho}{c\varepsilon_0}=\frac{J^0}{c\frac{1}{\mu_0c^2}}=\boxed{\mu_0cJ^0}\ ✓$$

Ley de Àmpere-Maxwell

$$F^{1\nu}{}_{,\nu}=F^{10}{}_{,0}+F^{11}{}_{,1}+F^{12}{}_{,2}+F^{13}{}_{,3}=-\frac{1}{c}\frac{\partial E^x}{\partial t}+\cancel{0}+c\frac{\partial B^z}{\partial y}-c\frac{\partial B^y}{\partial z}$$ Factorizando c

$$=c\left(\frac{-1}{c^2}\frac{\partial E^x}{\partial t}+\underbrace{\frac{\partial B^z}{\partial y}-\frac{\partial B^y}{\partial z}}_{\vec{\nabla}\times\vec{B}\ \text{rotacional}}\right)=c\left(\frac{-1}{c^2}\frac{\partial\vec{E}}{\partial t}+\nabla\times\vec{B}\right)_x=c\mu_0J^1\ ✓$$

De manera análoga para $F^{2\nu}{}_{,\nu}$ y $F^{3\nu}{}_{,\nu}$ se obtienen las componentes y y z respectivamente.

Así se concluye que $F^{\mu\nu}{}_{,\nu}=c\mu_0J^\mu$ contiene la ley de Gauss y la ley de Ampère-Maxwell

c) Identidad de Bianchi

Demostrar:
$\vec{\nabla}\cdot\vec{B}=0$, $\vec{\nabla}\times\vec{E}+\frac{\partial}{\partial t}\vec{B}=0$
se contienen en $F^{\mu\nu}{}_{,\nu}=c\mu_0J^\mu$

$F_{\mu\nu,\gamma}+F_{\nu\gamma,\mu}+F_{\gamma\mu,\nu}=0$

Tomando en cuenta que:
$\mu=1\quad\gamma=2\quad\nu=3$

$$F_{13,2}+F_{32,1}+F_{21,3}=0\rightarrow -c\frac{\partial B^y}{\partial y}-c\frac{\partial B^x}{\partial x}-c\frac{\partial B^z}{\partial z}=0$$

$$-c\underbrace{\left(\frac{\partial B^x}{\partial x}+\frac{\partial B^y}{\partial y}+\frac{\partial B^z}{\partial z}\right)}_{0}=0 \rightarrow \vec{\nabla}\cdot\vec{B}=0 \ ✓✓$$

Ahora bien, haciendo: $F_{\mu\nu,0}+F_{\nu 0,\mu}+F_{0\mu,\nu}=0$ se trabaja con:

$\mu=2,\ \nu=3 \rightarrow F_{23,0}+F_{30,2}+F_{02,3}=0$

Rotacional

$$\hookrightarrow \frac{\cancel{c}\,\partial B^x}{\cancel{c}\,\partial t}+\left(\frac{\partial E^z}{\partial y}-\frac{\partial E^y}{\partial z}\right)=0 \rightarrow (\vec{B}+\vec{\nabla}\times\vec{E})_x=0 \ ✓$$

$\mu=1,\ \nu=3 \rightarrow F_{13,0}+F_{30,1}+F_{01,3}=0$

Rotacional

$$\hookrightarrow \frac{-\cancel{c}\,B^y}{\cancel{c}\,\partial t}+\left(\frac{\partial E^z}{\partial x}-\frac{\partial E^x}{\partial z}\right)=0 \rightarrow (-\vec{B}+\vec{\nabla}\times\vec{E})_y=0 \ ✓✓$$

$\mu=2,\ \nu=1 \rightarrow F_{21,0}+F_{10,2}+F_{02,1}=0$

Rotacional

$$\hookrightarrow \frac{-\cancel{c}\,B^z}{\cancel{c}\,\partial t}+\left(\frac{\partial E^x}{\partial y}-\frac{\partial E^y}{\partial x}\right)=0 \rightarrow (-\vec{B}+\vec{\nabla}\times\vec{E})_z=0 \ ✓✓$$

De acuerdo a lo anterior se demuestra que la identidad de Bianchi contiene las ecuaciones: $\vec{\nabla}\cdot\vec{B}=0$ y $\vec{\nabla}\times\vec{E}+\frac{\partial}{\partial t}\vec{B}=0$