

- (a) Write a function that takes as input two Euclidean vectors $\vec{u} = \{u_1, u_2, \dots, u_N\} \in \mathbb{R}^N$ and $\vec{v} = \{v_1, v_2, \dots, v_N\} \in \mathbb{R}^N$ (of type `valarray<long double>`) and returns their inner product (also known as inner product or Hadamard product)

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^N u_i v_i \quad (1)$$

as a `long double` number. Your function may use `(u*v).sum()` to compute the dot product of the `valarrays` `u` and `v`. Create a constant `valarray<long double>` equal to `u={0.1, 0.1, ..., 0.1}` with $N = 10^6$ elements. Demonstrate that your program works by computing the dot product $\vec{u} \cdot \vec{u}$ for this constant vector. Display the difference $\vec{u} \cdot \vec{u} - 10^4$ on the screen.

- (b) Repeat Question 2a using Kahan compensated summation to compute the sum.
- (c) Write code for a function object that has a member variable `m` of type `int`, a suitable constructor, and a member function of the form
- ```
double operator()(const valarray<double> u) const {
```
- which returns the weighted norm

$$\ell_m(\vec{v}) = \sqrt[m]{\sum_{i=0}^N |v_i|^m} \quad (2)$$

Use this function object to calculate the norm  $\ell_2(\vec{u})$  for the vector in Question 2a. Does the quantity  $\ell_2(\vec{u})^2$  equal the inner product  $\vec{u} \cdot \vec{u}$  that you obtained above? [Note: half marks awarded if you use a regular function instead of a function object.]