# Rational Conceptual Change

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## 1. Introduction

One of the salient features of Carnap's systems of inductive logic is a conditionalization learning model, which also plays a fundamental role in the orthodox Bayesian account of rationality.[1] This learning model does not allow for revision of previously accepted evidence. It is, therefore, not adequate to represent all rational learning by an agent who accepts corrigible propositions as evidence. Recently [10] I used a generalization of the concept of conditional belief to extend the model so that rational revision of previously accepted evidence can be accommodated. My generalization of conditional belief carries with it a natural way of representing an agent's conceptual framework. In this paper I exploit this representation to produce learning models that can accommodate rational conceptual change.

The idea of conceptual change has received considerable attention in recent work of philosophers of science. The following quotation from Hilary Putnam is instructive.

> Whatever the nature of the conceptual revolution involved in the shift from Newtonian to relativistic cosmology may have been, it was not simply a matter of attaching the old labels, e.g., 'straight line', to new curves. What seemed *a priori* before the conceptual revolution was precisely that there are paths in space which behave in Euclidean fashion; or, to drop reference to 'paths', what seemed *a priori* was precisely that there were infinitely many non-overlapping places (of, say, the size of an ordinary room) to get to. What turned out to be the case (or, rather, what will turn out to be the case if the universe in the large has compact spatial cross-sections) is precisely that there are only finitely many disjoint places (of the size of an ordinary room) in space to get

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> to, travel as one will. Something literally inconceivable has turned out to be true; and it is not just a matter of attaching the old labels ('place', 'straight line') to different things. ([23], p. xv).

The important point here is that the change in geometry changed the possibility structure for reasoning about space and was not just a change in terminology. Before the change it was inconceivable that certain Euclidean propositions about space should fail to be true. Such propositions had a special status that went beyond mere belief. Nevertheless, a rational transition required revision of these claims. An account of belief adequate to model such transitions must

(a) provide an appropriate representation for this kind of special status;

and

(b) allow for changes in what gets such status.

I shall attempt to provide such an account within the framework of a natural extension of the Bayesian model.

The idea that alternative fundamental theories, such as Newtonian particle mechanics and the particle mechanics of special relativity are incommensurable plays a prominent role in the discussions provoked by the writing of Kuhn and Feyerabend.[2] This idea has been used for many different purposes ranging from criticism of overly simplistic versions of the thesis that science is an ever-increasing accumulation of knowledge to suggestions that neither empirical evidence nor rational argument of any other kind can apply to the transitions involved in scientific revolutions.[3] Sometimes the radical skeptical version of incommensurability is defended by the thesis that there is a difference in conceptual framework between the dislodged and dislodging theory and that empirical tests, or other arguments, cannot decide between frameworks without begging the question in favor of one or the other.[4] I believe that this argument for the skeptical version of the incommensurability thesis can be disarmed if one has adequate accounts of conceptual framework and conceptual change. I shall attempt to provide such accounts.

One feature of my model is that it can represent transitions from a framework where nothing can count as evidence against one theory to a different framework where nothing can count as evidence against an incompatible theory. This feature seems to capture the kind of framework clash that the conceptual change argument for skeptical incommensurability appeals to. Another feature of my model is that on it this kind conceptual change can be rational and the choice between frameworks can be based on empirical evidence without begging the question in favor of one or the other. The combination of these two features allows one to grant the conceptual change premise without accepting the skeptical conclusion.

## 2. Background

The objects of belief are propositions construed set theoretically, rather than sentences. The domain of a belief function will be a field $E$ of subsets of a fundamental set T of alternative possible states of the world. The propositional operations AB, $\bar{A}$, and AvB are represented by the corresponding set theoretical operations, $A \cap B$, T-A, and $A \cup B$. Entailment is represented by inclusion so that the basic set T of alternatives is a proposition entailed by every proposition in $E$ and the empty set $\phi$ is a proposition that entails every proposition in $E$.

The point of constructing a model of rational belief is to illuminate some problem of decision or inquiry. This problem ought to determine what counts as the fundamental set of alternative possible states and what is to be the appropriate field of propositions. The states are to be specified in whatever detail is required to capture all the distinctions relevent to the problem. According to this version of the set theoretical treatment, propositions correspond exactly to Savage's events. ([28], pp. 8-12, 87-90).

The models we shall work with will be ones where the agent can accept corrigible propositions and in which he assigns degree of belief 1 to every proposition he accepts. Part of the motivation for the extensions I have already produced [10] is to provide rational models where one can have acceptance without great restrictions on the kinds of learning situations that can be represented. This motivation also applies to the work developed here. I hope that the flexibility and usefulness of these models will help to support the idea that acceptance can be a key concept for investigating rational belief change.

We shall assume that our agent is semantically omniscient with respect to the field of propositions in the model.[5] At any time the set $\nabla$ of propositions he accepts is consistent and closed under semantical consequence, so that

(1) $\bigcap\nabla \neq \phi$ , and

$\bigcap\nabla \subseteq A$ only if $A \in \nabla$ , for all $A \in E$ .

This makes $\bigcap\nabla$ act as a single proposition that corresponds to the total content of all the propositions the agent accepts. We shall call $\bigcap\nabla$ the agents "acceptance context".[6] If K is the acceptance context for an agent relative to $E$ , then

(3) The agent accepts A iff $K \subseteq A$ , for all $A \in E$

and

(4) $K = \bigcap\{A \in E : K \subseteq A\}$ .

Perhaps the most fundamental idealization in the orthodox Bayesian model for rational belief is coherence. Carnap makes coherence relative to possible bodies of evidence. We shall make it relative to possible acceptance contexts. An agent's degree of belief function P is <u>coherent</u>

with respect to acceptance context K and field of propositions $E$ just in case:

(1) P is a probability function on $E$[7]

(2) $P(A) = 1$ , if $K \subseteq \mathbf{A}$[8]

(3) $P(B|A) = P(A \cap B)/P(A)$ , if $P(A)>0$ ,

where A and B are any propositions in $E$ and $P(B|A)$ is the agent's conditional degree of belief in B on A.

The idea behind coherence is that belief is a functional state given by its role in guiding decisions. Where K is the agent's acceptance context the states in K are all and only the states he treats as live options. His degree of belief function, therefore, ought to be appropriate to guide decisions where the relevant outcomes are just the states in K. If $P(A)>0$ then the agent's conditional degrees of belief on A ought to be appropriate to guide decisions relative to an alternative context $K \cap A$, which can be regarded as the minimal revision of K needed to have a context where A is accepted.

The extensions are based on the same sort of rationale that supports the classical conditionalization model in the cases where it works. Before the shift the agent's conditional degrees of belief on A, represented by ratio conditional probabilities, guide his hypothetical reasoning relative to assumption A. Such reasoning is just reasoning appropriate to an alternative acceptance context $K \cap A$. These conditional beliefs allow the agent to plan how he would reason were he to undergo a learning experience in which the total epistemic input was to shift his acceptance context from K to $K \cap A$. Conditionalizing on A when such learning experience actually comes about is no more than to put into practice reasoning he could have already planned hypothetically. The generalizations of conditional belief our extensions are built on are simply ways in which a rational agent can reason hypothetically with respect to ways of modifying his acceptance context that correspond to different kinds of learning experiences.

## 3. Conditional Belief and Assumption

The first step is to generalize conditional probability so that $P_A$ is defined even when A is not compatible with what the agent accepts. Just as with orthodox conditional probability, the extended conditional probability function $P_A$ is to be a degree of belief function appropriate for the agent's hypothetical reasoning relative to assumption A. Suppose K is the agent's acceptance context. Hypothetical reasonsing relative to assumption A, for this agent, is to be reasoning relative to an alternative acceptance context K(A) which he treats as the minimal revision of K required to assume A. We shall call K(A) the agents "A-assumption context". The idea that K(A) is to be as much like K as is compatible with being a context where A is accepted motivates the following general constraints on assumption contexts.[9]

(0) $K(A) = \bigcap\{B\in E : K(A)\subseteq B\}$

(1) $K(A)\subseteq A$

(2) If $K\subseteq A$ then $K(A) = K$

(3) If $B\subseteq A$ and $K(A)\cap B \neq \phi$ then

$$K(B) = K(A)\cap B$$

We want $P_A$ to be a conditional belief function appropriate for guiding decisions where the relevent outcomes are exactly the states in K(A).

(a) If $K(A) \neq \phi$ , then $P_A$ is coherent with respect to K(A).

Given our constraints on assumption contexts, and the orthodox treatment of coherence, the consequent of (a) may be replaced by the following conjunction:

(a) (i) $P_A$ is a probability function on $E$ such that $P_A(B) = 1$ for all B where $K(A)\subseteq B$ , and

(a) (ii) If $B\subseteq A$ and $P_A(B) > 0$, then $P_B(C) = P_A(B\cap C)/P_A(B)$ .

The second conjunct, (a)(ii), is Renyi's main transition constraint on his extension of conditional probability [25]. It insures that, where $P_A(B) > 0$ and $B\subseteq A$, extended conditional probability on B will agree with the result of first moving to extended conditional probability on A and then conditionalizing on B by orthodox ratio conditional probability.

Where K(A) is empty the agent regards A as impossible in that, from his standpoint, no possible state is consistent with all the propositions he would be committed to were he to assume A. If every B-state is an A-state and our ideally rational agent treats A as impossible then he ought to treat B as impossible as well.

(4) If $B\subseteq A$ and $K(A) = \phi$ then $K(B) = \phi$ .

An empty K(A) is not a viable acceptance context at all, and cannot be used as a base from which to make further assumptions non-trivially.

(5) If $K(A) = \phi$ then $K(A)(B) = \phi$ .

It is convenient to have a convention to cover conditional belief relative to impossible assumptions.

(b) If $K(A) = \phi$ , then $P_A(B) = 1$ for every B .

This convention reflects the idea that when K(A) is empty the agent regards A as so absurd that accepting A would require accepting anything whatever.

We want to be able to iterate shifts by extended conditionalization. Our extension of conditional probability lends itself naturally to this demand. Where K(A) is non-empty it is, itself, a possible acceptance context and can be a base for further revisions to accommodate new assumptions. For example, K(A)(B) is to be regarded as the result of first minimally revising K to assume A, and then minimally revising K(A) to assume B. Similarly, $P_{(A)(B)}$ is to be regarded as the belief function corresponding to this sequence of assumptions.

In earlier work ([8] and [9]) I argued that extended conditional belief functions ought to satisfy Karl Popper's characterization of conditional probability. Popper takes conditional probability as primitive and has it defined for all pairs. Any function P mapping $E \times E$ into the reals such that $P(B|A) \neq 1$ for some A and B in $E$ and satisfying the following axioms for all A, B and C in $E$ is a Popper function on $E$.

(1) $0 \leq P(B|A) \leq P(A|A) = 1$

(2) If $P(C|A) \neq 1$ then $P(\bar{B}|A) = 1 - p(B|A)$

(3) $P(AB|C) = P(A|C)\ P(B|AC)$ .[10]

Absolute probabilities are defined as conditional probabilities relative to T so that $P(A) = P(A|T)$. What makes Popper functions useful for representing extended conditional belief is that $P(B|A)$ can be non-trivial even when $P(A) = 0$.

The special values $P(A|\bar{A}) = 1$ and $P(\bar{A}|A) = 1$ are also of interest.

(a) $P(A|\bar{A}) = 1$ iff $P(A|C) = 1$ for all C

(b) $P(\bar{A}|A) = 1$ iff $P(C|A) = 1$ for all C .

The first of these special values characterizes what we may call P-necessity. The second characterizes what we may call P-impossibility. Popper's axioms guarantee that whenever $P(\bar{A}|A) = 1$ then $P_A$ is a classical probability function on $E$.

Suppose $E$ is a field of subsets of T, K is a possible acceptance context for $E$ (ie., $K \neq \phi$ and $K = \bigcap\{A \in E : K \subseteq A\}$), for all finite sequences $C_0 \ldots C_{n-1}$ drawn from $E$ and $A, B \in E$, $K(C_0) \ldots (C_{n-1})(A)$ and $K(C_0) \ldots (C_{n-1})(B)$ satisfy conditions (0) - (5) on assumption contexts and $P_{(C_0) \ldots (C_{n-1})}$ satisfies conditions (a) and (b) with respect to $K(C_0) \ldots (C_{n-1})$.

Theorem 3.1 If $K(C_o) \ldots (C_{n-1}) \neq \phi$ , then the function

$P_{<C>_n}(B|A) = P_{(C_o) \ldots (C_{n-1})(A)}(B)$ for

$A, B \in E$ is a Popper function on $E \times E$ such

that[11]

(1) $P_{<C>_n}(A|T) = P_{(C_o) \ldots (C_{n-1})}(A)$

(2) $P_{<C>_n}(A|\bar{A}) = 1$ iff $K(C_o) \ldots (C_{n-1})(\bar{A}) = \phi$

(3) $P_{<C>_n}(\bar{A}|A) = 1$ iff $K(C_o) \ldots (C_{n-1})(A) = \phi$ .

The proof of this theorem is a straight forward variation on that of a similar theorem in ([8], pp. 236-239).

Suppose $E$ T K and P are as in the hypothesis of Theorem 1 and

$$K^* = \bigcap\{A \in E : K(\bar{A}) = \phi\} .$$

This set of worlds K* acts as an accessibility region around K which determines $P_{<C>_n}$ - possibility for all the $P_{<C>_n}$'s corresponding to non-empty contexts $K(C_o) \ldots (C_{n-1})$.

Theorem 3.2 If $K(C_o) \ldots (C_{n-1}) \neq \phi$ then

(1) $K(C_o) \ldots (C_{n-1})(A) = \phi$ iff $K^* \cap A = \phi$

(2) $K(C_o) \ldots (C_{n-1}) \subseteq K^*$

(3) $P_{<C>_n}(A|\bar{A}) = 1$ iff $K^* \subseteq A$

(4) $P_{<C>_n}(\bar{A}|A) = 1$ iff $K^* \cap A = \phi$ .

These all follow easily from condition (5) on assumption contexts, the definition of K* and Theorem 1.

This fixed K* determines whether or not we allow for non-trivial revisions where there is a clash with previously accepted evidence. A system where K* = K is no better than a ratio conditional probability because $P_A$ will be trivial whenever $K \cap A$ is empty.

## 4. Construction of Assumption Contexts[12]

We want a way to construct non-trivial sequences of assumption contexts. The basic idea will be that K(A) ought to be the acceptance context generated by the set of states in A that differ minimally from

K. We shall want to define the acceptance context ac Z generated by any non-empty subset Z of T, the set K<A> of nearest possible A-states to K and have K(A) = ac K<A>. Suppose $E$ is a field of subsets of T, $K = \bigcap\{A \in E : K \subseteq A\}$ and Z is a non-empty subset of T.

D 4 (1) $\text{ac } Z = \bigcap\{A \in E : Z \subseteq A\}$

(2) $K\langle A\rangle = \{u \in A \cap K^* : u \leq_K v \text{ all } v \in A \cap K^*\}$

(3)(a) $K(A) = \text{ac } K\langle A\rangle$, if $K \neq \phi$

(b) $K(A) = \phi$, if $K = \phi$,

where $K^*$ is the set of states deemed possible relative to K and $u \leq_K v$ is some appropriate relation of comparative similarity of states to K. The main task before us is to produce an appropriate relation of comparative similarity of states to acceptance contexts.

Think of the difference between an acceptance context K and a state u as the amount of information contained in K that u conflicts with. We want the amount of information in K that u conflicts with to be an increasing function of the set of propositions accepted relative to K that u conflicts with. This motivation is sufficient to put some general constraints on any adequate method of comparing amounts of information on which states differ from K. The following notation will help formulate these constraints, and will be used again when we give detailed methods for making comparisons. For any subset $S$ of $E$, let

D 4 (4)(a) $SK = \{A \in S : K \subseteq A\}$, and

(b) $SKu = \{A \in SK : u \in \overline{A}\}$.

Here, $SK$ is the set of propositions in $S$ that are accepted relative to K, and $SKu$ is the set of propositions in $SK$ that u conflicts with. We are now ready to formulate our constraints on comparative similarity.

C 1 If $EKu \subseteq EKv$ then $u \leq_K v$.

If the set of all propositions accepted relative to K that u conflicts with is included in the set of all propositions accepted relative to K that v conflicts with then the amount of information on which u differs from K is not greater from the amount of information on which v differs from K.

C 2 If $u \in K$ and $v \notin K$ then $u <_K v$,

where $u <_K v$ iff $u \leq_K v$ and $v \not\leq_K u$.

Any state in K must agree with every proposition accepted relative to K; therefore, no state in K can conflict with any information contained in K.

Any state not in K, however, must conflict with some proposition accepted relative to K, because K = ac K. Thus, any state outside K must conflict with more information contained in K than any state that is in K.

An appropriate comparative similarity relation of states to K must also satisfy transitivity.

C 3 If $u \leq_K v$ and $v \leq_K w$ then $u \leq_K w$ .

This is a natural constraint on any adequate relation of overall comparative similarity.

These constraints are sufficient to insure that any K(A) and K(B) defined by D 4 with respect to an adequate $\leq_K$ satisfy conditions (0) - (3) on assumption contexts.

Theorem 4.1 If $E$ is a field of subsets of T, K is a non-empty possible acceptance context for $E$, $K \subseteq K^*$ and $\leq_K$ satisfies cl - c3, then K(A) and K(B) satisfy (0) - (3) for all A, $B \in E$ .[13]

This theorem helps to give additional motivation for conditions (0) - (3) on assumption contexts. It is natural to let the A-assumption context be the set of accessible A-states which conflict with the least amount of information contained in K. Any reasonable way of comparing such amounts of information ought to satisfy C 1 - C 3. Theorem 4.1, therefore generates the minimal revision conditions on assumption context from very general and basic ideas about comparisons of amounts of information on which states differ from acceptance contexts.

In addition to satisfying the minimal revision constraints (0) - (3), we need to insure that K* acts as a proper accessibility region and that conditions (4) and (5) on empty assumption contexts will be met. Condition (5) that $K(A)(B) = \phi$ when $K(A) = \phi$ will be trivially met if K(A)(B) is defined by iterated applications of D 4. If we insure that $K^* = acK^*$ as well as that $K \subseteq K^*$ the rest of what we want will follow from the following constraint on comparative similarity.

C 4 If $Z \subseteq T$ and $Z \neq \phi$ then some $u \in Z$ is such that $u \leq_K v$ for all $v \in Z$ .

This constraint requires that there be some $\leq_K$ minimal elements in every non-empty subset of T. We regard as adequate only those comparative similarity relations that satisfy it.[14]

Condition (4) on empty contexts,

(4) $K(B) = \phi$ , if $B \subseteq A$ and $K(A) = \phi$ ,

and the proper performance of K* as an accessibility region,

$$K^* = \cap\{A \in E : K(\bar{A}) = \phi\}$$

both follow trivially once C 4 is added to what we already have. Non-trivial sequences of assumption contexts can, therefore, be generated by iterated applications of D 4 if we have available appropriate non-trivial comparative similarity relations.

We want a detailed way of comparing amounts of information on which states differ from an acceptance context that will yield interesting comparative similarity relations that satisfy C1 - C4. Our comparison will be made relative to a sequence $S$ of subsets of the set of accepted propositions. Think of this sequence as an ordering according to importance, where $S_o$ is the set of accepted propositions the agent regards as most important to preserve, the propositions in $S_1$ are regarded as next most important, and so on. We consider only finite sequences where the intersection of the union of the range of $S$ is K.

For any stage $S_i$ in the sequence we can compare the cardinality of the set of K-accepted propositions in $S_i$ with which u conflicts to the cardinality of the set of K-accepted propositions in $S_i$ with which v conflicts. The comparative similarity of u and v to K relative to sequence $S$ is given lexicographically by the sequence of these cardinality comparisons. Where $S$ is a sequence with n as its domain such that $\bigcap[\bigcup_{i<n} S_i] = K$:

D 5 (i) $u \overset{S}{\underset{K}{<}} v$ iff for some $i<n$, $|S_i Ku| < |S_i Kv|$

and $|S_j Ku| = |S_j Kv|$ for all $j<i$ .

(ii) $u \overset{S}{\underset{K}{=}} v$ iff $|S_i Ku| = |S_i Kv|$ for all $i<n$ .

Theorem 4.2 If $E$ is a field of subsets of T, $K \neq \phi$ and K = ac K,

and $S$ is a sequence with domain n such that

$$\bigcap[\bigcup_{i<n} S_i] = K$$

then $\overset{S}{\underset{K}{<}}$ satisfies C1 - C4.[15]

Sequences that correspond to agent's who treat some propositions as more important to preserve than others will lead to interesting assumption contexts in which K(A) is significantly stronger than $K^* \cap A$. Such non-trivial assumption contexts preserve significant portions of the information that was accepted relative to K.

## 5. Question Openings

The extension of the Bayesian learning model presented in [9] provided for hypothetical reasoning relative to question openings. A question opening revision occurs when an agent moves to a weaker acceptance context in order to open up for investigation some proposition that conflicts with what he previously accepted.[16] Where K is the agent's acceptance context and A is a proposition,

$$K(?A) = K \cup K(A)$$

is the agent's question opening context with respect to A. It is the context he treats as the minimal revision of K needed to avoid begging the question against A.

## 6. Conceptual Frameworks

One of the most interesting features of a Popper function on $E$ is the equivalence relation it generates.

$$A \underset{P}{\sim} B \text{ iff } P(A|C) = P(B|C) \text{ all } C \in E \ .$$

We say that A and B are "P-equivalent" when $A \underset{P}{\sim} B$. This relation induces an algebra of equivalences classes in just the same way that a logic induces a Lindenbaum algebra on a calculus.

$$[A]_P = \{B \in E : A \underset{P}{\sim} B\}$$

$$E/_P = \{[A]_P : A \in E\}$$

$$[A]_P \wedge [B]_P = [AB]_P$$

$$\neg [A]_P = [\bar{A}]_P$$

$$E//_P = (E/_P, \wedge, \neg) \ .$$

The maximum element is the equivalence class of all A such that $P(A|\bar{A}) = 1$, so that P-necessity in the Popper function algebra corresponds to theoremhood in the Lindenbaum algebra. The minimum element is the equivalence class of all A such that $P(\bar{A}|A) = 1$, so that P-impossibility corresponds to inconsistency. The relation of logical equivalence underlies the Lindenbaum algebra just as P-equivalence does the Popper function algebra.

These parallels suggest that the structure imposed by the agent's extended conditional belief function underlies his inductive reasoning with respect to propositions in $E$ in a way similar to that in which a logic would underlie deductive inference relations among sentences. This suggestion is strengthened when one looks more closely at the roles played by various parts of the P-equivalence structure.

We have already noted that to treat A as P-impossible is to treat it as though assuming A would commit one to a contradiction. A proposition A is P-necessary just in case

$$P(A|C) = 1 \text{ for all } C \in E \ ;$$

therefore, an agent who treated A as P-necessary would count no proposition whatever as evidence against A. This is just the attitude one would take toward a proposition he treated as *a priori*. Propositions A and B are P-equivalent just in case

$$P(A|C) = P(B|C) \text{ for all } C \in E \ ,$$

so that the agent would count no assumption whatever as evidence that distinguishes situations where one holds from situations where the other holds. Any assumption will be regarded as support for one to exactly the same degree as it is regarded as support for the other. When A and B are P-equivalent we also have

$$P(C|A) = P(C|B) \text{ for all } C \in E$$

so that P-equivalent propositions are no more distinguishable in their role as possible assumptions from which to evaluate other propositions than they are on the basis of assumptions used to evaluate them.

I propose to identify the agent's conceptual framework with respect to field of propositions $E$ and the decision or inquiry problem for which belief is modeled with the structure imposed on $E$ by propositions he treats as P-equivalent.[17] This makes the agent's conceptual framework reflect whatever *a priori* constraints there are on his inductive reasoning.

We have already noted that the accessibility region K* determines which propositions are treated as P-necessary and which are treated as P-impossible. It is not hard to show that K* also determines P-equivalence. Suppose $E$, K, K* and P are as in the hypothesis of Theorem 3.2.

Theorem 6.1 $\quad A \underset{P}{\approx} B$ iff $K^* \cap A = K^* \cap B$ .[18]

The accessibility region K* is such that any state in the symmetric difference $A\bar{B} \cup \bar{A}B$ of A and B is regarded as impossible if A and B are P-equivalent.

7. Postulating Necessity and Possibility[19]

We want to extend our agent's resources for hypothetical reasoning so that he can entertain hypothetical revisions of his conceptual framework. The accessibility region $K^*$ provides the key to this extension. In order to focus on this role let us represent the agent's acceptance context with the pair

$$(K,K^*) ,$$

where $K$ is the intersection of all propositions he accepts and $K^*$ is the intersection of all the propositions he treats as P-necessary. On this notation assumptions and question openings only modify the first constituent of the acceptance pair. The assumption pair $(K,K^*)(A)$ and question opening pair $(K,K^*)(?A)$ are, therefore, as follows:

$$(K,K^*)(A) = (K(A), K^*)$$

$$(K,K^*)(?A) = (K(?A), K^*) .$$

Modifications of the agent's conceptual framework can be represented by modifications of the second constituent of the acceptance pair.

One kind of revision we want the agent to be able to entertain is that of postulating P-necessary status for a proposition A. Let

$$(K,K^*)(\Box A)$$

be an alternative context pair $(X,X^*)$ which the agent treats as the minimal revision of $(K,K^*)$ required to have a context pair where A is P-necessary. We shall call $(X,X^*)$ the "$\Box$A-<u>postulate pair</u> with respect to $(K,K^*)$". Hypothetical reasoning relative to postulating P-necessity for A is just hypothetical reasoning relative to $(X,X^*)$.

We want the accessibility region $X^*$ in the $(K,K^*)$ agent's $\Box$A-postulate pair to act as a minimal revision of $K^*$ to have an A-region. This suggests that $X^*$ ought to satisfy

$$X^* = ac\ \{u \in A : u \leq_{K^*} v \ \text{ all } v \in A\} ,$$

where $\leq_{K^*}$ is a relation of comparative similarity of states to $K^*$ appropriate to reflect the relative importance the agent attaches to preserving the special status of the various propositions he treats as P-necessary. We want the acceptance context $X$ of the $\Box$A-context pair to act as a minimal revision of $K$ required to have a context with $X^*$ as its accessibility region. This suggests that $X$ ought to satisfy

$$X = ac\ \{u \in X^* : u \leq_K v \text{ all } v \in X^*\} ,$$

where $\leq_K$ is a relation of comparative similarity of states to $K$ that reflects the relative importance the agent attaches to preserving his acceptance of the various propositions he accepts.

Another kind of revision we want our agent to be able to entertain is that of opening up his accessibility region to make proposition A possible. Let,

$$(K,K^*)\ (\Diamond A)$$

be an alternative context pair (Y,Y*) which the agent treats as the minimal revision of (K,K*) to have a context pair where A is P-possible. We shall call (Y,Y*) the agents $\Diamond$A-postulate pair with respect to (K,K*). Hypothetical reasoning relative to postulating P-possibility for A is just hypothetical reasoning relative (Y,Y*).

The relation between K* and Y* is to be exactly analogous to the relation between K and K(?A). We want Y* to be an expansion of K* which lets in all and only the A-states which differ minimally from K*. This gives the condition

$$Y^* = K^* \cup X^* \ ,$$

where $$X^* = \{u \in A : u \leq_{K^*} v \text{ all } v \in A\} \quad \text{, as before.}$$

There is no need to change the acceptance context, because all we have done to the accessibility region is to enlarge it; therefore the old acceptance context K is appropriate for the new accessibility region Y*. This gives the condition

$$Y = K\ .$$

Where $G$ is a finite set of propositions we want to be able to hypothetically reason relative to a move to simultaneously postulate possibility to all of them. We also want to be able to simultaneously postulate necessity for finite sets of propositions. Postulating simultaneous necessity for all the propositions in $G$ should take us from accessibility region K* to accessibility region $K^*(\bigcap G)$ which is the accessibility region for $(K,K^*)\ (\Box \bigcap G)$. Simultaneous postulation of possibility for all A in $G$ should take us to accessibility region

$$K^* \cup \bigcup_{A \in G} K^*(A) \ ,$$

where each K*(A) is the accessibility region for $(K,K^*)(\Box A)$. This need not correspond to any move to postulate possibility, or anything else, to any single proposition.

Where $G$ is a finite set of propositions let

$$(K,K^*)(\Diamond G) = (K,\ K^* \cup \bigcup_{A \in G} K^*(A)) \ ,$$

so that a $\Diamond G$ move is a revision to simultaneously postulate possibility to all the propositions in $G$.

We shall also want to represent a move to postulate necessity for proposition A combined with simultaneous possibility for the propositions in finite subset $G$ of $E$. This requires another addition to our representation.

Let

$$(K,K^*)(\Box A, \Diamond G) = (Z,Z^*)$$

where

$$Z^* = ac\ \{u \in \bigcup_{B\in G}(A \cap B) : u \leq_{K^*} v \text{ all } v \in \bigcup_{B\in G}(A\cap B)\}$$

$$Z = ac\ \{u \in Z^* : u \leq_K v \text{ all } v \in Z^*\}\ , \text{ if } K \neq \phi$$

$$= \phi\ , \text{ if } K = \phi\ .$$

We now have all the facilities for modal postulation we need.

## 8. Kinematics of Conceptual Change

We want to model sequences of revision which include question openings, and all three kinds of modal postulations, as well as assumption makings. We can use an occurrence of proposition A to represent a move to assume A in a sequence of revisions. We need a distinct way to represent a move to postulate necessity for A. Let $\Box$ be a one-one function with domain $E$ and range disjoint from $E \cup T$. An occurrence of $\Box A$ can represent a move to postulate necessity for A in a sequence of revisions. For each finite subset $G$ of $E$ we need distinct ways of representing a move to simultaneously open the question for all the propositions in $G$ and a way to represent a move to postulate simultaneous possibility for all the propositions in $G$. Let ? and $\Diamond$ be one-one functions each with the set of all finite subsets of $E$ as its domain and such that their ranges are disjoint from one another and both disjoint from $T \cup E \cup \text{Rng}\ \Box$. We can use pairs drawn from $E \times \text{Rng}\ ?$ and $\text{Rng}\ \Box \times \text{Rng}\ \Diamond$ respectively to represent moves to combine assumption with question opening and moves to combine necessity and possibility postulation.

Let $H = E \cup \text{Rng}\ \Box \cup \text{Rng}\ ? \cup \text{Rng}\ \Diamond \cup (E \times \text{Rng}\ ?) \cup (\text{Rng}\ \Box \times \text{Rng}\ \Diamond)$.

Sequences drawn from H can represent all the sequences of revision we want.

Where $(C)_n$ is an n-term sequence drawn from H and Q is an element of H, $(C)_n(Q)$ is the n + 1 term sequence resulting from adding Q to the end of $(C)_n$. The sequence $(C)_n\ (\Diamond G)$, for example, represents a move to postulate simultaneous possibility for all the propositions in finite subset $G$ of $E$ as the next revision following the sequence of revisions indicated by $(C)_n$.

We shall define a system of sequences of possible acceptance pairs that can represent various possible sequences of revision. The acceptance function $X$ will assign a pair (K,K*) to each finite sequence drawn from H. The pair $X\phi = (K_o, K_o^*)$, which $X$ assigns to the empty sequence $\phi$, is the acceptance context $K_o$ and accessibility region $K_o^*$ from which all the sequences of revision originate. We require $K_o = \text{ac}\ K_o$, $K_o^* = \text{ac}\ K_o^*$, $K_o \neq \phi$ and $K_o \subseteq K_o^*$, in order to have $(K_o, K_o^*)$ play the role of a non-trivial acceptance pair.

We shall also have a function $\leq$ on the range of $X$ which assigns to each pair $(K,K^*)$ a pair

$$(\leq_K\ ,\ \leq_{K^*})$$

of similarity relations such that $\leq_{K^*}$ is appropriate to $K^*$ and $\leq_K$ is appropriate to K if $K \neq \phi$.

D 8.1 $(E\mathrm{T}?\Box\Diamond X\leq)$ <u>is an acceptance system</u> iff $E$ is a field of subsets of T; $\Box$ is a one-one function with doman $E$ and range disjoint from $E \cup \mathrm{T}$, ? and $\Diamond$ are one-one functions each with the set of all finite subsets of $E$ as domain and such that their ranges are disjoint from one another and both disjoint from $E \cup \mathrm{T} \cup \text{Rng}\ \Box$; $X$ maps $\bigcup_{n\in\omega} H^n$ into $P\mathrm{T} \times P\mathrm{T}$, where $H = E \cup \text{Rng}\ \Box \cup \text{Rng}\ ? \cup \text{Rng}\ \Diamond \cup (E \times \text{Rng}\ ?) \cup (\text{Rng}\ \Box \times \text{Rng}\ \Diamond)$, and for $(K_o, K_o^*) = X\phi$, $K_o = \text{ac}\ K_o$, $K_o^* = \text{ac}\ K_o^*$, $K_o \subseteq K_o^*$ and $K_o \neq \phi$; $\leq$ is a function on the range of $X$ which assigns a pair $(\leq_K, \leq_{K^*})$ of comparative similarity relations appropriate respectively to K and $K^*$ to each $(K,K^*)$ in the range of $X$ where $K \neq \phi$, finally, for all $(C)_n \in \text{Dom}X$, $A \in E$, finite subsets $G$ of $E$, $(K,K^*) = X(C)_n$ and $(\leq_K, \leq_{K^*})$

(a) $X(C)_n(A) = (K(A)\ ,\ K^*)$ , where

$$K(A) = \text{ac}\ \{u \in A \cap K^* : u \leq_K v \text{ all } v \in A \cap K^*\}\ ,\ \text{if } K \neq \phi$$

$$= \phi\ ,\ \text{if } K = \phi$$

(b) $X(C)_n(?G) = (K \cup \bigcup_{A\in G} K(A)\ ,\ K^*)$

(c) $X(C)_n\ (\Box A) = (K^o\ ,\ K^*(A))$ , where

$$K^*(A) = \text{ac}\ \{u \in A : u \leq_{K^*} v \text{ all } v \in A\} \text{ and}$$

$$K^o(A) = \text{ac}\ \{u \in K^*(A) : u \leq_K v \text{ all } v \in K^*(A)\}\ ,\ \text{if } K \neq \phi$$

$$= \phi\ ,\ \text{if } K = \phi$$

(d) $X(C)_n(\Diamond G) = (K\ ,\ K^* \bigcup_{A\in G} K^*(A))$

(e) $X(C)_n\ (\Box A\ , \Diamond G) = (Z\ ,\ Z^*)$ , where

$$Z^* = \text{ac } \{u \epsilon \bigcup_{B \epsilon G} (A \cap B) : u \leq_{K^*} v \text{ all } v \epsilon \bigcup_{B \epsilon G} (A \cap B)\} \quad \text{and}$$

$$Z = \text{ac } \{u \epsilon Z^* : u \leq_K v \text{ all } v \epsilon Z^*\} \text{ , if } K \neq \phi$$

$$= \phi \text{ , if } K = \phi$$

This definition is also a recipe for constructing acceptance systems. Given a pair $(K_o, K_o^*)$ satisfying our conditions on $X\phi$, any pair of appropriate comparative similarity relations $\leq K_o$ and $\leq K_o{}^*$ will generate all the pairs $(K_o, K_o^*)(C)$ for $C \epsilon H$. Such pairs with non-empty first constraints will, in turn, satisfy our starting conditions, and comparative similarity relations appropriate to them will generate the next layer of pairs. In this way all the pairs can be generated by recursion on a length lexicographical ordering of the sequences in the domain of $X$.

To get our belief systems we simply add degree of belief functions that satisfy appropriate coherence constraints with respect to acceptance systems.

D 8.2 $(E T \square ? \Diamond X \leq P)$ <u>is a belief system</u> iff $(E T \square ? \Diamond X \leq)$ is an acceptance system, and $P$ is a function on the range of $X$ such that for each pair $(K,K^*)$

(1) $P_{(K,K^*)}$ is coherent with respect to K , if $K \neq \phi$

(2) $P_{(K,K^*)}(A) = 1$ for all $A \epsilon E$ , if $K = \phi$ .

These are the same conditions we used for conditional belief relative to assumptions.

It will be convenient to have a standard way of referring to first and second constituents of ordered pairs. For any pair $(K,K^*)$

$$1 \ [(K,K^*)] = K \quad \text{and} \quad 2 \ [(K,K^*)] = K^* \ .$$

We are now ready to state the main consequences of our definitions.

Theorem 8.1 Suppose $(E T \square ? \Diamond X \leq P)$ is a belief system, $(C)_n \epsilon \text{Dom} X$[20]

$X(C)_n = (K,K^*)$ , $K' \epsilon \{K,K^*\}$ ; for all $A \epsilon E$ and finite subsets $G$ of $E$ ,

$K(A) = 1[X(C)_n(A)]$ , $K(?G) = 1[X(C)_n(?G)]$ ,

$K^*(A) = 2[X(C)_n(\square A)]$ , $K^*(?G) = 2[X(C)_n(\Diamond G)]$ ;

and, for all $A \epsilon E$ and $Q \epsilon H = E \cup \text{Rng } \square \cup \text{Rng} ? \cup \text{Rng} \Diamond$

$\cup (E \times \text{Rng}?) \cup (\text{Rng } \square \times \text{Rng} \Diamond)$ ,

$P(A|Q) = P_{(K,K^*)(Q)}(A)$ and $P(A) = P_{(K,K^*)}(A)$ .

If all this holds, $Q \in H$ ; $G$ is a finite subset of H and A, $B \in E$, then all the following hold:

I (a) If $K = \phi$ then $1[X(C)_n(Q)] = \phi$

(b) $K \subseteq K^*$ and, $K^* = \bigcap \{A \in E : K(\bar{A})\}$ , if $K \neq \phi$

(c) If $A \neq \phi$ then $K^*(A) \neq \phi$ ;

II (0) $K'(A) = \text{ac}\, K'(A)$

(1) $K'(A) \subseteq A$

(2) If $K' \subseteq A$ then $K'(A) = K'$

(3) If $B \subseteq A$ and $K'(A) \cap B \neq \phi$ then $K'(B) = K'(A) \cap B$

(4) If $B \subseteq A$ and $K'(A) = \phi$ then $K'(B) = \phi$ ;

III $K'(?G) = K' \cup \bigcup_{B \in G} K'(B)$ ;

IV (a) $(K,K^*)(\Box A) = (K,K^*)$ iff $K^* \subseteq A$

(b) $K^*(A) = K^* \cap A$ , if $K^* \cap A \neq \phi$

(c) $(K,K^*)(\Box A) = (K \cap A , K^* \cap A)$ iff $K \cap A \neq \phi$

(d) $(K,K^*)(\Diamond G) = (K,K^*)$ iff $K^* \cap B \neq \phi$ all $B \in G$

(e) $(K,K^*)(\Box A, \Diamond G) = (K,K^*)$ iff $K^* \subseteq A$ and $K^* \cap B \neq \phi$ , for all $B \in G$ ;

V If $K \neq \phi$ then

(a) $P(-|-)$ as a function in $E \times E$ is a Popper function

(b) $P(A|\bar{A}) = 1$ iff $K^* \subseteq A$ ;

VI (a) If $K \cap A \neq \phi$ then $P(B|?\{A\}) = P(B)$

(b) $P(B|?\{A\}) = P(B) \cdot P(\bar{A}|?\{A\}) + P(B|A) \cdot P(A|?\{A\})$

(c) $P(B|?G) = P(B) \cdot P(\bar{G}|?G) + \sum_{A \in G} P(B|A) \cdot P(A|?G)$ , if propositions in $G$ are pairwise disjoint and all

incompatible with K and $\bar{G} = \bigcap\{\bar{A} : A \epsilon G\}$ .

The second group of consequences shows that necessity postulations with respect to an accessibility region, as well as assumptions with respect to an acceptance context, satisfy the basic conditions on assumptions. The third group contains the single result that possibility postulations with respect to an accessibility region behave just like question openings behave with respect to an acceptance context. The fourth group gives some of the desirable special consequences of the second and third groups.

The last group gives constraints on extended conditional probability with respect to question openings. Suppose $G$ is a finite set of pairwize disjoint propositions all of which are incompatible with K, and $\bar{G} = \bigcap\{\bar{A} : A \epsilon G\}$. Then $P(B|?G)$ is a weighted sum of the old probability of B and the extended conditional probabilities $P(B|A)$ for $A \epsilon G$, where the respective weights are $P(\bar{G}|?G)$ for $P(B)$ and $P(A|?G)$ for each A. When one sees that each $P(A|?G)$ acts just like the probability of K(A) relative to K(?$G$) and that $P(\bar{G}|?G)$ acts just like the probability of K relative to K(?$G$) he sees that this condition (c) on $P(B|?G)$ is appropriate. Condition (b) holds as a special case of (c) when $K \cap A = \phi$. It also holds when $K \cap A \neq \phi$, in which case it reduces to condition (a). All these are plausible properties of conditional probability with respect to question openings.

## 9. Confirmational Commitment

Belief systems correspond to families of confirmation systems where each accessibility region K* has its own confirmation function $C_{K*}$ such that for each acceptance pair (K,K*) and $A \epsilon E$,

$$P_{(K,K*)}(A) = C_{K*}(A|K) \text{ if } K \neq \phi$$
$$= 1 \text{ otherwise .}$$

If each $P_{(K,K*)}$ satisfies this condition with respect to $C_{K*}$ then the role played by $C_{K*}$ for the accessibility region is that of expressing what Issac Levi calls "Confirmational Commitment".[21] It assigns to each non-empty acceptance context the degree of belief function which the agent treats as the appropriate one for reasoning relative to that context.

Having confirmational commitment is to conform to a kind of path independence. On it the belief function you arrive at after a series of transitions depends only on what acceptance context you reach, not on the specific transitions that got you there.

We need confirmation systems somewhat more general than Carnap's. The following definition is suggested by Alfred Renyi's generalization of conditional probability [25].

D 8.1 $(ETKC)$ is a confirmation system iff $E$ is a field of subsets of T, $K$ is a set of non-empty subsets of T, and $C$ maps $E \times K$ into the reals such that for all A, B$\in E$ and K$\in K$.

(1) $C(-|K)$ is a probability function on $E$, and

$C(A|K) = 1$ if $K \subseteq A$

(2) If $K \cap A \in K$ and $C(A|K) > 0$, then

$C(B|K \cap A) = C(A \cap B|K) / C(A|K)$ .[22]

A belief system will determine a confirmation system for each accessibility region, and a family of confirmation functions corresponding to the accessibility regions of an acceptance system will determine a belief system.

Suppose $(ET\square?\Diamond X\leq)$ is an acceptance system and for each of its accessibility regions K*, let

$$K_{K^*} = \{K : (K,K^*) \in \text{Rng } X \text{ and } K \neq \phi\} .$$

This is the set of non-empty acceptance contexts with respect to K*.

Theorem 9.1 (1) If $(ET\square?\Diamond X\leq P)$ is a belief system, K* is an accessibility region for $X$, and $C_{K^*}$ is a function on $E \times K_{K^*}$ such that

$$C_{K^*}(A|K) = P_{(K,K^*)}(A)$$

for $K \in K_{K^*}$ and $A \in E$ , then

$(ETK_{K^*}C_{K^*})$ is a confirmation system.

(2) If for each accessibility region K* there is a function $C_{K^*}$ such that $(ETK_{K^*}C_{K^*})$ is a confirmation system and $P$ is a function on the range of $X$ such that

$$P_{(K,K^*)}(A) = C_{K^*}(A|K) \text{ if } K \neq \phi$$
$$= 1 \text{ otherwise}$$

for each (K,K*) in the range of $X$ and $A \in E$ ,

then $(ET\square?\Diamond X\leq P)$ is a belief system.

## 10. Theory Change[23]

Pioneering work of Robb [27], Mckinsey and Suppes ([20], [21]) and later work by Suppes [37] and Sneed [30] suggested that the fundamental postulates of classical particle mechanics and of the particle mechanics of special relativity can be illuminatingly formulated as set theoretical predicates specifying sets of structures that can represent systems of particles. If one can find a set T of mathematical structures such that the particle systems of classical mechanics and special relativity respectively constitute subsets K* and Y* of T, then he can apply my model to represent the conceptual change from classical theory to special relativity as a shift from acccessibility region K* to accessibility region Y*. I believe that such formulations and such a T can be found for almost any example of dislodged theory and dislodging theory that has been cited as a scientific revolution.

Useful things can be learned from such applications. It has been suggested to me, for example, that Robb's formulation might be used to render the change from classical particle kinematics to the particle kinematics of special relativity as the result of a single revision[24] to postulate that the speed of light is invariant with respect to inertial frames. One interest in modelling such a transition in my system would be to read off what classical propositions have to be regarded as more important and which as less important to preserve in order to obtain a comparative similarity relation that makes the kinematics of special relativity result from this postulation move. This sort of thing could be profitable in other cases where historical or systematic reasons could establish what revisions one wants to have result in the transformation of one theory into another.

## 11. Contextual *a priori*

Our systems illustrate an important point about the relation between certainty and corrigibility. This is that certainty, even in the highest degree where one would count nothing as evidence that he is mistaken, need not be confused with incorrigibility. $P$-necessity is a kind of contextual *a priori*, where which propositions get this special status can depend on circumstances that are subject to change.

## 12. Hypothetically Postulating the Impossible

Our notion of belief system and conceptual framework are rich enough to allow for an agent who can reason hypothetically in a way appropriate to a framework incompatible with his own. Such reasoning is something most of us are quite capable of doing. When, in the 18th century, Lambert attempted to demonstrate that violation of the parallels postulate would violate other axioms of Euclidean geometry he did not have to be less than absolutely certain that such an attempt could succeed in order to rationally pursue it.

## 13. Dynamics of Conceptual Change

The kind of clash of conceptual framework involved in a scientific revolution need not be based on misunderstanding. One advantage of our systems is that they can model framework clashes in which agent's understand each other and yet continue to disagree. The main disadvantage of these systems, as we have developed them so far, is that no mechanism is provided for rational resolution of such disagreement. We can model a transition that takes place; but the revisions that lead to the transition are treated as unanalysed inputs. Such a treatment of conceptual change is like a treatment of motion without forces. It can describe transitions that take place, but it can't explain why other transitions did not take place instead. This is why we entitled Section 8 "Kinematics of Conceptual Change". We want to extend our account to allow for rational resolution of conflict between frameworks. Such an extension will be part of what might be called "Dynamics of Conceptual Change".[25]

Suppose $(E T ? \Box \Diamond X \leq P)$ is a belief system, $(K,K^*) = X\phi$ is an acceptance pair where $K^*$ is the accessibility region corresponding to one fundamental theory $T_o$ and $(Y,Y^*) = (K,K^*)\ (C)_n$ is an acceptance pair where $Y^*$ is the accessibility region corresponding to an alternative fundamental theory $T_1$. We want to model situations where the agent can use evidence to decide whether or not to make the transition from accessibility region $K^*$ to accessibility region $Y^*$.

Let $k$ and $y$ be one-one functions with $K^*$ and $Y^*$ as their respective domains, and be such that their ranges are disjoint from one another and each disjoint from $E \cup T \cup \mathrm{Rng}\Box \cup \mathrm{Rng}? \cup \mathrm{Rng}\Diamond$. We think of the ranges of $y$ and $k$ as sets of new states. For state u in $K^*$, $k(u)$ is a new counterpart state; similarly, $y(w)$ is a counterpart of w if w is in $Y^*$. For each subset G of T, let

$$G^+ = G \cup \mathrm{Rng}\ (k{\uparrow}G) \cup \mathrm{Rng}\ (y{\uparrow}G)\ ,$$

where $\mathrm{Rng}\ (k{\uparrow}G)$ and $\mathrm{Rng}\ (y{\uparrow}G)$ are respectively the image of G under $k$ and the image of G under $y$. The set $G^+$ contains all the original G-states together with all $y$ and $k$ counterparts of there original G-states.

Let

$$E^+ = \{A^+ : A \in E\}\ .$$

This is a field of subsets of $T^+$ isomorphic to $E$, and represents the proposition field $E$ in the expanded system we are constructing.

We now add representatives for $\Box$, ?, $\Diamond$ , $X$ and $\leq$ to complete our expanded acceptance system. Let $\Box^+$ be a function with domain $E^+$ such that for all A in $E$,

$$\Box^+(A^+) = \Box A\ .$$

Let $?^+$ and $\Diamond^+$ be functions with the set of finite subsets of $E^+$ as domain, and be such that for each subset $G^+$ of $E^+$ and corresponding subset $G$ of $E$, where $G^+ = \{A^+ : A \epsilon G\}$,

$$?^+(G^+) = ?(G)$$

$$\Diamond^+(G^+) = \Diamond(G) \ .$$

For any sequence $(C)_n{}^+$ drawn from $E^+ \cup \mathrm{Rng}\Box \cup \mathrm{Rng}? \cup \mathrm{Rng}\Diamond$ the corresponding $(C)_n$ is one just like $(C)_n{}^+$ except for having an occurrence of A at every place where $(C)_n{}^+$ has an occurrence of $A^+$. Let $X^+$ be a function with all finite sequences $(C)_n{}^+$ as its domain and be such that

$$X^+(C)_n{}^+ = (Z^+, Z^{*+}) \text{ , where}$$

$(Z, Z^*) = X(C)_n$ for the corresponding sequence $(C)_n$.

To each state in $T^+$ there corresponds an unique state $u_o$ in T, such that $u = u_o \vee u = k(u_o) \vee u = y(u_o)$ holds. We call this state $u_o$ the "T-representative" of u. Comparative similarity in our expanded model goes by comparative similarity of T-representatives in our original model. Let $\leq^+$ be a function on the range of $X^+$ which assigns a pair of relations on $T^+$ to each $X^+(C)_n{}^+$ such that

$$u \underset{K^+}{\leq^+} v \quad \text{iff} \quad u_o \underset{K}{\leq} v_o$$

where K is either the first or second constituent of $X(C)_n$.

Theorem 13.1 $(E^+ T^+ ?^+ \Box^+ \Diamond^+ X^+ \leq^+)$ is an acceptance system such that $E^+$ is isomorphic to $E$ and for all

$$(C)_n{}^+ \in \mathrm{Dom} X^+ \text{ , } A^+ \epsilon E^+$$

$$1[X^+(C)_n{}^+] \subseteq A^+ \quad \text{iff} \quad 1[X(C)_n] \subseteq A$$

$$2[X^+(C)_n{}^+] \subseteq A^+ \quad \text{iff} \quad 2[X(C)_n] \subseteq A \text{ .}$$ [26]

The expanded system represents all the aspects of acceptance that are represented in the original system. It goes beyond the original system in that it allows agents to treat the difference between the accessibility regions K* and Y* as an empirically testable difference about the nature of physical necessity. In order to exploit this capability we introduce a new primitive notion of accessibility. For each $u \epsilon T^+$, let

$$
\begin{aligned}
R(u) &= T^{+}\ , \text{ if } u \in T \\
&= (K^{*})^{+}\ , \text{ if } u \in \mathrm{Rng}\ k \\
&= (Y^{*})^{+}\ , \text{ if } u \in \mathrm{Rng}\ y\ .
\end{aligned}
$$

$R(u)$ is the set of all states in $T^{+}$ that are accessible from u. All states are accessible from any of our old T-states, a $k$-counterpart has accessible from it all and only states in $(K^{*})^{+}$, while a $y$-counterpart has accessible from it all and only states in $(Y^{*})^{+}$. Accessibility is given objective content in the model by making it a property which can differ from state to state. In this way the expanded models are suitable for representing accessibility as a property of the world, rather than merely a property of the agent's own framework.

We introduce new necessity and possibility operations $\boxplus$ and $\diamondplus$ such that $\boxplus A^{+}$ and $\diamondplus A^{+}$ function as propositions.[27] For $A^{+} \in E^{+}$, let

$$\boxplus A^{+} = \{u \in T^{+} : R(u) \subseteq A^{+}\}$$

$$\diamondplus A^{+} = \{u \in T^{+} : R(u) \cap A^{+} \neq \phi\}$$

Our original context pair $(K, K^{*})$ can now be represented by the single context

$$\hat{K} = \mathrm{Rng}\ (k \upharpoonright K)\ ,$$

and our competing alternative context pair $(Y, Y^{*})$ can be represented by

$$\hat{Y} = \mathrm{Rng}\ (y \upharpoonright Y)\ .$$

For all propositions A in $E$, we have:

$$
\begin{aligned}
\hat{K} \subseteq \boxplus A^{+} &\quad \text{iff} \quad K^{*} \subseteq A \\
\hat{K} \subseteq \diamondplus A^{+} &\quad \text{iff} \quad K^{*} \cap A \neq \phi \\
\hat{Y} \subseteq \boxplus A^{+} &\quad \text{iff} \quad Y^{*} \subseteq A \\
\hat{Y} \subseteq \diamondplus A^{+} &\quad \text{iff} \quad Y^{*} \cap A \neq \phi
\end{aligned}
$$

Consider the context $\hat{K} \cup \hat{Y}$. This is neutral between the accessibility regions $K^{*}$ and $Y^{*}$. In certain respects it acts like the acceptance pair $(K \cup Y\ ,\ K^{*} \cup Y^{*})$. For all $A \in E$,

$$
\begin{aligned}
\hat{K} \cup \hat{Y} \subseteq A^{+} &\quad \text{iff} \quad K \cup Y \subseteq A \\
\hat{K} \cup \hat{Y} \subseteq \boxplus A^{+} &\quad \text{iff} \quad K^{*} \cup Y^{*} \subseteq A\ .
\end{aligned}
$$

It differs in the important respect that there can be propositions A and B in $E$ such that $A^{+}$ and $B^{+}$ are each compatible with $\hat{K} \cup \hat{Y}$ and

$$(\mathring{K}\cup\mathring{Y})\cap A \subseteq k\uparrow K^*$$

$$(\mathring{K}\cup\mathring{Y})\cap B \subseteq y\uparrow Y^*.$$

Any propositions A, B$\in E$ such that

$$A\cap Y^* = \phi \quad \text{and} \quad A\cap K^* \neq \phi$$

$$B\cap K^* = \phi \quad \text{and} \quad B\cap Y^* \neq \phi$$

will do the job.

Any experiment with $\mathring{K}\cup\mathring{Y}$ as background assumptions such that its outcome can decide between propositions A and B will be crucial enough to give rational grounds for the choice between the competing conceptual frameworks K* and Y*. In the transition from Newtonian mechanics to the special theory of relatively, for example, almost any of the experiments that were actually used to test special relativity can be put into a $\mathring{K}\cup\mathring{Y}$ with outcome B form, where $\mathring{K}\cup\mathring{Y}$ constitutes background assumptions that do not beg the question either way and B is an outcome that, given this context of assumptions, counts conclusively for relativity theory when tested against Newtonian theory.

On the extended system fundamental theory change can be represented as conceptual change without giving up the idea that choice between theories can be made rationally on the basis of outcomes of experiments. The system is, nevertheless, only part of a dynamical account of such transitions. One thing missing is an explanation for the transition to the neutral context from which experiments can be made. My idea is that this kind of transition would occur only after an alternative has been developed far enough to be a genuine competitor. What is to count as "developed enough", however, is not something that my system is designed to decide.

## Notes

[1] A Carnapian confirmation system can be represented as a quadruple $(ETKC)$ in which $E$ is a field of subsets of T, $K = E - \{\phi\}$, and $C$ maps $E \times K$ into the reals such that for all A, B$\in E$, K$\in K$;

(1) $C(-|K)$ is a probability function on $E$, and $C(A|K) = 1$ if $K\subseteq A^*$

(2) If $K\cap A \in K$, then $C(B|K\cap A) = C(A\cap B|K) / C(A|K)$.

(This is equivalent to the characterization given in Carnap-Jeffrey [2], pg. 38.) $E$ is a field of propositions, T is a fundamental set of

*We require only finite additivity of probability functions. A probability function on $E$ is a map from $E$ into the reals such that $P(T) = 1$ and $P(A\cup B) = P(A) + P(B)$, if $A\cap B = \phi$.

alternative possible states, $K$ is a set of possible bodies of knowledge, and $C$ is to be adequate to capture the confirmational commitment of an ideally rational agent with respect to $E$ and $K$. For each $A \in E$ and $K \in K$

$$C(A|K)$$

is to be the creedal state such an agent would deem appropriate for possible body of knowledge K. For Carnap's ideal agent creedal states are coherent degree of belief functions. Conditions (1) and (2) are jointly necessary and sufficient for having each $C_K$ be coherent with respect to its K.

If $C_K(A) > 0$ the agent conditionalizes on A by shifting his creedal state from $C_K$ to $C_{K \cap A}$. Condition (2) on $C$ insures that this is a shift from $C_K$ to the ratio conditional probability of $C_K$ with respect to A. When $C_K(A) = 0$ this ratio conditional probability is not defined and Carnap's system does not allow conditionalization on A.

On Carnap's system, and the orthodox Bayesian model, rational learning is only represented by conditionalization of new evidence with ratio conditional probabilities. This commits such models to assigning degree of belief 1 to new evidence when it comes in and does not allow any way to rationally lower the agent's degree of belief in any proposition that he has previously accepted as evidence. On such a representation evidence can only accummulate. No way is provided to give up previously accepted evidence on the basis of surprizing new developments.

[2] Kuhn [14], Feyerabend [6 ], Lakatos and Musgrave (eds.) [16], Stegmüller [36 ].

[3] I am unable to tell exactly what version of the incomensurability thesis Kuhn [14] holds. Some passages, eg., pp. 101-102, seem to be primarily criticisms of simplistic accounts of accummulation of scientific knowledge, others strongly suggest a skeptical view of the transitions in scientific revolution as non-rational, eg., pp. 109, 110, pg. 148. Passages such as that on pg. 152, on the other hand, seem to suggest that Kuhn holds the transitions in scientific revolutions to be rational and guidable by empirical evidence, and that he takes issue only with standard accounts of how this is done.

[4] I cannot be sure that Kuhn actually subscribes to this conceptual change argument for a skeptical version of incomensurability. It may well be that the answer my model provides is quite compatable with his position; nevertheless, the argument is initially plausible and strongly suggested by his writing.

[5] When the set theoretical treatment of propositions is combined with the idea that belief is a functional state determined by its role in guiding decisions, as is the case in Savage's classic presentation of the orthodox model [28], one can argue that semantical omniscience is not so much an attribution of super-reasoning ability to the agent as it is a criterion for having $E$ be an appropriate field to represent the objects of the agent's beliefs in the problem being modeled. Assent to sentences is only one among many kinds of evidence

bearing on what propositions an agent accepts. A situation where the agent assents to a sentence $S_0$ that would standardly assert A and refuses to assent to some logically equivalent sentence $S_1$ is evidence that the agent does not use both these sentences to express the proposition A. It cannot be evidence that he both accepts A and does not accept A, because such a belief state is not possible. Whatever irrationality the agent is displaying will apply to his failure to understand the sentences in the standard way, not to his attitudes toward propositions. (For an interesting exploration of some of the problems and benefits in this approach to semantical omniscience, see Robert Stalnaker [33],[34].)

[6]Stalnaker is most responsible for this use of "acceptance context". In Stalnaker [34] he uses acceptance contexts as part of an illuminating treatment of assertion.

[7]Recently much attention in the literature has been devoted to generalizations of the Bayesian model which give up coherence in favor of representing rational belief with indeterminate probabilities. (A few of the many available examples are: Kyburg [15], Dempster [5], Levi [18], Shafer [29], I. J. Good and P. Suppes are also prominant among those involved). The present extension is not in conflict with these developments and can be adapted to such systems in a relatively straightforward manner. Such an adaptation together with a critical examination of the representation of rational belief with indeterminate probabilities will be developed in future work. In this paper restriction to coherent models simplifies the presentation and keeps it as close as possible to the orthodox model presented in the work of Ramsey and Savage.

[8]If we replaced (2) with the stronger condition

$$(2') \quad P(A) = 1 \text{ iff } K \subseteq A$$

we would be demanding strict coherence relative to K. This would force the identification of acceptance with assigning degree of belief 1. Such an identification is very plausible when one thinks of probability primarily as degree of belief; however, the identification also rules out many probability functions based on standard measure theory. I am not sure which side of this tradeoff is more valuable; but, since a fully adequate development of the (2') version would require finding some way to substitute for all the neat applications of measure theory it rules out, I do not restrict the present extensions of the Bayesian model to versions where (2') holds. (This is an important difference between the present treatment and the related work in [8] and [9].)

[9]See Harper [10] for a more detailed account of how this motivation generates the constraints.

[10]Popper wants to define his conditional probability functions on minimal algebras where there is no guarantee that the binary operation AB and the unary operation $\bar{A}$ behave as a boolean meet and complement.

(Popper [22], pp. 318-358.) This requires an additional axiom

$$(4)\quad P(AB|C) \le P(B|C) \;,$$

which is redundant for Popper functions on boolean domains such as our field $E$.

In Harper [8] I also used an axiom

$$(2^*)\quad P(B|A) = 1 = P(A|B) \text{ only if } P(C|A) = P(C|B) \;.$$

This axiom is derivable from 1-3 if the domain is boolean and from 1-4 if the domain is an arbitrary minimal algebra. Therefore, it is not needed in either case. We may also replace (3) with Renyi's transition rule (Condition (a)(ii)). These improvements were suggested to me by the formulation given in van Fraassen [39].

[11] The idea of using Popper functions to represent extended conditional belief functions was first due to Robert Stalnaker [32].

[12] In this section I am indebted to the work on conditionals of Robert Stalnaker and David Lewis. (Stalnaker [31], Lewis [17].) This section has also benefited from their advice and the advice and criticism of Nuel Belnap, Allan Gibbard, Rich Thomason, and Ed Green.

[13] Condition (0) that $K(A) = ac\ K(A)$ is satisfied trivially because $ac\ K(A) = ac\ (ac\ K(A))$ whatever subset of $T\ K(A)$ might happen to be. Condition (1), that $K(A) \subseteq A$ is also an obvious consequence of D 4. Condition (2) that $K(A) = A$, when $K \subseteq A$, requires C 1 and C 2. To show Condition (3) that $K(B) = K(A) \cap B$ if $B \subseteq A$ and $K(A) \cap B \neq \phi$: note that $K(A) \cap B \subseteq K(B)$ follows from D 4 and the hypothesis that $B \subseteq A$. For the other inclusion we need transitivity of $\le_K$ and the hypothesis that $K(A) \cap B \neq \phi$, as well as D 4.

[14] Lewis calls this the limit assumption and argues against it as a constraint on overall comparative similarity of worlds to worlds. (Lewis [17], pp. 19-21.)

[15] Constraints C 1 - C 3 follow obviously from the relevant definitions. To show C 4 assume Z is a non-empty subset of T and define by recursion

$$\Gamma_o = \{u \in Z : |\mathcal{S}_o Ku| \le |\mathcal{S}_o Kv| \text{ all } v \in Z\}$$

$$\Gamma_{i+1} = \{u \in \Gamma_i : |\mathcal{S}_{i+1} Ku| \le |\mathcal{S}_{i+1} Kv| \text{ all } v \in \Gamma_i\}$$

If Z is non-empty then $\bigcap_{i<n} \Gamma_i$ is non-empty, and its members are all $\overset{\mathcal{S}}{\underset{K}{\le}}$ minimal elements of Z.

[16] In 1973 Richard Jeffrey suggested to me in private conversation that it would be useful to provide for question openings. In public

addresses at Pittsburgh and Boston, which I attended in 1975, Issac Levi argued that question openings are an essential part of any rational revision of evidence.

[17] An earlier treatment of this theme is to be found in Harper [8].

[18] From right to left note that $P(A|C) = P(B|C)$ for all C only if both $A\bar{B}$ and $\bar{A}B$ are $P$-impossible, so the symmetric difference $A\bar{B} \cup \bar{A}B$ is incompatable with K*. From right to left note that $P(AB \cup \bar{A}\bar{B} \mid C) = 1$ only if $P(A|C) = P(B|C)$.

[19] The ideas in this section have benefitted from discussions with Nuel Belnap, Ed Green and Bas van Fraassen.

[20] Group I all follow directly from the relevent definitions. Group II follows from Theorems 4.1 and 4.2 together with appropriate definitions. III is obvious. IV follows easily from II and III, V follows from Theorems 3.1 and 3.2. VI follows from the fact that $K = K(?G) \cap \bar{G}$ and each $K(A) = K(?G) \cap A$.

[21] Levi has been developing systems for representing rational belief change that also allow for acceptance of corrigible evidence and revision of previously accepted evidence. (Levi [18] and further developed in lectures.) One important difference between Levi's system and ours is that he treats sentences rather than propositions as objects of belief. For Levi confirmational commitment determines a convex set of probability functions, not a single degree of belief function (see note 1). Levi's system does not model conceptual change, though he does provide a special set of *a priori* sentences that plays a role similar to a fixed K*.

[22] In this definition $K$ need not be a subset of $E$. This is a generalization on Renyi as well as Carnap. It is convenient, because it allows for acceptance contexts corresponding to filters $\nabla$ where $\nabla$ is not a proposition in $E$.

A more important generalization on Carnap is that not every nonempty proposition in $E$ need be a possible acceptance context. This is Renyi's innovation. It allows him to use an unbounded measure $\mu$ where $\mu(T) = \infty$ to generate $C$. In his later work [26] he restricts himself to exactly those confirmation systems where $C$ can be generated by a single $\sigma$-finite, but perhaps unbounded, measure. In his earlier work [25] he also provides for systems where $C$ can only be generated by a whole family of measures. Csaszar in comments on Renyi 1955 [3] and van Fraassen [39] have extended Renyi's work on using families of measures to generate conditional probabilities. van Fraassen also compares Renyi's generalization of conditional probability with Popper.

Adding to the antecendent of (2) the hypothesis that $C_K(A) > 0$ allows for cases where $C_K(A) = 0$, but $C_{K \cap A}$ is still well defined. This is the essential difference between Renyi's more general early systems and his [26] system. Allowing this generality of Renyi's earlier systems is essential to make use of dimensionally ordered families of measures discussed by Csaszar and van Fraassen.

Finally, the present definition agrees with Carnap in requiring only finite additivity. This is a generalization over all of Renyi's systems, since he only considers countably-additive probabilities. De Finetti argues, at length, [4] that countable additivity is not an appropriate constraint on belief functions. He also develops extensive and ingenous methods for using probability functions that are only finitely additive in order to do jobs that most work in mathematical probability does with σ-additive measures. We want to allow for belief functions corresponding to finitely additive confirmation functions.

[23]The ideas in this section benefited from discussion with Roger Jones, Bill Demopoulis, Jeff Bub, John Nicholas, Bas van Fraassen, and Allan Gibbard.

[24]This suggestion is due to Roger Jones.

[25]I first came across the idea of using "Kinematics" to refer to descriptive models of belief change in a chapter entitled "Probability Kinematics" in Jeffrey [13]. More recently I noticed Patrick Suppes article "The Kinematics and Dynamics of Concept Formation" [38].

[26]This all follows fairly straightforwardly from the fact that relations between worlds u of $T^+$, propositions $A^+$ of $E^+$ and subsets $K^+$ of $T^+$ mirror the corresponding relations between representative states $u_o$ of T, propositions A of $E$ and subsets K of T. Of particular importance are the facts that

$$[\mathrm{ac}\ K]^+ = \mathrm{ac}\ [K]^+$$

$$u \leq_{K^+} v \quad \text{iff } u_o \leq_K v_o$$

$$[A \cap B]^+ = [A]^+ \cap [B]^+$$

$$[A \cup B]^+ = [A]^+ \cup [B]^+$$

$$[\bar{A}]^+ = T^+ - [A]^+$$

$$K^+ \subseteq A^+ \quad \text{iff } K \subseteq A\ .$$

[27]An earlier treatment of $\boxplus$ and $\oplus$ as propositional operators along these lines benefited from discussion with Nuel Belnap, Ed Green, and Robert Stalnaker. It is tempting to extend the representation further to have counterparts for all of the possible accessibility regions. This temptation should be resisted. Having only the accessibility regions of an old theory and developed competitors allows us to represent the insight that a theory is only given up if a viable alternative replaces it. The role of evidence in testing a theory depends on what theory it is being tested against.

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