

C-13.2 Show that every language  $L$  in  $P$  is polynomial-time reducible to the language  $M=\{5\}$ , that is, the language that simply asks if the binary encoding of the input is equal to 5.

- Since  $L$  is in  $P$  which is polynomial time, and it's reducible by language  $M$ .  
Assume that there is an algorithm  $AC(x)$  that will return yes if the input is equal to 5

```
if AC(x) = yes then
    return 5
else
    return 10
```

A. Show that the MST decision problem is polynomial-time reducible to the Subset Sum problem.

```
Final > go MST2SubSetSum.go
1  Algorithm MST2SubSetSum(G, k):
2      T = MST(G)
3      for e in T.Edges()
4          w = w + e.weight()
5
6      for e in G.Edges()
7          S = S + e.weight()
8
9      if w <= k:
10         return (S, w)
11     else:
12         return (S, k)
13
```

B. Show the shortest path decision problem is polynomial-time reducible to the MST decision problem. Hint: convert the shortest path problem to a decision problem, then reduce to MST problem.

```
Final > go ShortestPath2MST.go
1  Algorithm ShortestPath2MST(G, s,t,k)
2      newGraph := new empty graph()
3
4      for each (u,v) in G.edges()
5          if u is reachable from s && v is reachable from t
6              newGraph.insertVertex(u)
7              newGraph.insertVertex(v)
8              newGraph.insertEdge(u, v, weight(u, v))
9
10     shortestPathWeight <- ShortestPath(G, s, t)
11
12     if shortestPathWeight <= k:
13         return (newGraph, shortestPathWeight)
14     Else:
15         return (newGraph, k)
```

Decision problem

∴ Given a graph  $G = (V, E)$  and two vertices  $s, t$  and a max  $K$ , Does there exist a path from  $s$  to  $t$  with total weight  $\leq K$  ?