1. Let us simulate a geometric Brownian motion (GBM), which is the most fundamental model of the value of a financial asset. The process $\{S(t)\}$ is a GBM with drift parameter μ, volatility parameter σ, and initial value $S(0)$ if

$$S(t) = S(0) \exp\left(\left[\mu - \frac{1}{2}\sigma^2\right] t + \sigma W(t)\right),$$

where $\{W(t)\}$ is a standard BM. As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < \cdots < t_n$ as:

$$S(t_{i+1}) = S(t_i) \exp\left(\left[\mu - \frac{1}{2}\sigma^2\right](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}\; Z_{i+1}\right), \; i = 0, 1, \ldots, n-1,$$

where $Z_1, Z_2, \ldots, Z_n$ are independent $\mathcal{N}(0,1)$ variates. In the interval $[0,5]$, taking both positive and negative values for μ and for at least two different values of σ^2, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$.

Now, taking $S(0) = 5, \mu = 0.06, \sigma = 0.3$, first simulate $S(1)$ by taking $n = 50$ steps to reach time 1 (i.e., with time intervals of length 0.02). Then, compute the expectation of $S(1)$ (i.e., $E(S(1))$) by simulating N values of $S(1)$. By varying values for N from 1 to 1000000, plot the values of $E(S(1))$ as a curve. Simulate and plot 10 such curves (N in the x-axis and $E(S(1))$ in the y-axis).

2. Use the following Monte Carlo estimator to approximate the expected value $I = E[\exp(\sqrt{U})]$ where, $U \sim \mathcal{U}[0,1]$:

$$I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i, \text{ where } Y_i = \exp(\sqrt{U_i}) \text{ with } U_i \sim \mathcal{U}[0,1].$$

Take the values of M to be $10^2, 10^3, 10^4$ and 10^5. Determine the 95% confidence interval for I_M for all the four values of M that you have taken.

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