Assignment 5 U2010038

① joint probability mass function

$y \backslash x$	0	1	2
0	$\frac{25}{102}$	$\frac{13}{51}$	$\frac{1}{17}$
1	$\frac{13}{51}$	$\frac{13}{102}$	0
2	$\frac{1}{17}$	0	0

a) $X=0 \quad Y=0 \Rightarrow C_2^{26} / C_2^{52} = 325/1326 \Rightarrow 25/102$

$X=0 \quad Y=1 \Rightarrow C^{13} \cancel{\&} C^{26} / C_2^{52} \Rightarrow 13 \cdot 26 / 1326 \Rightarrow 13/51$

$X=0 \quad Y=2 \Rightarrow C_2^{13} / C_2^{52} \Rightarrow 78/1326 \Rightarrow 1/17$

$X=1 \quad Y=1 \Rightarrow C^{13} C^{13} / C_2^{52} \Rightarrow 13 \cdot 13 / 1326 \Rightarrow 13/102$

$X=2 \quad Y=0 \Rightarrow C_2^{13} / C_2^{52} \Rightarrow 78/1326 \Rightarrow 1/17$

b) $X=0 \Rightarrow \frac{25}{102} + \frac{13}{51} + \frac{1}{17} = \frac{19}{34}$ $\qquad$ $Y=0 = \frac{25}{102} + \frac{13}{51} + \frac{1}{17} = \frac{19}{34}$

$X=1 \Rightarrow \frac{13}{51} + \frac{13}{102} + 0 = \frac{13}{34}$ $\qquad$ $Y=1 = \frac{13}{51} + \frac{13}{102} \Rightarrow \frac{13}{34}$

$X=2 = \frac{1}{17}$ $\qquad$ $Y=2 = \frac{1}{17}$

c) no because   U2010038

$X=0, Y=0 \Rightarrow \frac{25}{102} \neq$ and $(X=0)\cdot(y=0) = \frac{19}{34} \cdot \frac{19}{34}$

d) $E(X) = 0 + \frac{13}{34} + \frac{2}{17} = 0{,}5$ Variance$(X) = \left(0+\frac{13}{34}+\frac{4}{17}\right) - \left(\frac{1}{2}\right)^2 = \frac{25}{68}$

e) Covariance of X and Y

$E(XY) - E(X)\cdot E(Y)$

$E(XY) = 0+0+0+0+\frac{13}{102}+0+0+0+0 \Rightarrow \frac{13}{102}$

therefore $(X,Y) = \frac{13}{102} - \frac{1}{4} \Rightarrow -\frac{25}{204}$

f) Correlation between X and Y

$\sqrt{\frac{\text{Covariance}}{\text{Var}(X)\,\text{Var}(Y)}} \Rightarrow \frac{-\frac{25}{204}}{\sqrt{\frac{25}{68}\cdot\frac{25}{68}}} = -\frac{1}{3}$

g) $Y=0 \quad X=0 \quad \Rightarrow \frac{P(X=0, Y=0)}{P(X=0)} \Rightarrow \frac{25}{67}$

$Y=1 \quad \cancel{X=0} \; X=0 \quad \Rightarrow \frac{P(X=0, Y=1)}{P(X=0)} = \frac{25}{67}$

$Y=2 \quad X=0 \quad \Rightarrow \frac{P(X=0)\, Y=2)}{P(X=0)} = \frac{2}{19}$

$Y=0 \quad X=1 \quad \Rightarrow \frac{P(X=1, Y=0)}{P(X=1)} = 2/3$

$Y=1 \quad X=1 \quad \Rightarrow \frac{P(X=1, Y=1)}{P(X=1)} = 1/3$

$Y=2 \quad X=1 \quad \Rightarrow \frac{P(X=1, Y=2)}{P(X=1)} = 0$

(2) $f(x) = Ax(L-x) \quad 0 \le x \le L$

a) $\int_0^L Ax(L-x)\,dx = 1 \Rightarrow A\int_0^L xL - x^2\,dx = A\left[\frac{x^2 L}{2} - \frac{x^3}{3}\right]_0^L \Rightarrow A\frac{L^3}{6} = 1 \Rightarrow \frac{6}{L^3}$

b) $f(x) = \begin{cases} \frac{3}{2L} - \frac{3x^2}{2L^3} & 0 \le x \le L \\ 0 & \ldots \end{cases}$

c) $\int_0^L x\left[\frac{3}{2L} - \frac{3x^2}{2L^3}\right]dy = \frac{3}{2L}\left[\frac{x^2}{2}\right]_0^L - \frac{3}{2L^3}\left[\frac{x^4}{4}\right]_0^L = \frac{3L}{8}$

③ $f(x,y) = 4x(2-y) \qquad 0 \le x \le 1 \qquad 1 \le y \le 2$ 1620100038

$$\int_0^1 \int_1^2 4x(2-y)\,dy\,dx = 1$$

a) $f_X(x) = \int_1^2 4x(2-y)\,dy = 4x\left[2y - \frac{y^2}{2}\right]_1^2 = 2x$

$f_Y(y) = \int_0^1 4x(2-y)\,dx = 4-2y$

b) $f(x,y) = f(x) + f(y)$

$4x(2-y) \qquad 2x(4-2y)$

$8x - 4xy \qquad 8x - 4xy$ Yes, independent

c) $Cov(x,y)$

$E(x,y) - E(x)E(y)$ independent.

d) $y = 1,5$ also independent.