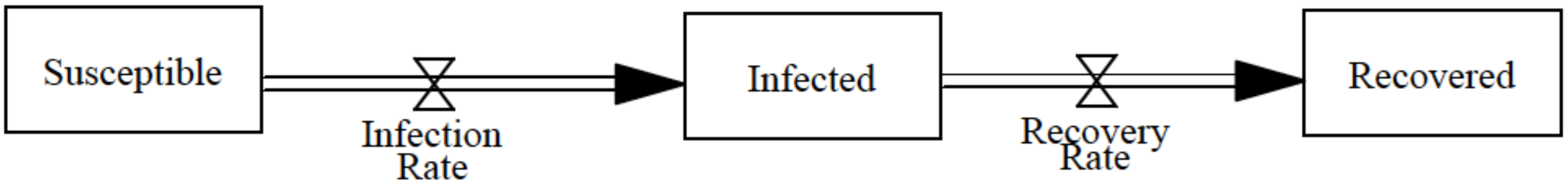
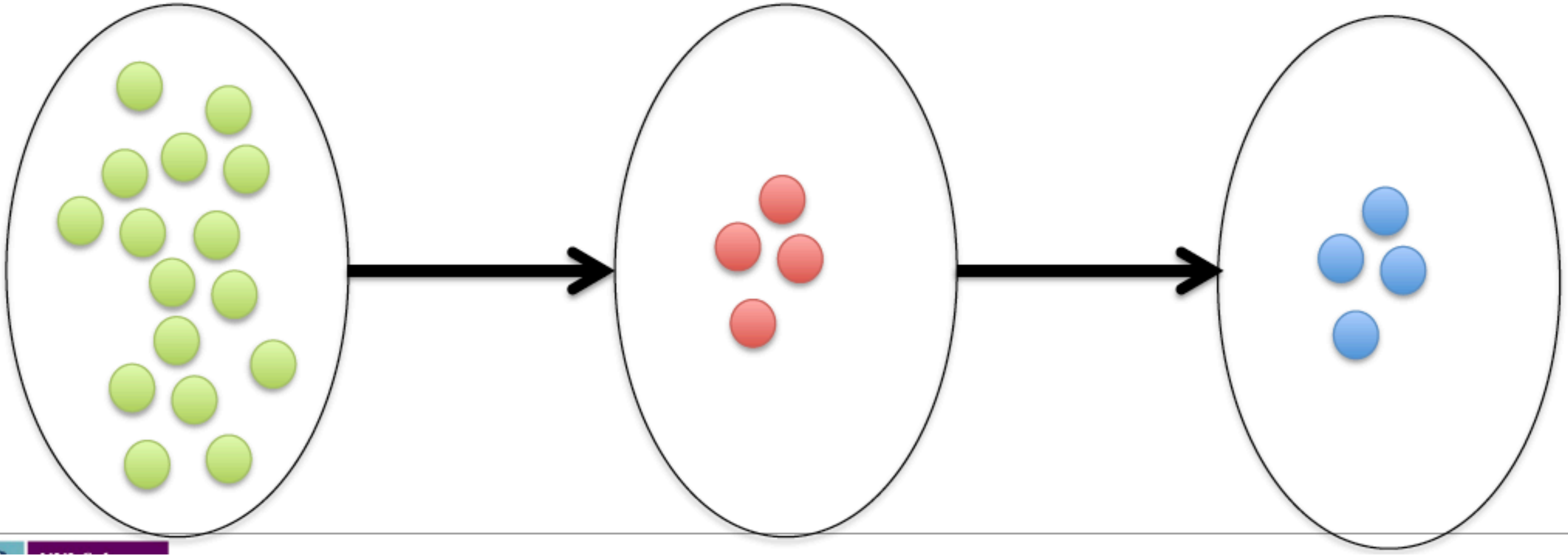
Need a model structure – Stocks & Flows





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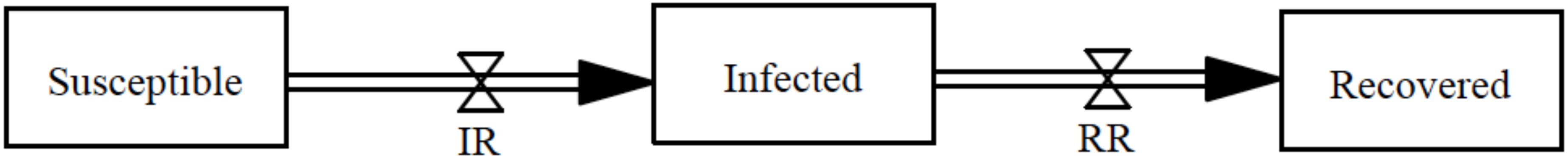
Stock Equations



$$Susceptible\ (S) = \ INTEGRAL(-IR, 99999)$$
$$Infected\ (I) = \ INTEGRAL(IR - RR, 1)$$
$$Recovered\,(R) = \ INTEGRAL(RR, 0)$$

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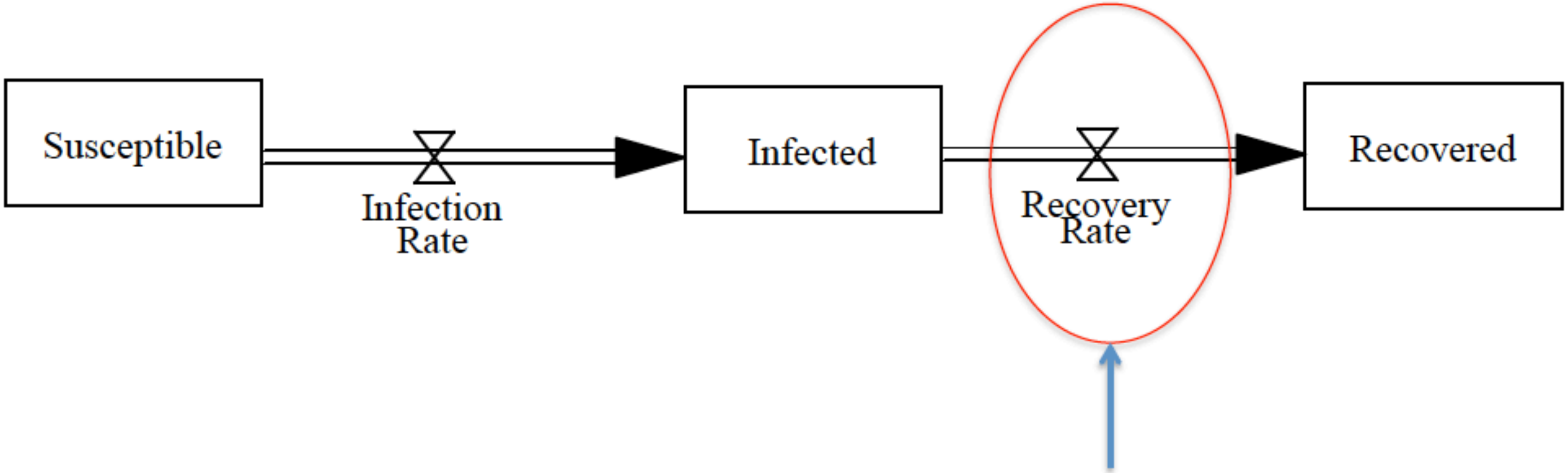
What about the flows?



First Order Delay Structure

$$RR = \frac{I}{D}$$

$$Delay\ (D) = 2$$

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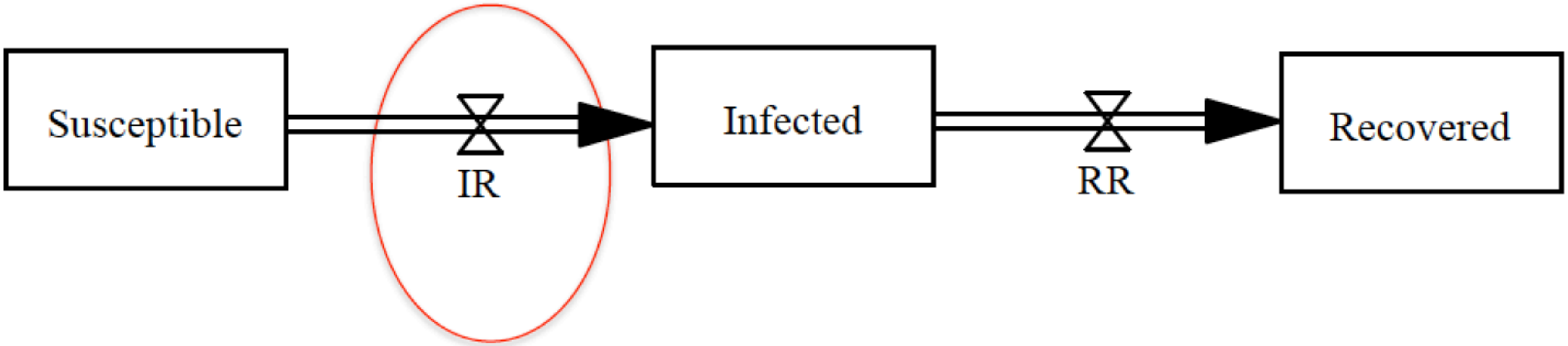
Infection rate?



- Infection spreads through contact
- As the number of infected increase, so to does the infection rate
- A positive feedback process

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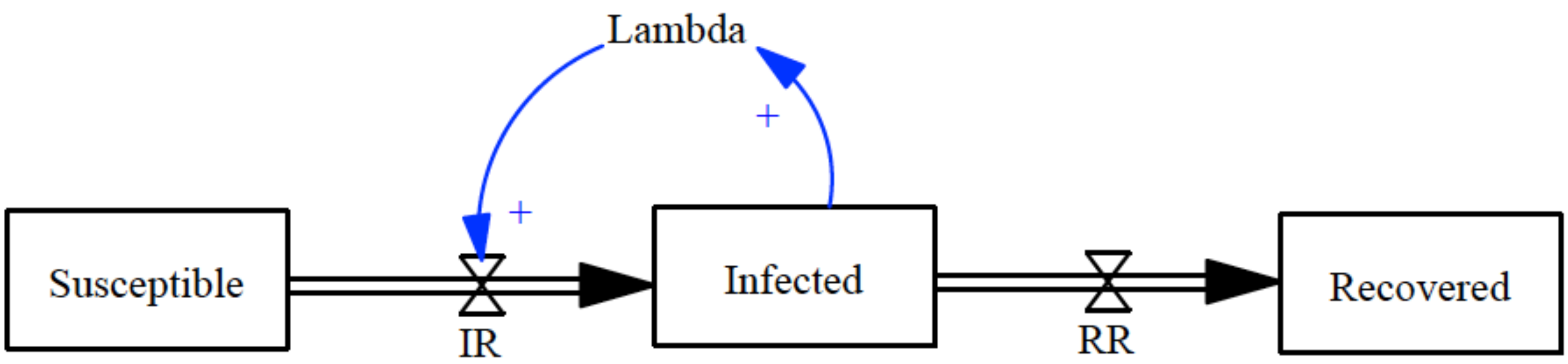
Lambda – force of infection (Attack Rate)



↑	Infected	→	Lambda	↑
↑	Lambda	→	IR	↑
↑	IR	→	Infected	↑

The rate at which susceptible individuals become infected per unit time

Proportional to the number infected

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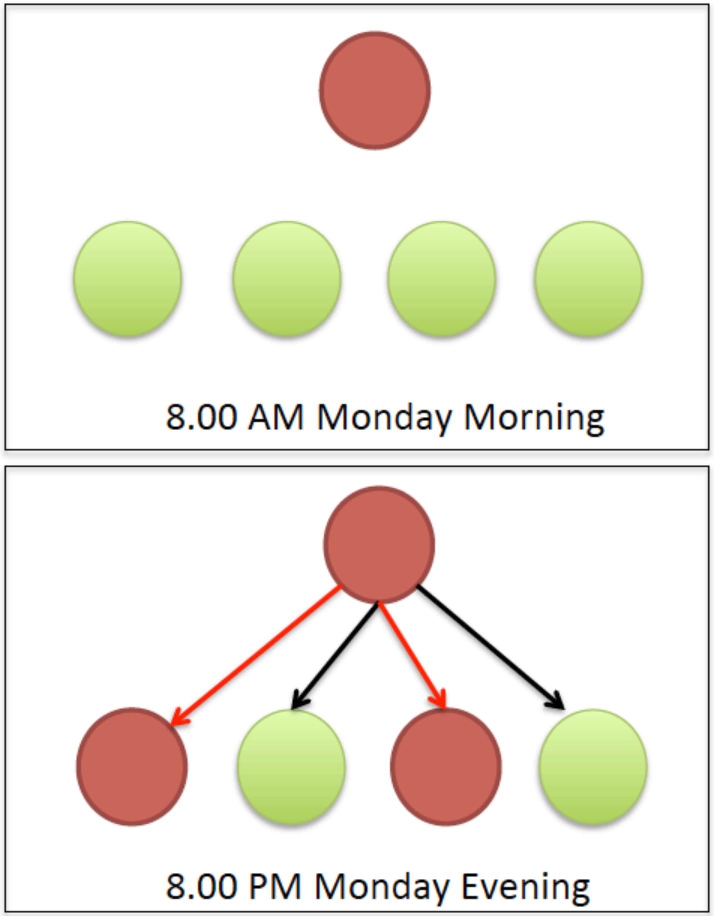
Effective Contact (Ce)

- Defined a one which is sufficient to lead to infection, were it to occur between a **susceptible** and **infectious** individuals
- For example, if $C_e = 2$
  - An infectious person will infect two susceptible people in one day
  - They could meet 4 people, and pass on the virus with probability (0.50)



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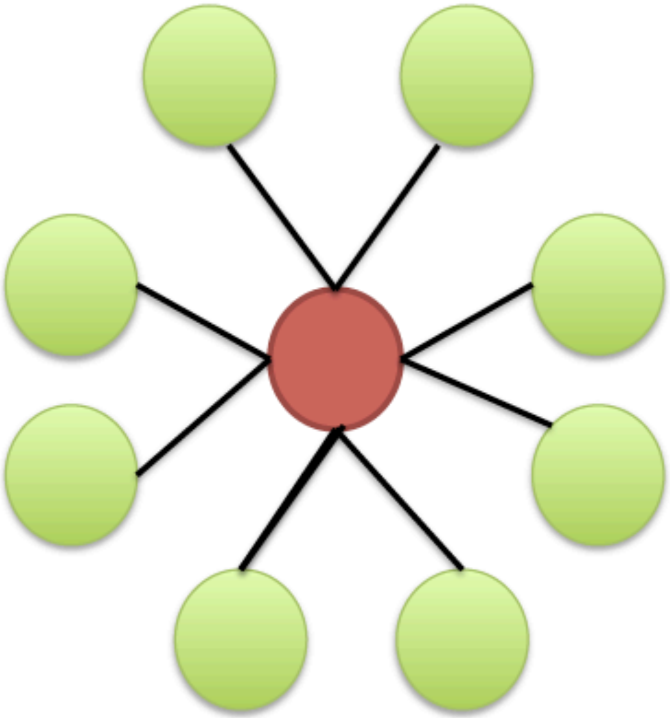
Beta

- **Per capita rate** at which two specific individuals come into effective contact per unit time
- An important parameter used to model disease transmission



$$\beta = \frac{c_e}{N}$$

$$\beta = \frac{2}{10000} = 0.0002$$

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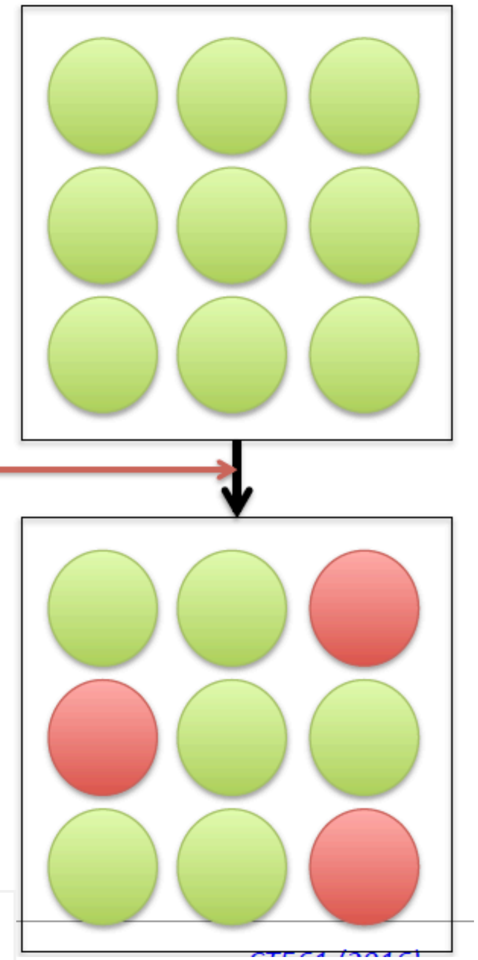
Lambda – force of infection

- The rate at which susceptible individuals become infected per unit time
- Also know as the hazard rate or incidence rate

$$\lambda = \beta I$$

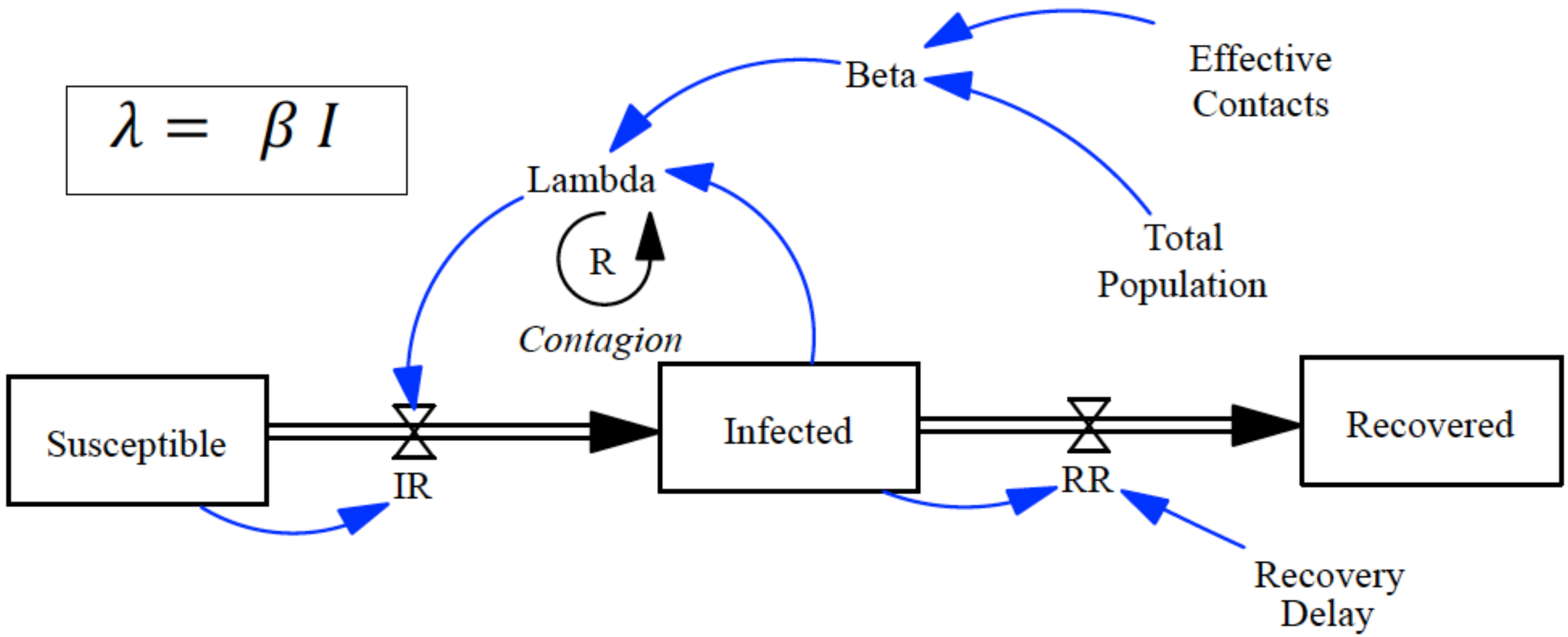
$$\lambda = \frac{1}{3}$$



A stock and flow model

$$\beta = \frac{c_e}{N}$$

$$\lambda = \beta I$$



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Model equations

Total Population = 10000

Effective Contacts=2

Beta= Effective Contacts/Total Population

Lambda = Beta*Infected

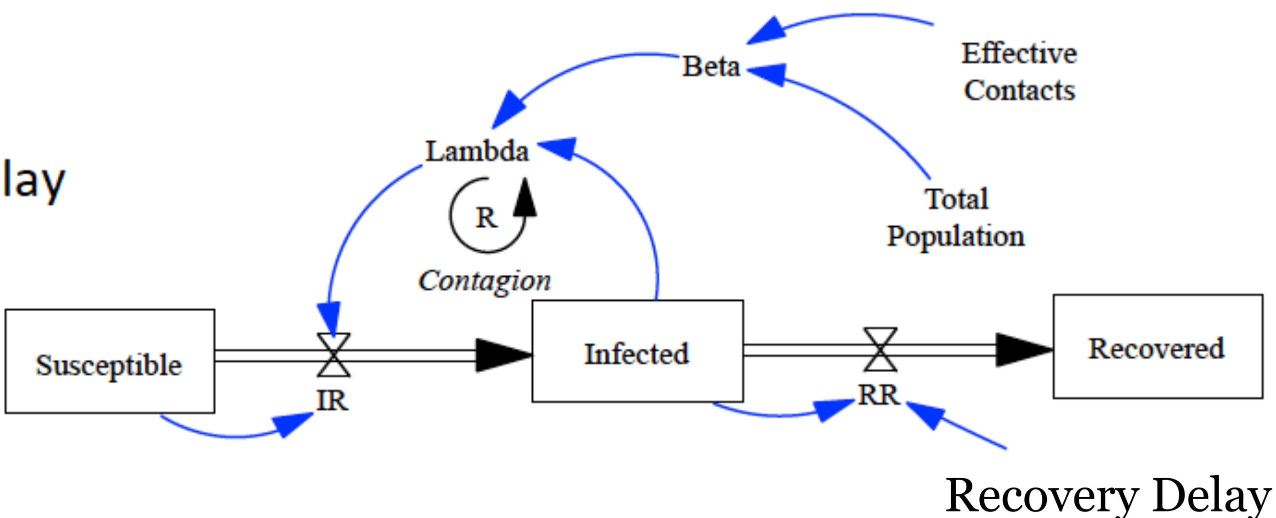
IR=Lambda*Susceptible

RR=Infected/Recovery Delay

Susceptible= INTEG (-IR, 9999)

Infected= INTEG (IR-RR, 1)

Recovered= INTEG (RR, 0)



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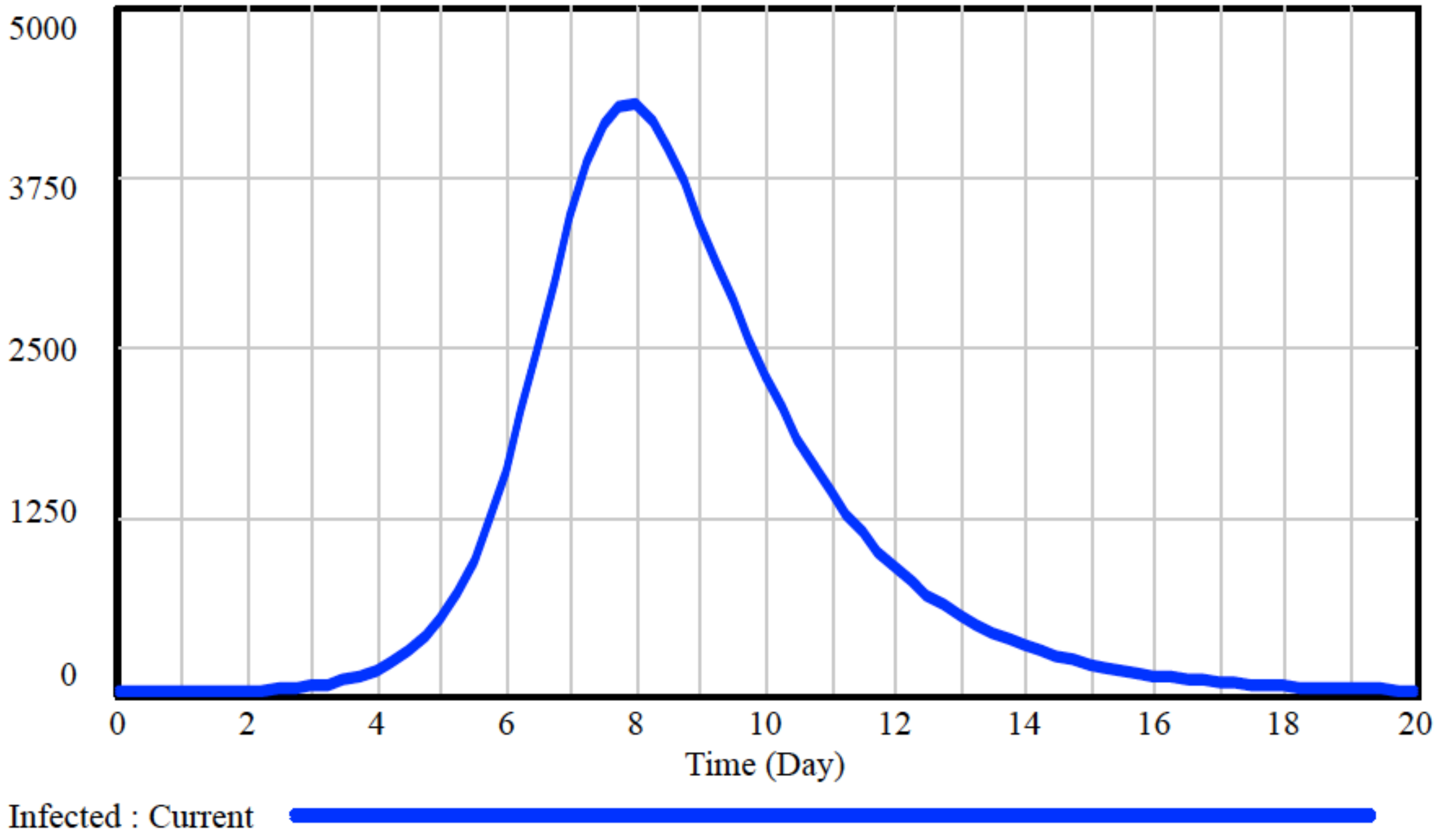
Simulation Output



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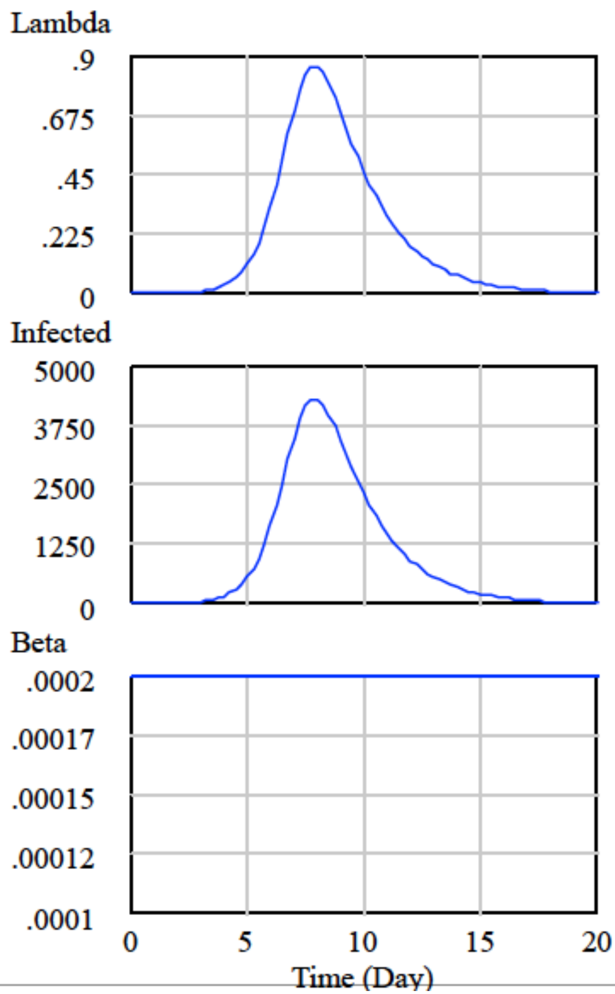
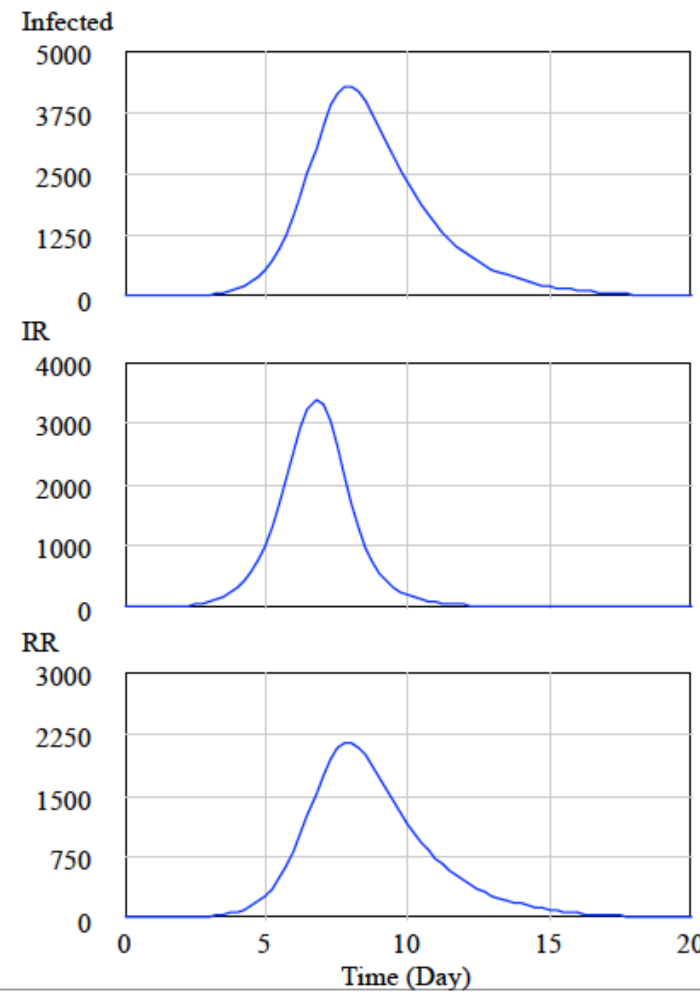
Exploring Variables





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Challenge 2

- Draw the stock and flow model the corresponds to the following equations

$$\frac{dS}{dt} = -\lambda S \qquad \frac{dI}{dt} = fE - rI$$

$$\frac{dE}{dt} = \lambda S - fE \qquad \frac{dR}{dt} = rI$$

The rate at which something occurs is 1 / Average time to the event
f and *r* are rates.

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article

OPEN ACCESS Freely available online PLOS MEDICINE

Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases

Joël Mossong[1,2*], Niel Hens[3], Mark Jit[4], Philippe Beutels[5], Kari Auranen[6], Rafael Mikolajczyk[7], Marco Massari[8], Stefania Salmaso[8], Gianpaolo Scalia Tomba[9], Jacco Wallinga[10], Janneke Heijne[10], Malgorzata Sadkowska-Todys[11], Magdalena Rosinska[11], W. John Edmunds[4]

Methods and Findings

7,290 participants recorded characteristics of 97,904 contacts with different individuals during one day, including age, sex, location, duration, frequency, and occurrence of physical contact. We found that mixing patterns and contact characteristics were remarkably similar across different European countries. Contact patterns were highly assortative with age: schoolchildren and young adults in particular tended to mix with people of the same age. Contacts lasting at least one hour or occurring on a daily basis mostly involved physical contact, while short duration and infrequent contacts tended to be nonphysical. Contacts at home, school, or leisure were more likely to be physical than contacts at the workplace or while travelling. Preliminary modelling indicates that 5- to 19-year-olds are expected to suffer the highest incidence during the initial epidemic phase of an emerging infection transmitted through social contacts measured here when the population is completely susceptible.

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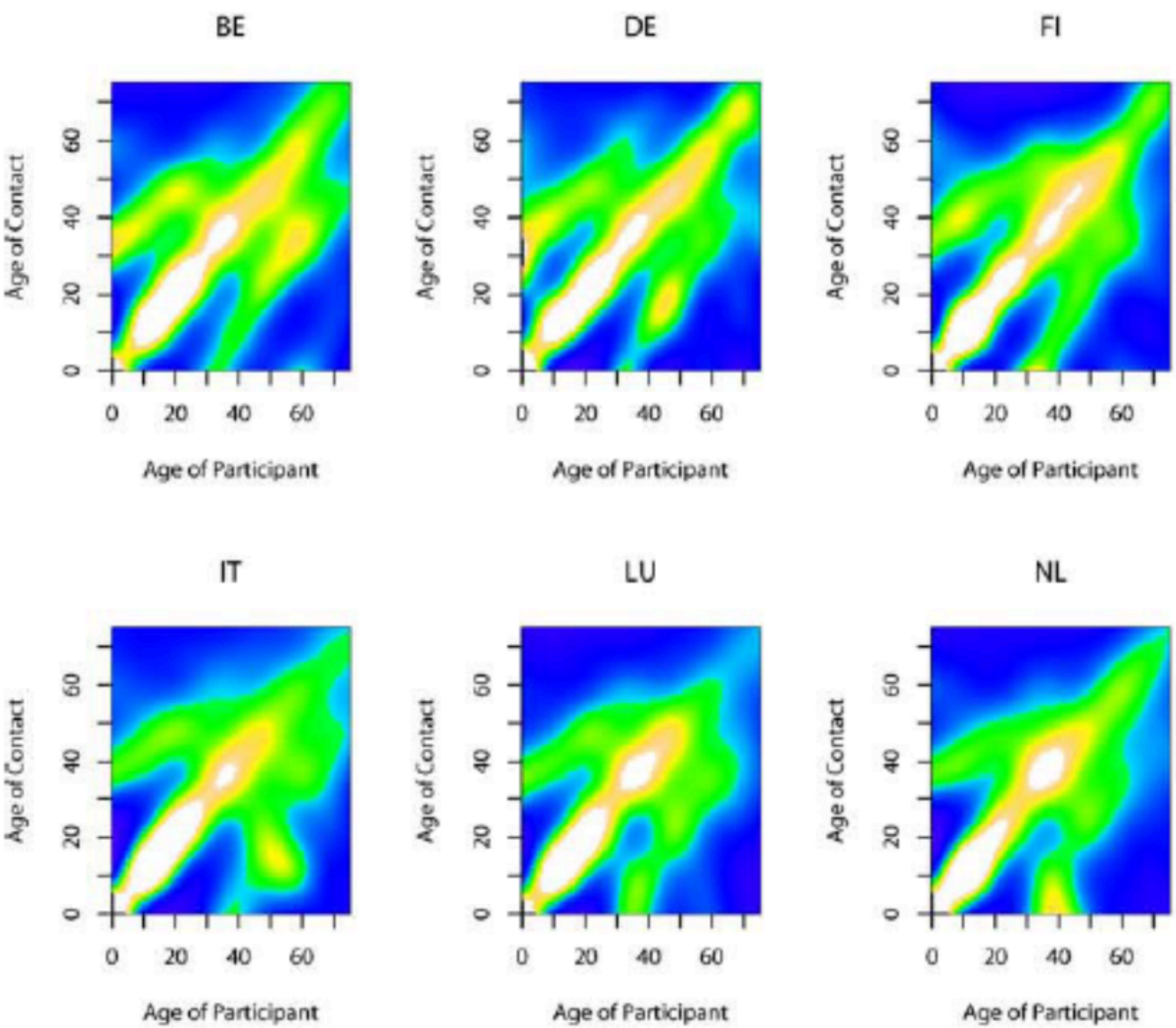
article



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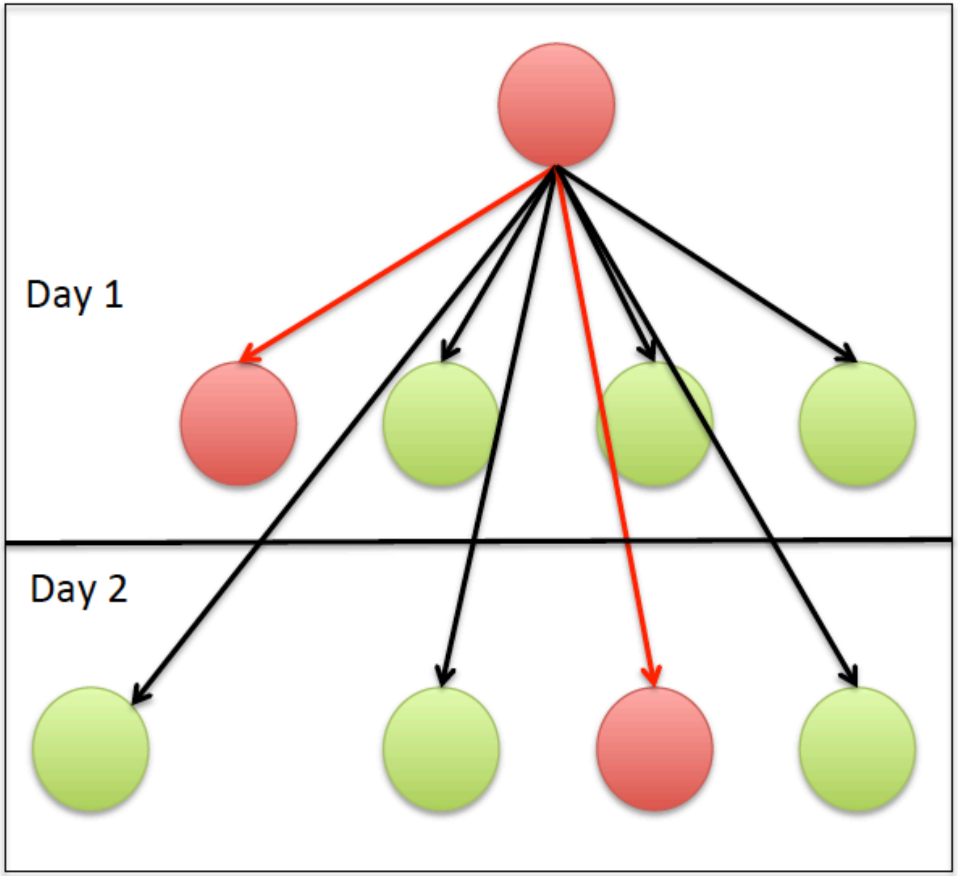
Reproduction rate R0

- Formally defined as the average number of secondary infectious resulting from a typical infectious person being introduced to a totally susceptible population

$$\boxed{R_0 = c_e\, D}$$



$D = 2 \quad c_e = 1 \quad R_0 = 2$

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Challenge 3

- Suppose we have a town with 10,000 (=N) individuals, of which 1% were infectious with measles, with R_0 = 13 and D=7 Days
- Calculate the force of infection λ

$$\boxed{R_0 = c_e D}$$

$$\boxed{\beta = \frac{c_e}{N}}$$

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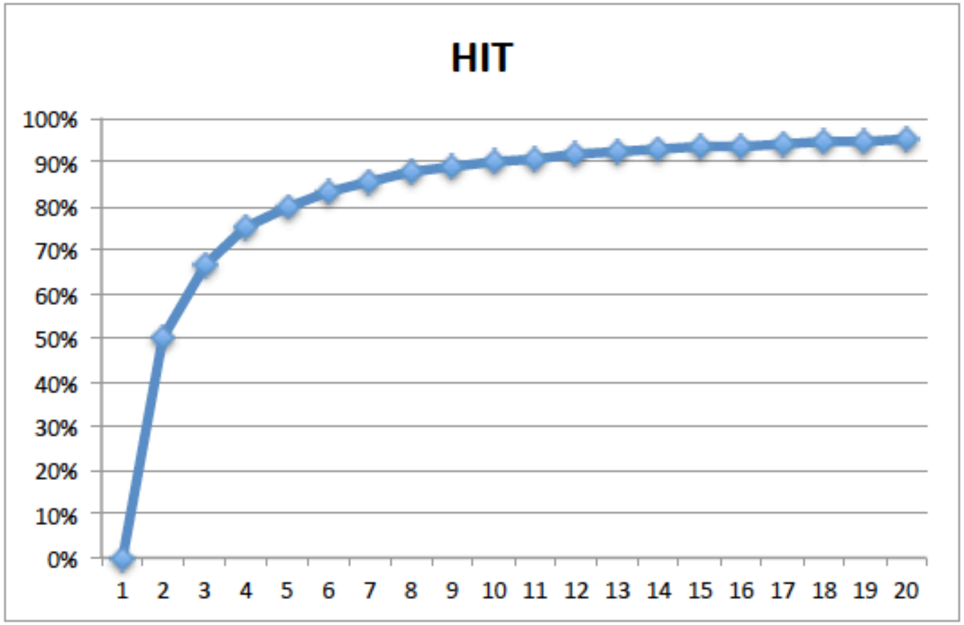
Herd Immunity Threshold

- Depends on R_0
- The proportion of the population which needs to be immune for the infection incidence to be stable
- To eradicate an infection, the proportion of the population that is immune must exceed this threshold value

$$HIT = 1 - \frac{1}{R_0}$$



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Vaccine preventable diseases

Infection	R_o	Herd Immunity
Diphtheria	6-7	85
Influenza	2-4	50-75
Malaria	5-100	80-99
Measles	12-18	83-94
Pertussis	12-17	92-94

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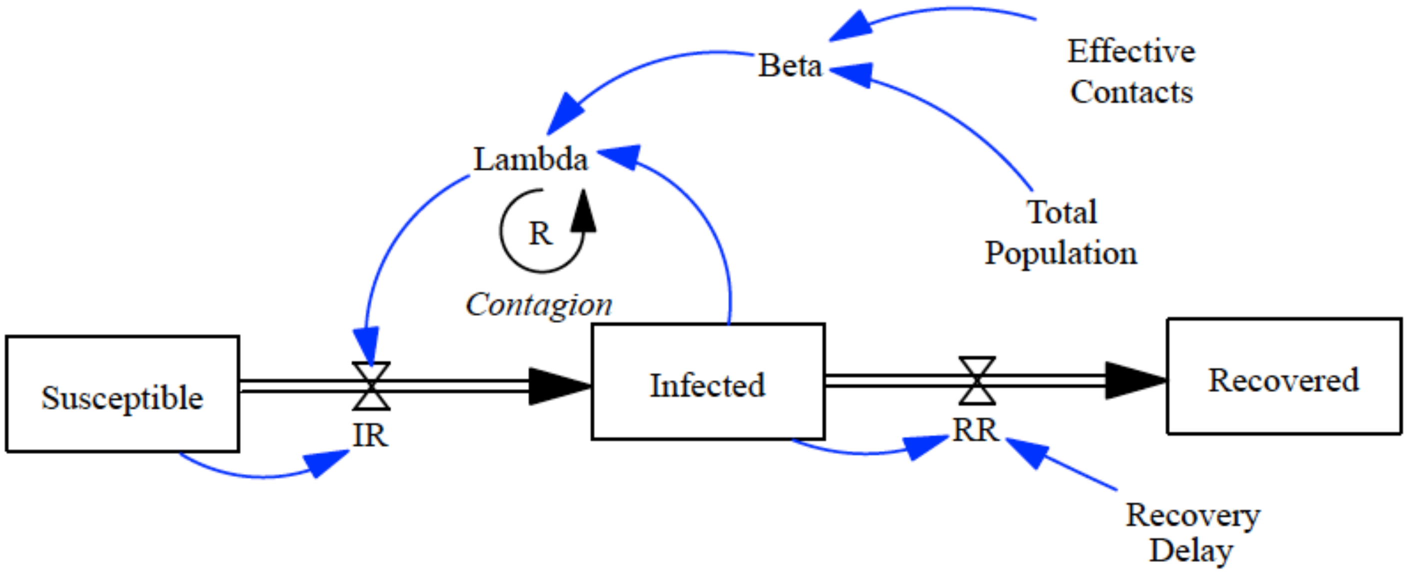
Challenge 4



- Explore the SIR Model
- What model conditions will stop disease spread?

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