Using a pocket calculator, one may notice that by applying the cosine key repeatedly to the value one obtains a sequence of real numbers

$$
\begin{aligned}
x_1 &= \cos x_0 = 1. \\
x_2 &= \cos x_1 = 0.54030230586814 \\
x_3 &= \cos x_2 = 0.857553215846393 \\
&\vdots \\
x_{21} &= \cos x_{20} = 0.739184399771494 \\
&\vdots
\end{aligned}
$$

which tends to the value $x_\infty = 0.739085\ldots$, which is the point where the functions x and $\cos x$ intersect. The iteration can be written as

$$x_{n+1} = \cos x_n \text{ for } n = 0, 1, 2, \ldots \text{ with } x_0 = 1.$$

The limit x_∞ satisfies the transcendental equation

$$\cos x = x.$$

Write a `for` loop that performs the iteration $x_{n+1} = \cos x_n$ starting from the initial condition $x_0 = 0$ and stops when the absolute value of the difference $|x_{n+1} - x_n|$ between two consecutive iterations is less than a prescribed tolerance $\epsilon = 10^{-12}$. Print out the final value x_{n+1} to 16 digits of accuracy. In how many iterations did your loop converge? What is the final error in the above transcedental equation? (Hint: use the final value to compute and print out the difference $x - \cos x$.)