Faculty of Enigeering **Cairo University**

Analog Electronics Project # 1

Design and Implementation of a Biquad

Presented for **ELC 3060** Cadence Project

Presented to:

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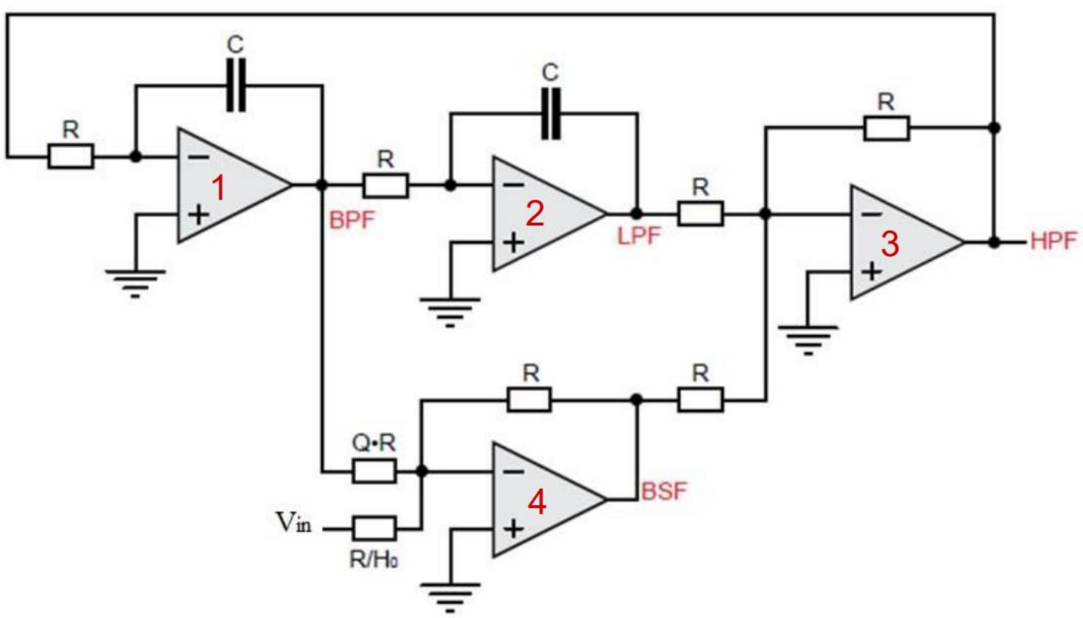
Universal Biquadratic Filter



Task 1

Derivations and Parameters

Assume for all Op-amps are ideal and in Negative Feedback $\rightarrow V_{out} = A_v(V_{in}^{+} - V_{in}^{-})$.

Since the output is Finite, While $A_v \rightarrow \infty$, then $(V_{in}^{+} - V_{in}^{-} = 0) \rightarrow V_{in}^{+} = V_{in}^{-}$

We use KCL at input nodes in S-domain to approach the derivations.

OP - AMP # 1 (BPF)

KCL: $\frac{V_{HPF} - 0}{R} = \frac{0 - V_{BPF}}{\frac{1}{SC}}$

$\therefore V_{HPF} = -V_{BPF}\, SCR \quad \rightarrow (1)$

OP - AMP # 2 (LPF)

KCL: $\frac{V_{BPF} - 0}{R} = \frac{0 - V_{LPF}}{\frac{1}{SC}}$

$\therefore V_{BPF} = -V_{LPF}\, SCR \quad \rightarrow (2)$

OP - AMP # 3 (HPF)

KCL: $\frac{V_{LPF} - 0}{R} + \frac{V_{BSF} - 0}{R} = \frac{0 - V_{HPF}}{R}$

$\therefore V_{HPF} = -V_{LPF} - V_{BSF} \quad \rightarrow (3)$

OP - AMP # 4 (BSF)

KCL: $\frac{V_{in}}{R/H_o} + \frac{V_{BPF}}{R.Q} = \frac{-V_{BSF}}{R}$

$\therefore H_o V_{in} + \frac{1}{Q} V_{BPF} = -V_{BSF} \quad \rightarrow (4)$

From (3): $-V_{BSF} = V_{HPF} + V_{LPF}$, Sub. In (4): $\therefore H_o V_{in} + \frac{1}{Q} V_{BPF} = V_{HPF} + V_{LPF} \quad \rightarrow (5)$

From (2): $V_{BPF} = -V_{LPF}\, SCR$, From (1): $V_{HPF} = -V_{BPF}\, SCR$

Substitute in (5) 4 times to get $H_{LPF}(S)\, , H_{HPF}(S)\, , H_{BPF}(S)\, , H_{BSF}(S)$:

$$H_o V_{in} + \frac{1}{Q}(-V_{LPF}\, SCR) = -(-V_{LPF}\, SCR)\, SCR + V_{LPF} \rightarrow H_o V_{in} = \left(1 + \frac{SCR}{Q} + S^2C^2R^2\right) V_{LPF}$$

2nd Order LPF Transfer Function

$$\boxed{\therefore H_{LPF} = \frac{V_{LPF}}{V_{in}} = \frac{H_o}{S^2C^2R^2 + \frac{SCR}{Q} + 1} = \frac{\frac{H_o}{C^2R^2}}{S^2 + \frac{S}{QCR} + \frac{1}{C^2R^2}}}$$

$$H_oV_{in} + \frac{1}{Q}\left(-\frac{V_{HPF}}{SCR}\right) = V_{HPF} + \frac{-\left(-\frac{V_{HPF}}{SCR}\right)}{SCR} \rightarrow H_oV_{in} = \left(1 + \frac{1}{S^2C^2R^2} + \frac{1}{QSCR}\right) V_{HPF}$$

2nd Order HPF Transfer Function

$$\boxed{\therefore H_{HPF} = \frac{V_{HPF}}{V_{in}} = \frac{H_o}{1+\frac{1}{S^2C^2R^2}+\frac{1}{QSCR}} = \frac{H_o\ S^2}{S^2+\frac{S}{QCR}+\frac{1}{C^2R^2}}}$$

$$H_oV_{in} + \frac{1}{Q}V_{BPF} = -V_{BPF}\ SCR + \frac{-V_{BPF}}{SCR} \rightarrow H_oV_{in} = -\left(\frac{1}{Q} + SCR + \frac{1}{SCR}\right) V_{BPF}$$

2nd Order BPF Transfer Function

$$\boxed{\therefore H_{BPF} = \frac{V_{BPF}}{V_{in}} = \frac{-H_o}{\frac{1}{Q}+SCR+\frac{1}{SCR}} = \frac{-H_o\frac{S}{CR}}{S^2+\frac{S}{QCR}+\frac{1}{C^2R^2}}}$$

From (3): $V_{BSF} = -\left((-V_{BPF}\ SCR) + \left(-\frac{V_{BPF}}{SCR}\right)\right) \rightarrow V_{BPF} = \frac{V_{BSF}}{SCR+\frac{1}{SCR}}$

From (4): $H_oV_{in} + \frac{1}{Q}\frac{V_{BSF}}{SCR+\frac{1}{SCR}} = -V_{BSF} \rightarrow -H_oV_{in} = \left(1 + \frac{1}{Q}\frac{1}{SCR+\frac{1}{SCR}}\right)V_{BSF}$

2nd Order BSF Transfer Function

$$\boxed{\therefore H_{BSF} = \frac{V_{BSF}}{V_{in}} = \frac{-H_o}{1+\frac{1}{Q}\frac{1}{SCR+\frac{1}{SCR}}} = \frac{-H_o\left(S^2+\frac{1}{C^2R^2}\right)}{S^2+\frac{S}{QRC}+\frac{1}{C^2R^2}}}$$

Parameters

H_o: DC gain (The Magnitude response be shifted vertically upwards or downwards by 20 log |H|).

Q : Quality factor of filter.

ω_o: Cut - off Frequency, From Transfer function $\omega_o = \left(\frac{1}{RC}\right)$.

BW: Bandwidth of the filter, represent the coefficient of S in denominator of the transfer function

$$\text{BW} = \frac{\omega_o}{Q} = \frac{1}{QRC}$$

Task 2

Components Design

Parameters	Its Values
$\boldsymbol{f_o}$	$1\ MHz$
$\boldsymbol{Q}$	2.2
$\boldsymbol{H_o}$	1
$\boldsymbol{\omega_o = 2\pi f_o}$	$6.283 \times 10^6\ rad/sec$

$$RC = \frac{1}{\omega_o} = 1.592 \times 10^{-7}\ \text{sec} \ \rightarrow \ (1)$$

In integrated circuit (IC) design, the values of components are converted into physical area on the chip. For instance, capacitance (C) is measured at ($fF/\mu m^2$), and capacitor sizes commonly ranges between 10 picofarads (pF) with dimensions of ($100\ \mu m \ \times 100\ \mu m$) to 100 femtofarads (fF) with dimensions of ($10\ \mu m \ \times 10\ \mu m$). For resistors, larger ones amplify thermal noise power, following the equation S(f) = 4KTR, where K is Boltzmann's constant, T is temperature in Kelvin, and R is resistance. Conversely, smaller resistors generate higher current and power consumption. Typically, resistors measure in the kilo ohms range.

To Summarize that $C \propto Area\ ,\ R \propto \frac{1}{Power\ Consumption}$, So we take the optimal values of R & C to optain small area and small power consumption

Let the nominal value for $R = 10\ k\Omega$ and from (1): $C = 15.92\ pF$.

Components Design	Its Values
R	$10\ k\Omega$
C	$15.92\ pF$
R/H	$10\ k\Omega$
Q.R	$22\ k\Omega$

Task 3

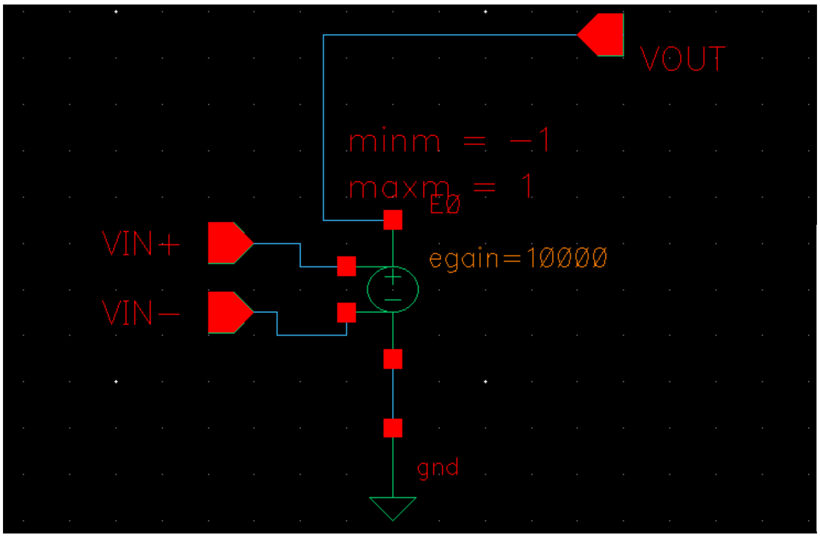
Cadence Simulation

Ideal Op - amp Schematic

Ideal Op - amp is modeled as a Voltage Controlled Voltage Source (VCVS) with Voltage Gain = 10000 , and with $|V_{out}| < 1 \quad (V_{max} = 1\,, V_{min} = -1)$

Note: Since we don't have any pole. Therefore, we don't need a Buffer Stage as $(R_{out} = 0)$.

Votage Source is Modeled as a Short Circuit in Small Signal Model.

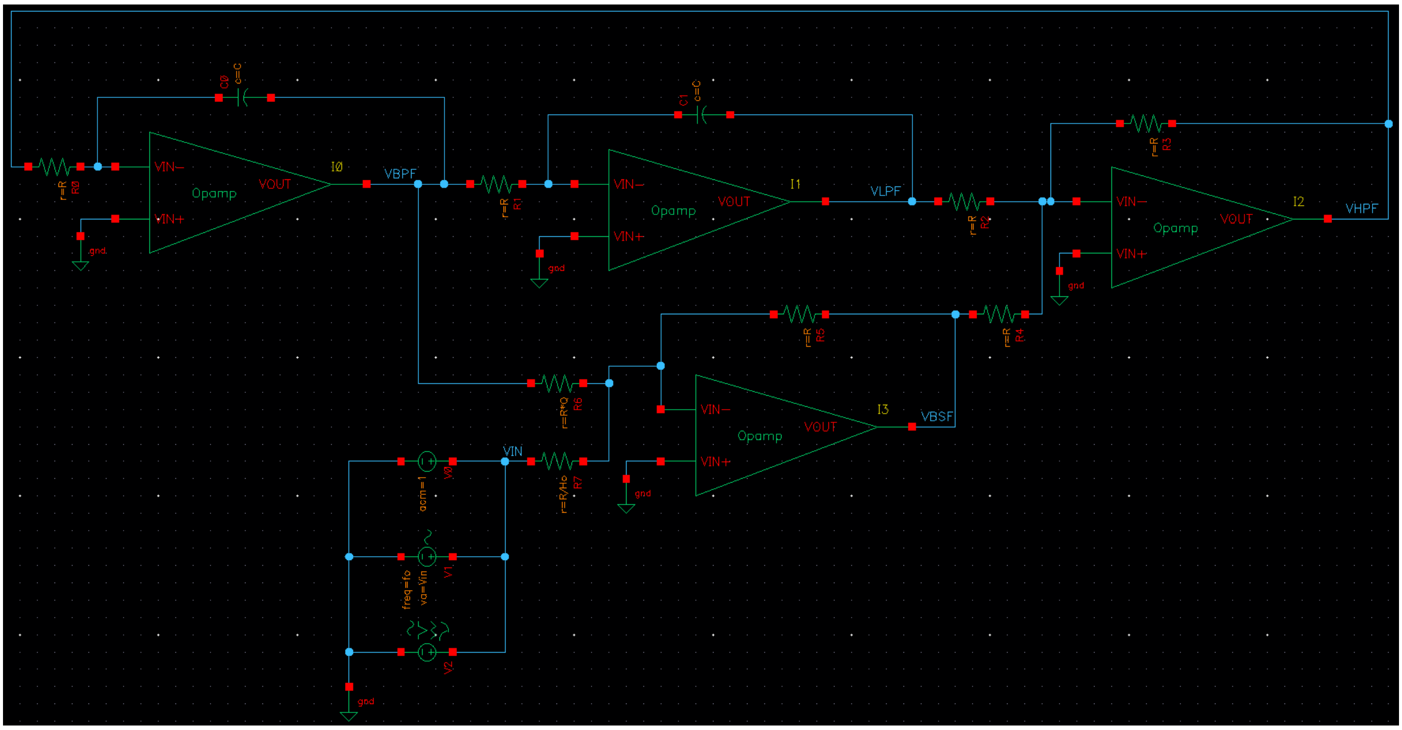


Global Variables

C	15.92p
Ho	1
Q	2.2
R	10k
fo	1M
Vin	100m

Click to add variable

Universal Biquadratic Filter



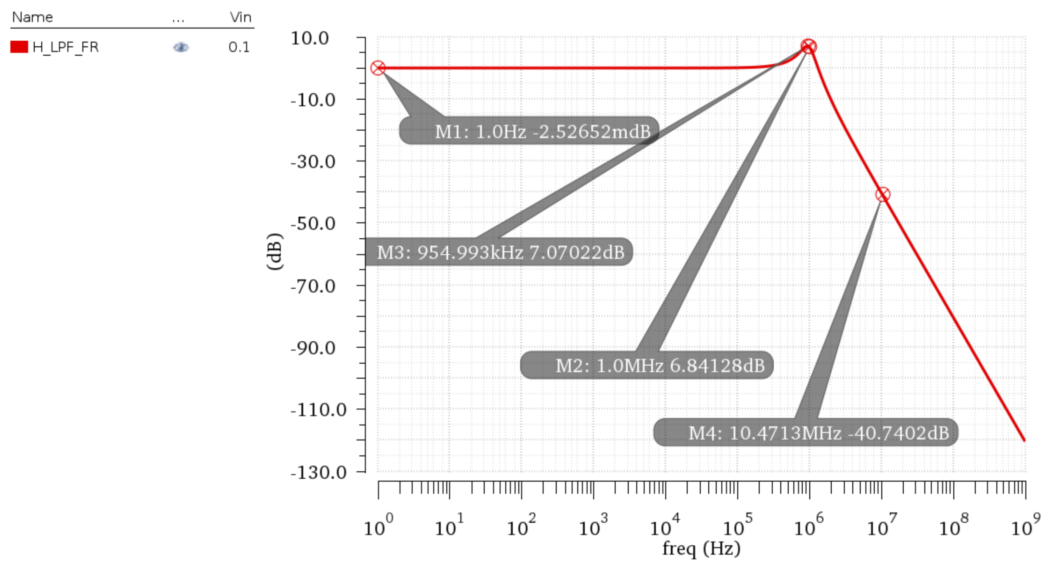
Frequency Response

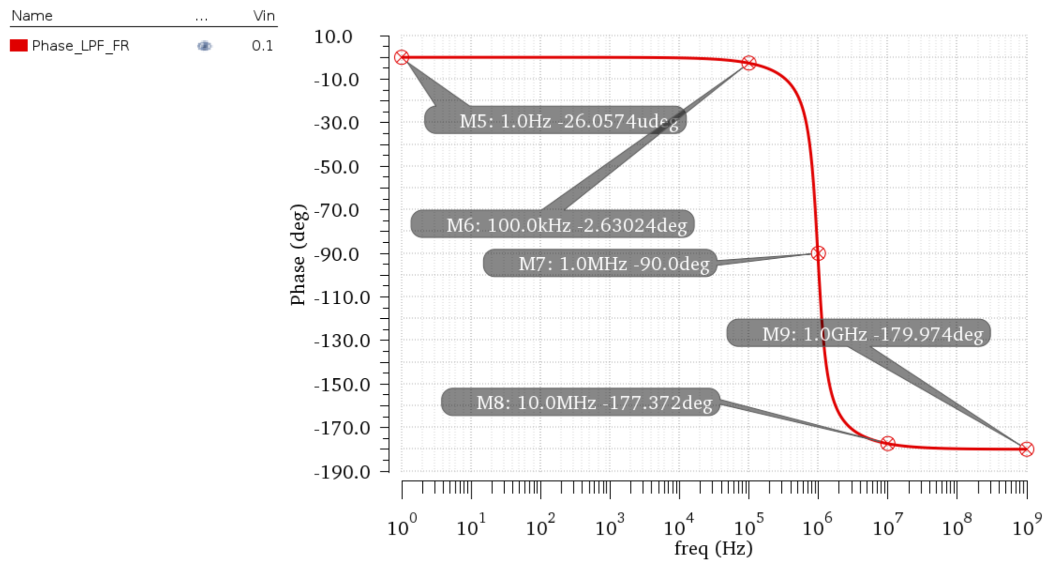
LPF Response

Replace $(S = j\omega)$, $H_{LPF}(S) = \frac{H_O\,\omega_O^2}{S^2+\frac{\omega_O}{Q}S+\omega_O^2} \rightarrow H_{LPF}(\omega) = \frac{\omega_O^2}{(\omega_O^2-\omega^2)+j\frac{\omega\omega_O}{Q}}$, $|H_{LPF}(\omega)| = \frac{\omega_O^2}{\sqrt{(\omega_O^2-\omega^2)^2+\left(\frac{\omega\omega_O}{Q}\right)^2}}$

For Max Magnitude: $\therefore \frac{d|H(\omega)|}{d\omega} = 0$, $2(\omega_O^2-\omega^2)(-2\omega)+2\left(\frac{\omega\omega_O}{Q}\right)\left(\frac{\omega_O}{Q}\right) = 0 \rightarrow \omega_{peak} = \omega_O\sqrt{1-\frac{1}{2Q^2}}$

Frequency (ω_o)	$\boldsymbol{\omega \rightarrow 0}$	$\boldsymbol{\omega \rightarrow \infty}$	$\boldsymbol{\omega \rightarrow \omega_o}\ \left(\boldsymbol{H_{LPF}(\omega)} = \frac{Q}{j}\right)$	$\boldsymbol{\omega \rightarrow \omega_{peak}}\ \left(\boldsymbol{Q > \frac{1}{\sqrt{2}}}\right)$
$\lvert\boldsymbol{H_{LPF}(\omega)}\rvert$	1	0	Q	$\frac{Q^2}{\sqrt{Q^2-\frac{1}{4}}} > Q_{slightly}$
$\angle\,\boldsymbol{H_{LPF}(\omega)}$	$0°$	$-180°$	$-90°$	$\frac{Q^2}{\sqrt{Q^2-\frac{1}{4}}} = 2.25 > 2.2$



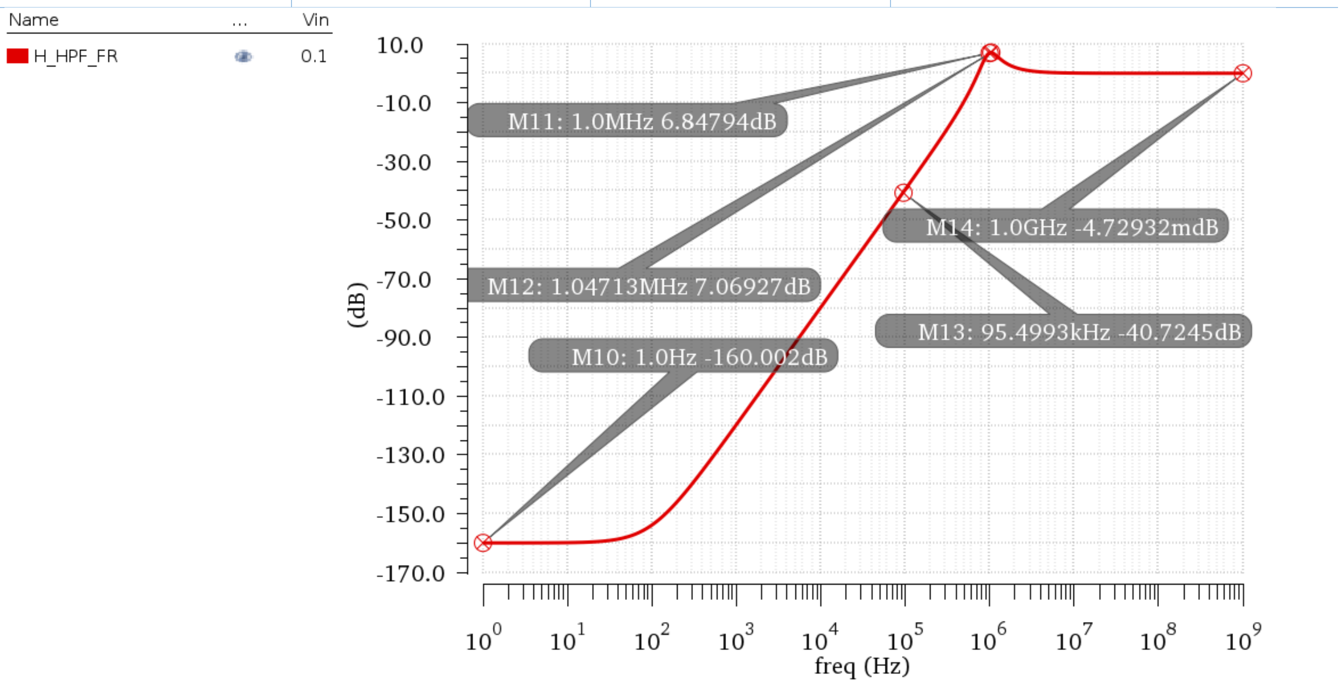


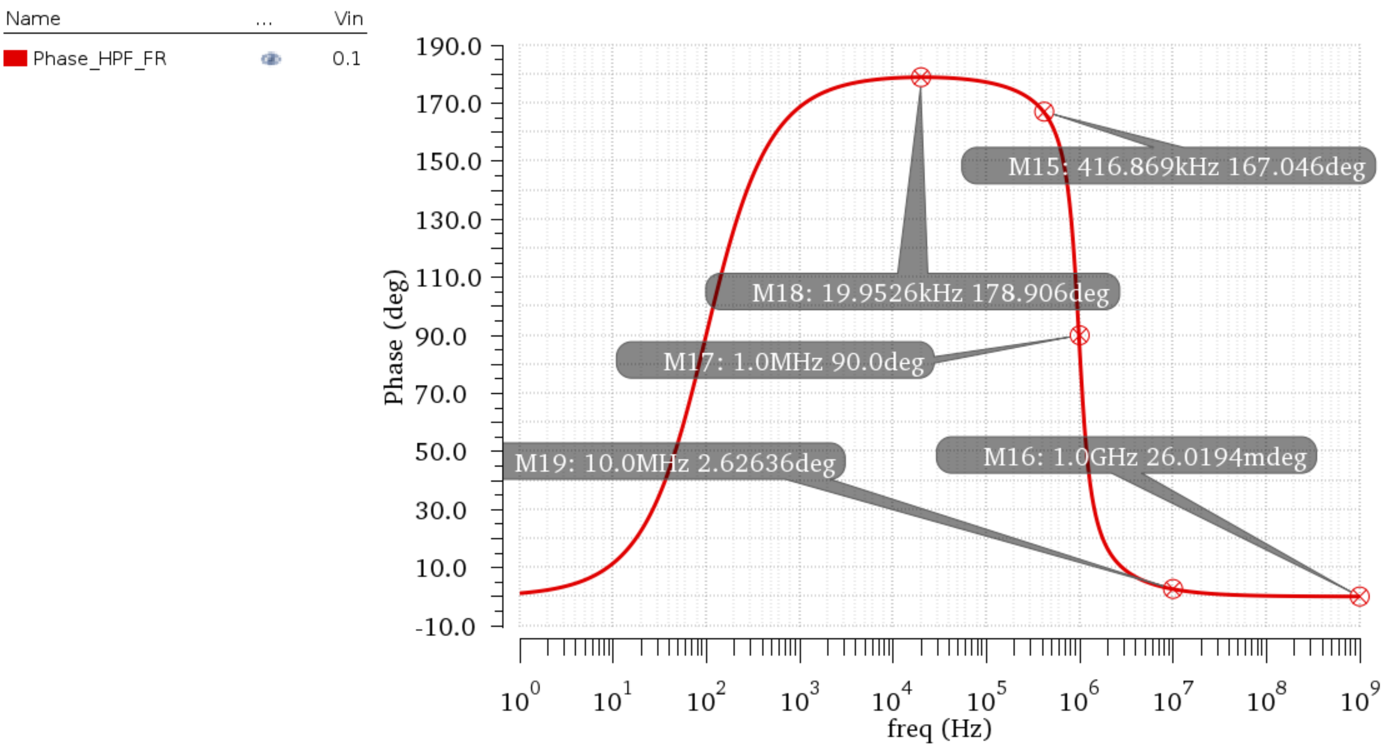
P.O.C	Simulation	Theoretically
Cut off Freq. (ω_o)	From Phase Plot: @ Phase $= -90°$ $f_o = 1\ MHz$ $\omega_o = 2\pi f_o$ $\omega_o = 6.3 \times 10^6\ rad/sec$	$C = 15.92\ pF\ , H_o = 1\ ,$ $Q = 2.2\ , R = 10\ k\Omega\ ,$ $\omega_o = \frac{1}{RC} = 6.3 \times 10^6\ rad/sec$
DC Gain (H)	@ Low Frequency $\lvert H_{LPF}(\omega_{low})\rvert = 20\log\lvert H\rvert$ $H_o = 0\ dB = 1$	$H_o = \left(\frac{10\ k}{10\ k}\right) = 1$
Quality Factor	From Magnitude Plot: $@\omega_o$ Cut off Freq. $\lvert H_{LPF}(\omega_o)\rvert = H_o Q = 6.84128\ dB$ $Q = 6.84128\ dB = 2.198$	$Q = \left(\frac{QR}{R}\right) = \frac{22\ k}{10k} = 2.2$
Peaking Freq $(\boldsymbol{\omega_{peak}})$	From Magnitude Plot: $@\ f_{peak} = 954.993\ kHz$ $V_{peak} = 7.07022\ dB$ $V_{peak} = 2.257$	$f_{peak} = f_o\sqrt{1-\frac{1}{2Q^2}} = 946.94\ kHz$ $V_{peak} = \frac{Q^2}{\sqrt{Q^2-\frac{1}{4}}} = 2.259$

HPF Response

Replace $(S = j\omega)$, $H_{HPF}(S) = \frac{H_o\,S^2}{S^2+\frac{\omega_o}{Q}S+\omega_o^2} \rightarrow H_{HPF}(\omega) = \frac{-\,\omega^2}{(\omega_o^2-\omega^2)+j\frac{\omega\omega_o}{Q}}$, $\lvert H_{HPF}(\omega)\rvert = \frac{\omega^2}{\sqrt{(\omega_o^2-\omega^2)^2+\left(\frac{\omega\omega_o}{Q}\right)^2}}$

Frequency (ω_o)	$\boldsymbol{\omega \rightarrow 0}$	$\boldsymbol{\omega \rightarrow \infty}$	$\boldsymbol{\omega \rightarrow \omega_o}\ \left(\boldsymbol{H_{HPF}(\omega) = \frac{-Q}{j}}\right)$
$\lvert\boldsymbol{H_{HPF}(\omega)}\rvert$	0	1	Q
$\angle\,\boldsymbol{H_{HPF}(\omega)}$	180°	0°	90°





P.O.C	Simulation	Theoretically
Cut off Freq. (ω_o)	From Phase Plot: @ Phase = 90° $f_o = 1\ MHz$ $\omega_o = 2\pi f_o$ $\omega_o = 6.3 \times 10^6\ rad/sec$	$C = 15.92\ pF\ , H_o = 1\ ,$ $Q = 2.2\ , R = 10\ k\Omega\ ,$ $\omega_o = \frac{1}{RC} = 6.3 \times 10^6\ rad/sec$
DC Gain $(\boldsymbol{H})$	@ High Frequency $\lvert H_{HPF}(\omega_{infinity})\rvert = 20\log\lvert H\rvert$ $H_o = 0\ dB = 1$	$H_o = \left(\frac{10\ k}{10\ k}\right) = 1$
Quality Factor	From Magnitude Plot: @ω_o Cut off Freq. $\lvert H_{HPF}(\omega_o)\rvert = H_o Q = 6.84794\ dB$ $Q = 6.84794\ dB = 2.199$	$Q = \left(\frac{QR}{R}\right) = \frac{22\ k}{10k} = 2.2$
Peaking Freq $(\boldsymbol{\omega_{peak}})$	From Magnitude Plot: @ $f_{peak} = 1.04713\ kHz$ $V_{peak} = 7.06927\ dB$ $V_{peak} = 2.257$	$f_{peak} = f_o\sqrt{1 - \frac{1}{2Q^2}} = 946.94\ kHz$ $V_{peak} = \frac{Q^2}{\sqrt{Q^2 - \frac{1}{4}}} = 2.259$

BPF Response

Replace $(S = j\omega)$, $H_{BPF}(S) = \frac{\frac{\omega_o}{Q}S}{S^2+\frac{\omega_o}{Q}S+\omega_o^2} \rightarrow H_{BPF}(\omega) = \frac{j\frac{\omega\omega_o}{Q}}{(\omega_o^2-\omega^2)+j\frac{\omega\omega_o}{Q}}$, $|H_{BPF}(\omega)| = \frac{\frac{\omega\omega_o}{Q}}{\sqrt{(\omega_o^2-\omega^2)^2+\left(\frac{\omega\omega_o}{Q}\right)^2}}$

Frequency (ω_o)	$\boldsymbol{\omega \rightarrow 0}$	$\boldsymbol{\omega \rightarrow \infty}$	$\boldsymbol{\omega \rightarrow \omega_o\ (H_{BPF}(\omega) = 1)}$
$\boldsymbol{\lvert H_{BPF}(\omega)\rvert}$	0	1	1
$\angle \boldsymbol{H_{BPF}(\omega)}$	$90°$	$-90°$	$0°$

$@\left(\omega \rightarrow \omega_{1,2\,(-3\,dBs\,from\,peaking)}\right)$

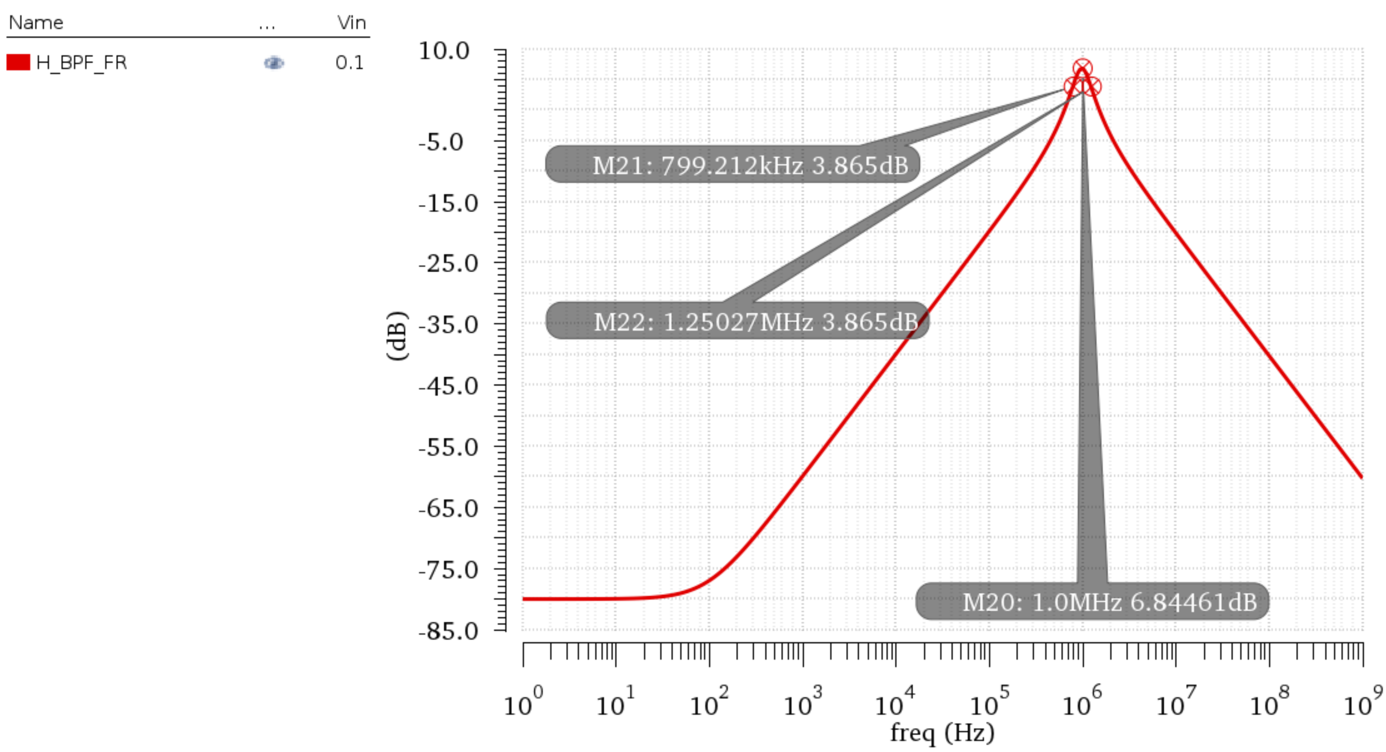
$$|H_{BPF}(\omega)| = \frac{\frac{\omega\omega_o}{Q}}{\sqrt{(\omega_o^2-\omega^2)^2+\left(\frac{\omega\omega_o}{Q}\right)^2}} = \frac{1}{\sqrt{2}} \rightarrow 2\left(\frac{\omega\omega_o}{Q}\right)^2 = (\omega_o^2-\omega^2)^2 + \left(\frac{\omega\omega_o}{Q}\right)^2 \rightarrow (\omega_o^2-\omega^2)^2 = \left(\frac{\omega\omega_o}{Q}\right)^2$$

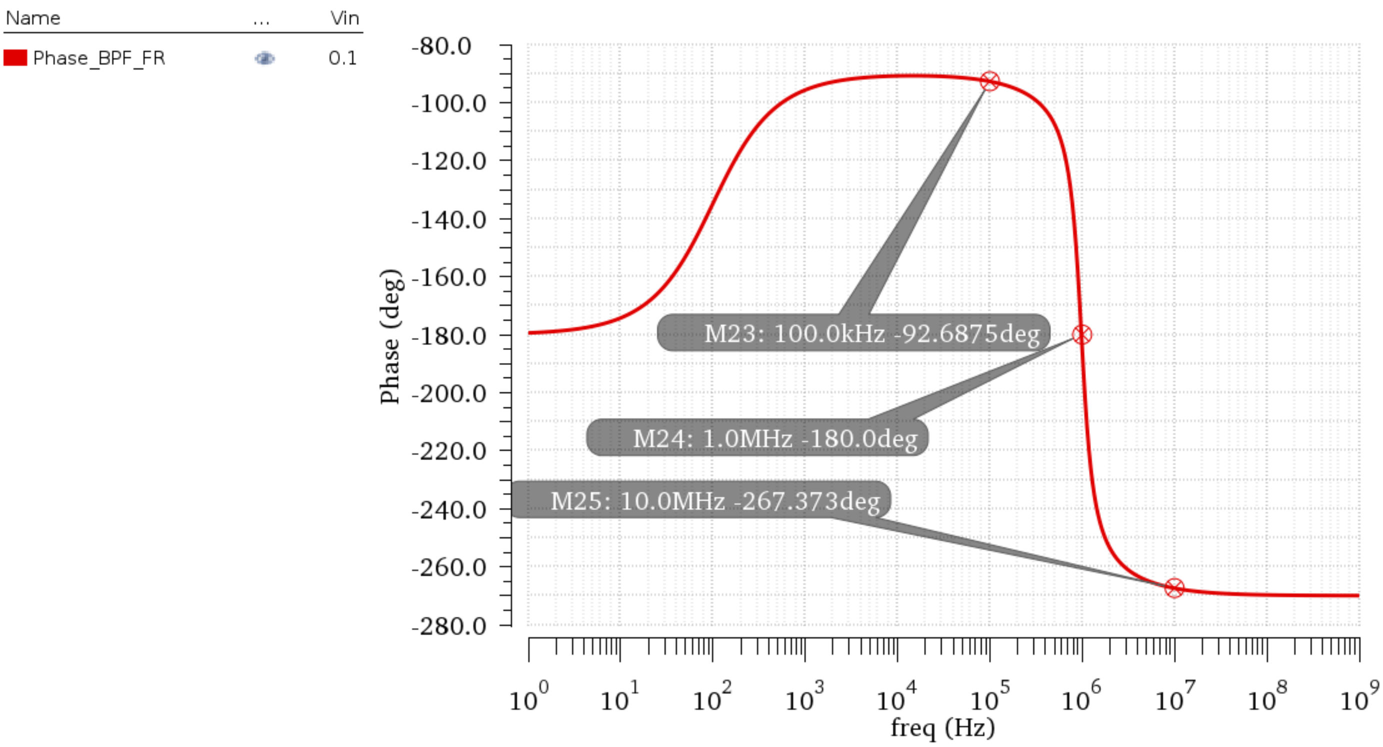
$$\omega^2 + \frac{\omega\omega_o}{Q} - \omega_o^2 = 0 \rightarrow \ \omega_{1,2} = \frac{\pm\frac{\omega_o}{Q} + \sqrt{\left(\frac{\omega_o}{Q}\right)^2+4\omega_o^2}}{2} = \pm\frac{\omega_o}{2Q} + \omega_o\sqrt{1+\frac{1}{4Q^2}}$$

$$\therefore BW = \omega_2 - \omega_1 = \frac{\omega_o}{Q} \qquad\qquad \therefore \omega_o^2 = \omega_2\,\omega_1$$

$H_{BPF}(S) = \frac{\frac{\omega_o}{Q}S}{S^2+\frac{\omega_o}{Q}S+\omega_o^2}$ But The Transfer function is $H_{BPF}(S) = \frac{-H_oQ\left(\frac{S}{RC}\right)}{S^2+\frac{S}{QRC}+\frac{1}{C^2R^2}}$

$\therefore$ Magnitude Shifts up by $20\log|QH_o| \cong 6.848\ dB$, Phase Shifts down by $-180°$





P.O.C	Simulation	Theoretically
Cut off Freq. (ω_o)	From Phase Plot: @ Phase $= -180°$ $f_o = 1\ MHz$ $\omega_o = 2\pi f_o$ $\omega_o = 6.3 \times 10^6\ rad/sec$	$C = 15.92\ pF$, $H_o = 1$, $Q = 2.2$, $R = 10\ k\Omega$, $\omega_o = \frac{1}{RC} = 6.3 \times 10^6\ rad/sec$
3 dB $f_{1,2}$	$f_1 = 799.212\ kHz$ $f_2 = 1.25027\ MHz$ Note: $\sqrt{f_1 f_2} = 0.99962\ MHz \cong 1\ MHz$	$f_{1,2} = \pm\frac{f_o}{2Q} + f_o\sqrt{1 + \frac{1}{4Q^2}}$ $f_1 = 1.253\ MHz$ $f_2 = 798.23\ kHz$
DC Gain (H)	From Magnitude Plot: @ ω_o Cut off Freq. $\lvert H_{BPF}(\omega_o)\rvert = H_o Q = 6.84461\ dB$ $20\log\lvert H_o\rvert + 20\log\lvert Q\rvert = 6.84461\ dB$ $20\log\lvert H_o\rvert = -\ 0.00384\ dB$ $H_o \cong 1$	$H_o = \left(\frac{10\ k}{10\ k}\right) = 1$
Bandwidth BW	$BW = f_2 - f_1 = 451.058\ kHz$	$BW = \frac{f_o}{Q} = 454.55\ kHz$
Quality Factor	$Q = \frac{f_o}{BW} = \frac{1\ MHz}{451.058\ MHz} = 2.217$	$Q = \left(\frac{QR}{R}\right) = \frac{22\ k}{10k} = 2.2$

BSF Response

Replace $(S = j\omega)$, $H_{BSF}(S) = \frac{S^2+\omega_o^2}{S^2+\frac{\omega_o}{Q}S+\omega_o^2} \rightarrow H_{BSF}(\omega) = \frac{(\omega_o^2-\omega^2)}{(\omega_o^2-\omega^2)+j\frac{\omega\omega_o}{Q}}$, $|H_{BSF}(\omega)| = \frac{(\omega_o^2-\omega^2)}{\sqrt{(\omega_o^2-\omega^2)^2+\left(\frac{\omega\omega_o}{Q}\right)^2}}$

Frequency (ω_o)	$\omega \rightarrow 0$	$\omega \rightarrow \infty$	$\omega \rightarrow \omega_o$	$\omega \rightarrow \omega_o^-$	$\omega \rightarrow \omega_o^+$
$\lvert H_{BSF}(\omega)\rvert$	1	1	0	0	0
$\angle H_{BSF}(\omega)$	$0°$	$0°$	$0°$	$-90°$	$90°$

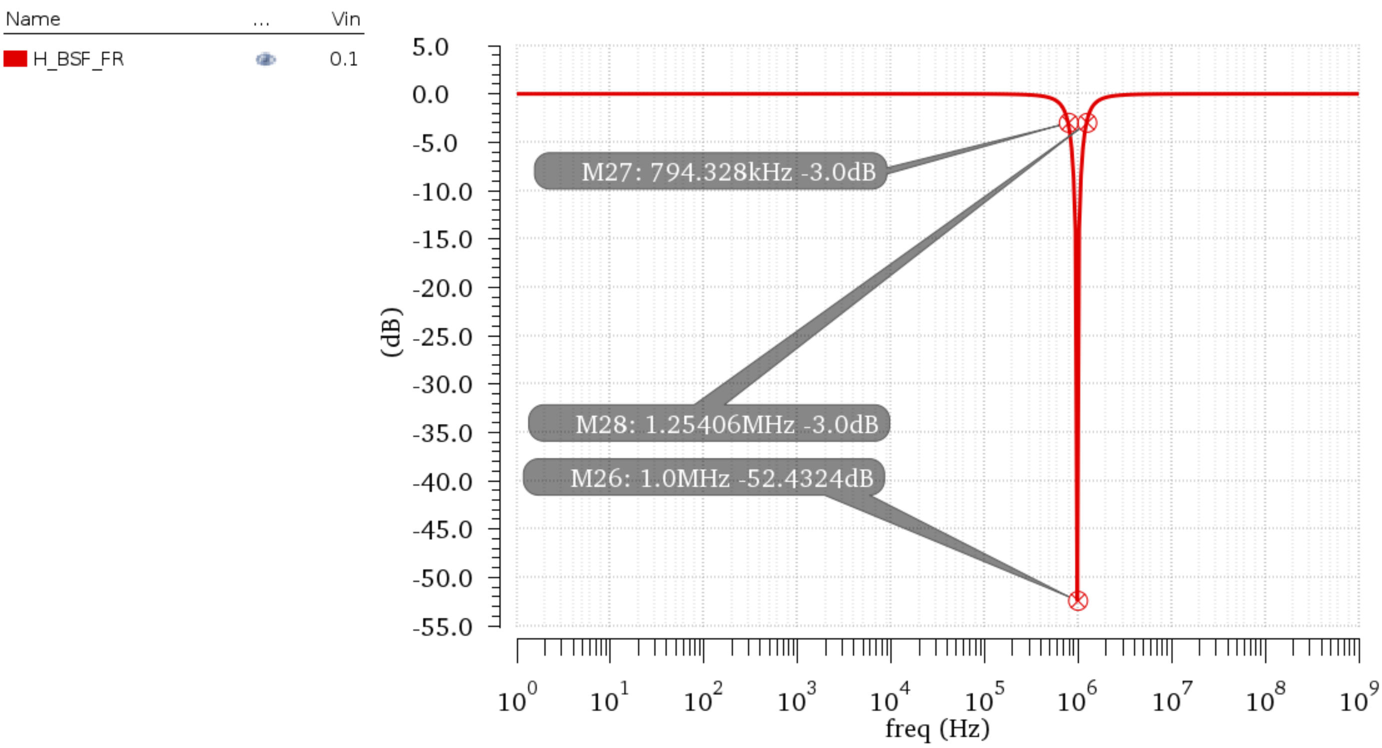
$@\left(\omega \rightarrow \omega_{1,2\,(-3\,dBs\,from\,peaking)}\right)$

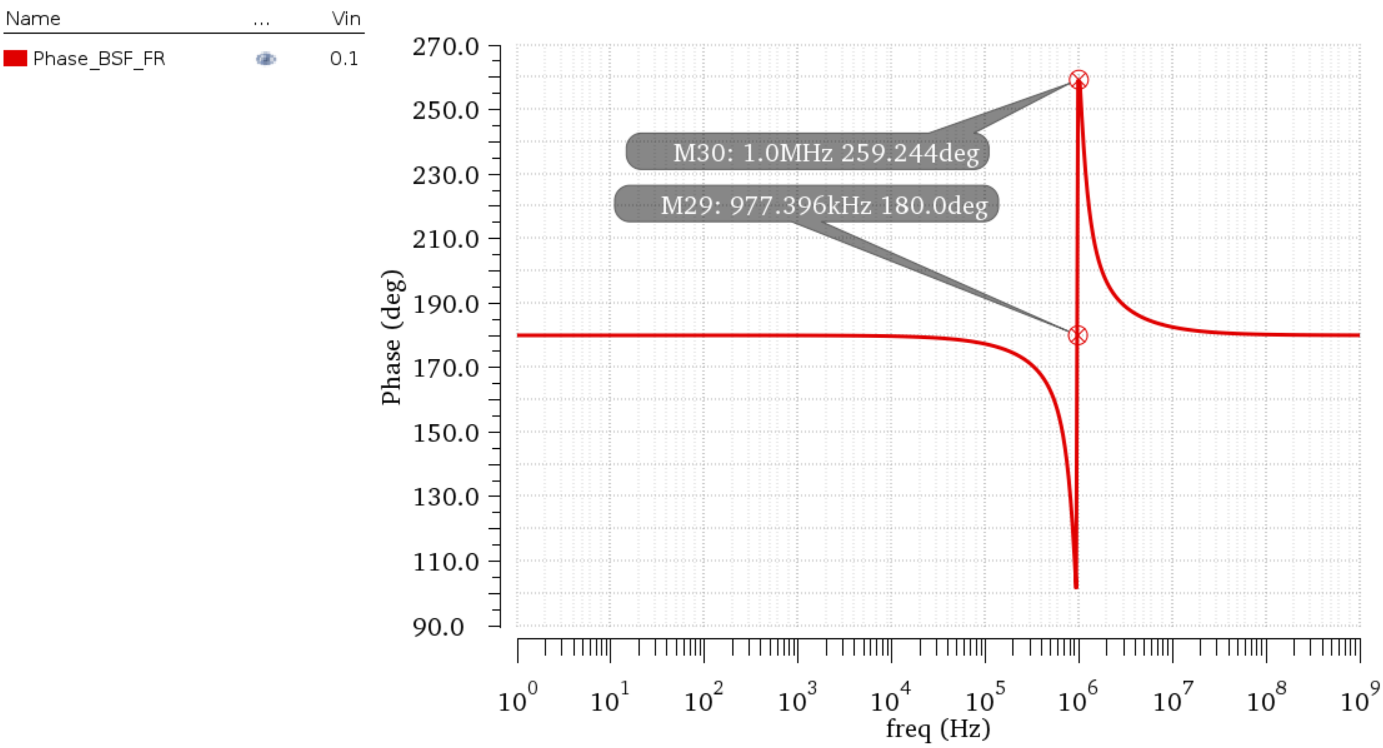
$$\omega_{1,2} = \frac{\pm\frac{\omega_o}{Q}+\sqrt{\left(\frac{\omega_o}{Q}\right)^2+4\omega_o^2}}{2} = \pm\frac{\omega_o}{2Q}+\omega_o\sqrt{1+\frac{1}{4Q^2}}$$

$$\therefore BW = \omega_2 - \omega_1 = \frac{\omega_o}{Q} \qquad \therefore \omega_o^2 = \omega_2\,\omega_1$$

$H_{BSF}(S) = \frac{S^2+\omega_o^2}{S^2+\frac{\omega_o}{Q}S+\omega_o^2}$ But The Transfer function is $H_{BSF}(S) = \frac{-H_o\left(S^2+\frac{1}{C^2R^2}\right)}{S^2+\frac{S}{QRC}+\frac{1}{C^2R^2}}$

$\therefore$ Magnitude Shifts by $20\log|H_o| \cong 0\ dB$, Phase Shifts down by $-180°$

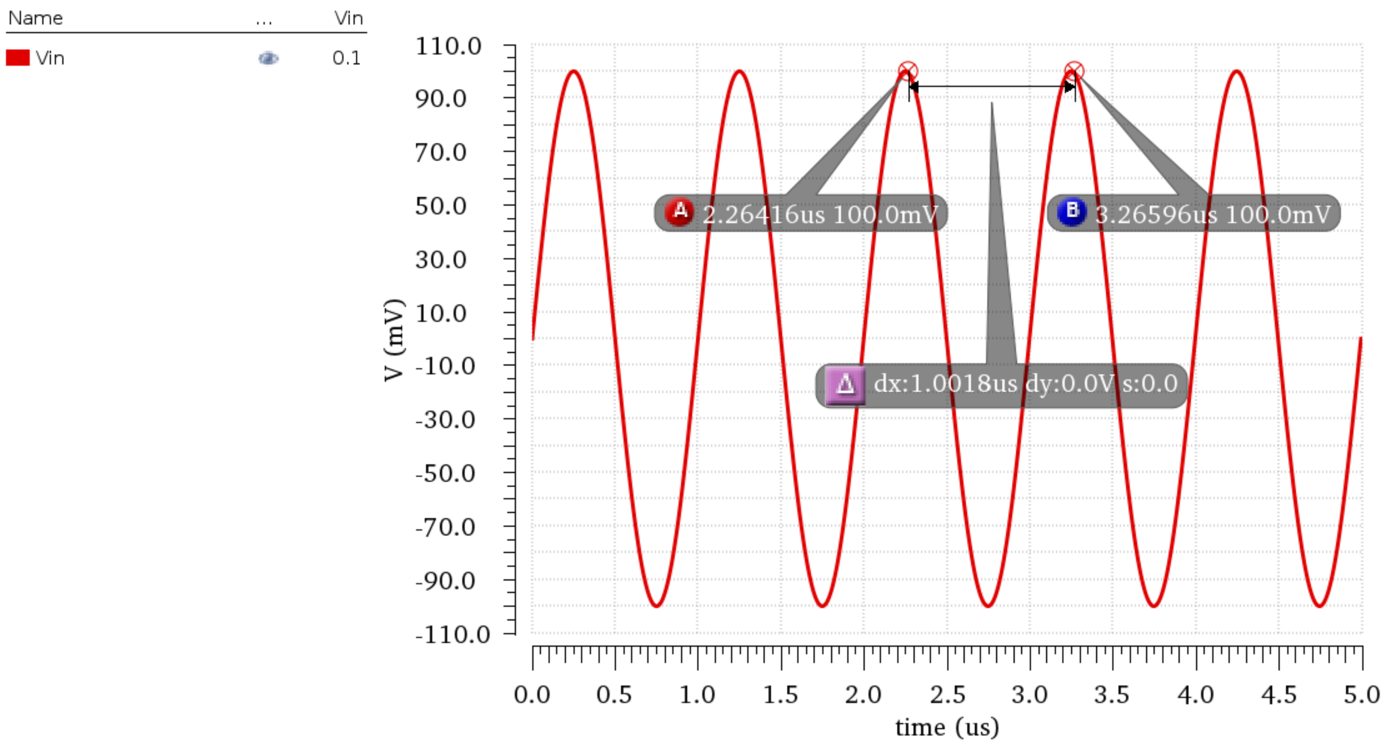




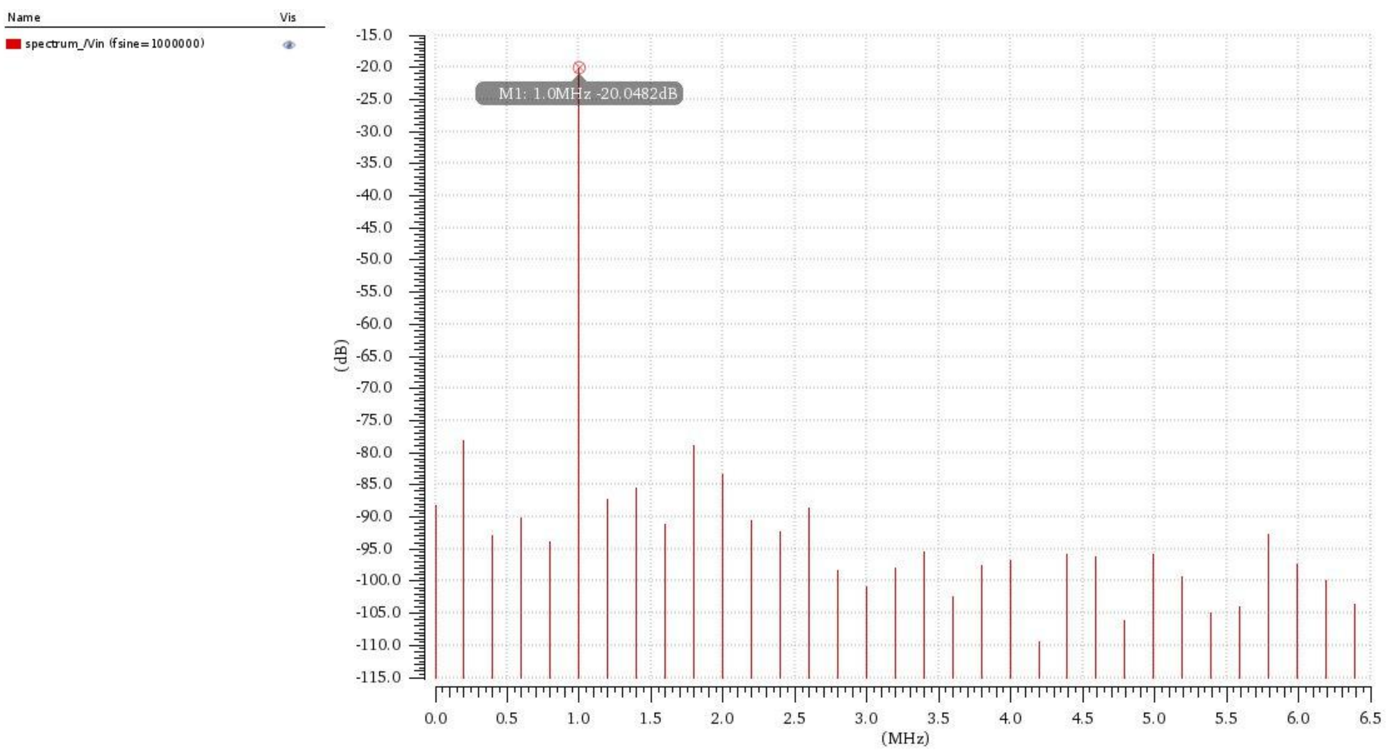
P.O.C	Simulation	Theoretically
Cut off Freq. $(\boldsymbol{\omega_o})$	From Magnitude Plot @ Notch: @ Magnitude $= -52.4324\ dB$ $f_o = 1\ MHz$ $\omega_o = 2\pi f_o$ $\omega_o = 6.3 \times 10^6\ rad/sec$	$C = 15.92\ pF\ , H_o = 1\ ,$ $Q = 2.2\ , R = 10\ k\Omega\ ,$ $\omega_o = \frac{1}{RC} = 6.3 \times 10^6\ rad/sec$
3 dB $\boldsymbol{f_{1,2}}$	$f_1 = 794.328\ kHz$ $f_2 = 1.25406\ MHz$ Note: $\sqrt{f_1 f_2} = 0.9981\ MHz \cong 1\ MHz$	$f_{1,2} = \pm\frac{f_o}{2Q} + f_o\sqrt{1 + \frac{1}{4Q^2}}$ $f_1 = 1.253\ MHz$ $f_2 = 798.23\ kHz$
DC Gain $(\boldsymbol{H})$	From Magnitude Plot: @ ω_o Cut off Freq. $\lvert H_{BSF}(\omega_{low})\rvert = H_o = 0\ dB$ $20\log\lvert H_o\rvert = 0\ dB$ $H_o \cong 1$	$H_o = \left(\frac{10\ k}{10\ k}\right) = 1$
Bandwidth BW	$BW = f_2 - f_1 = 459.732\ kHz$	$BW = \frac{f_o}{Q} = 454.55\ kHz$
Quality Factor	$Q = \frac{f_o}{BW} = \frac{1\ MHz}{459.732\ kHz} = 2.175$	$Q = \left(\frac{QR}{R}\right) = \frac{22\ k}{10k} = 2.2$

Transient Response
Sine Wave

$$V_{sine}\left(Amplitude = 100\ mV\ , Frequency = f_o = 1\ MHz\ , V_{out} = V_{in} \times H(\omega_o)\right)$$

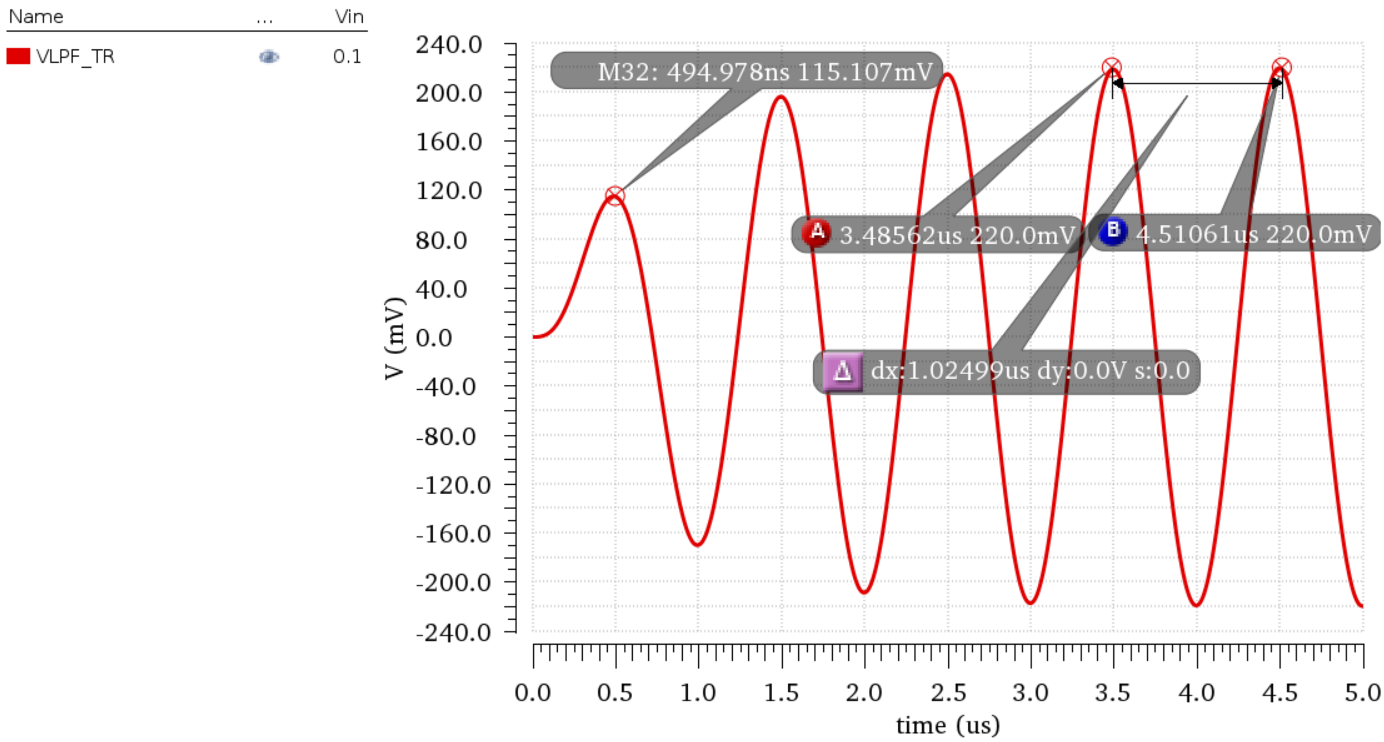


DFT Analysis For input Sine Wave



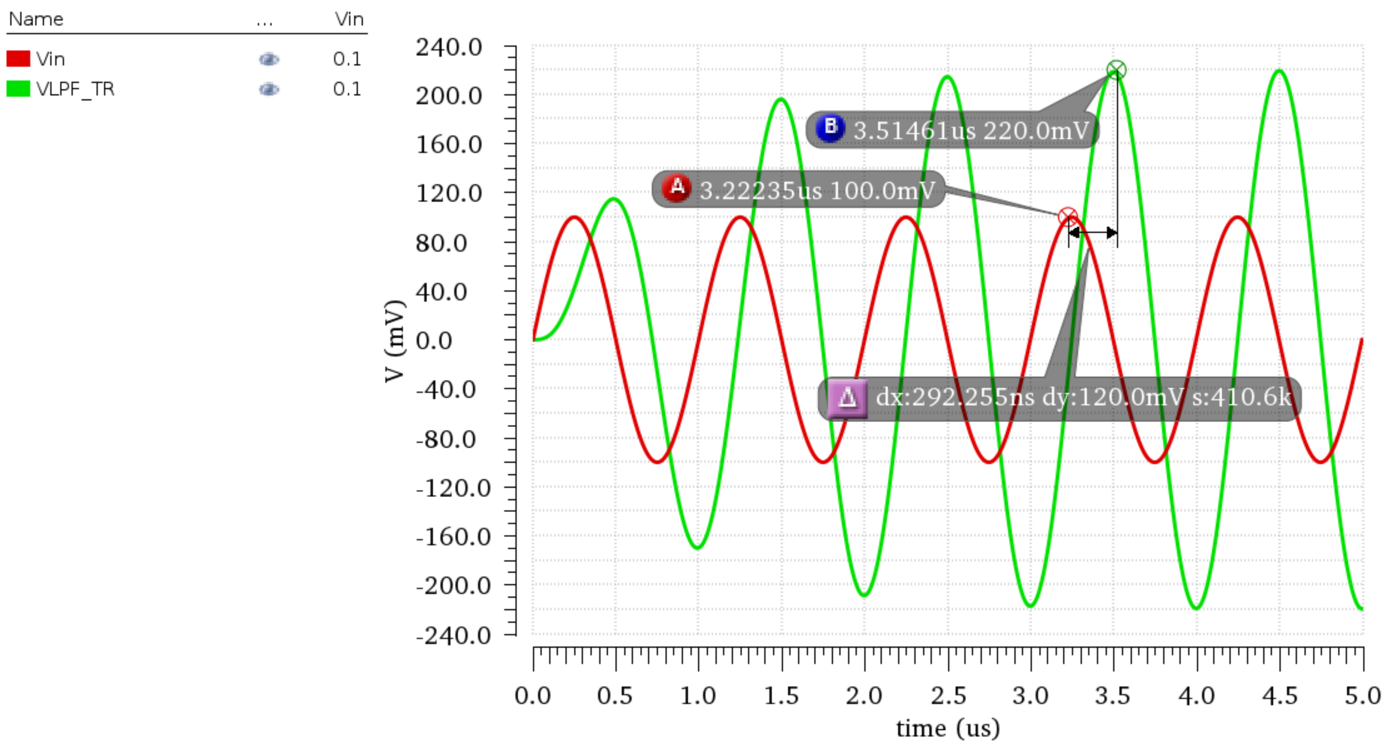
LPF Response

LPF Transfer Function for Sine input Waveform in Transient Response

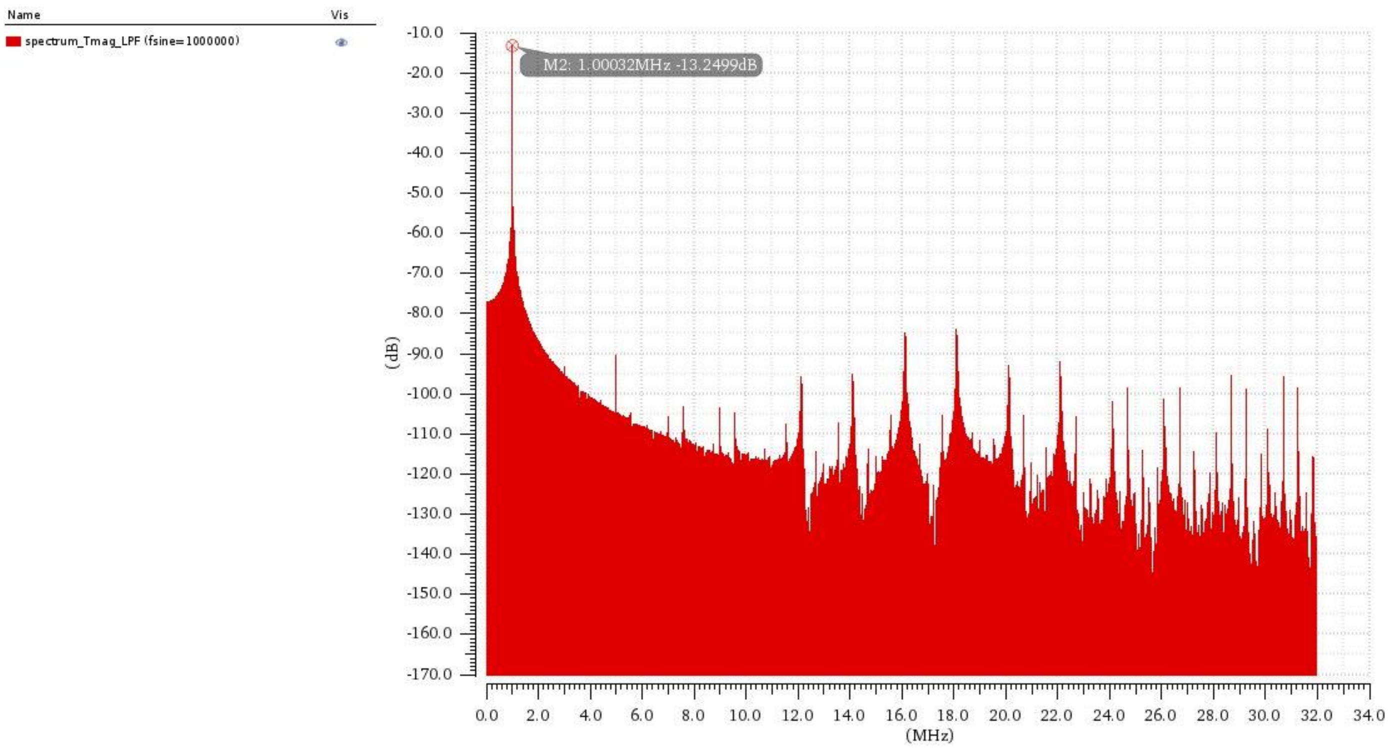


LPF in Transient Response Show the Phase difference

$$\Delta\theta = \frac{\Delta T\ (Between\ I/O\ peaks)}{T\ (period)} \times 2\pi = \frac{0.2923\ \mu\, s}{1\ \mu\, s} \times 360^\circ \rightarrow\approx \frac{1}{4}\ cycle \approx -90^\circ$$

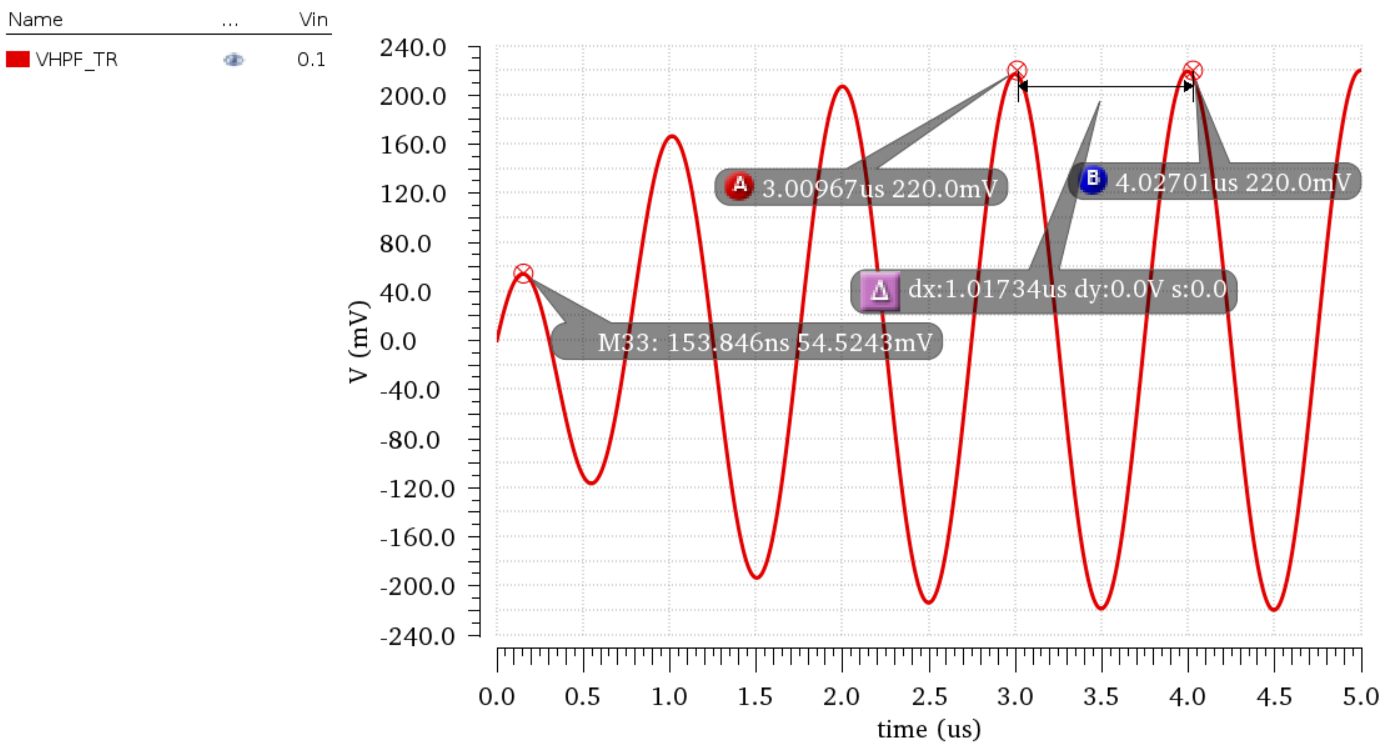


LPF DFT Analysis



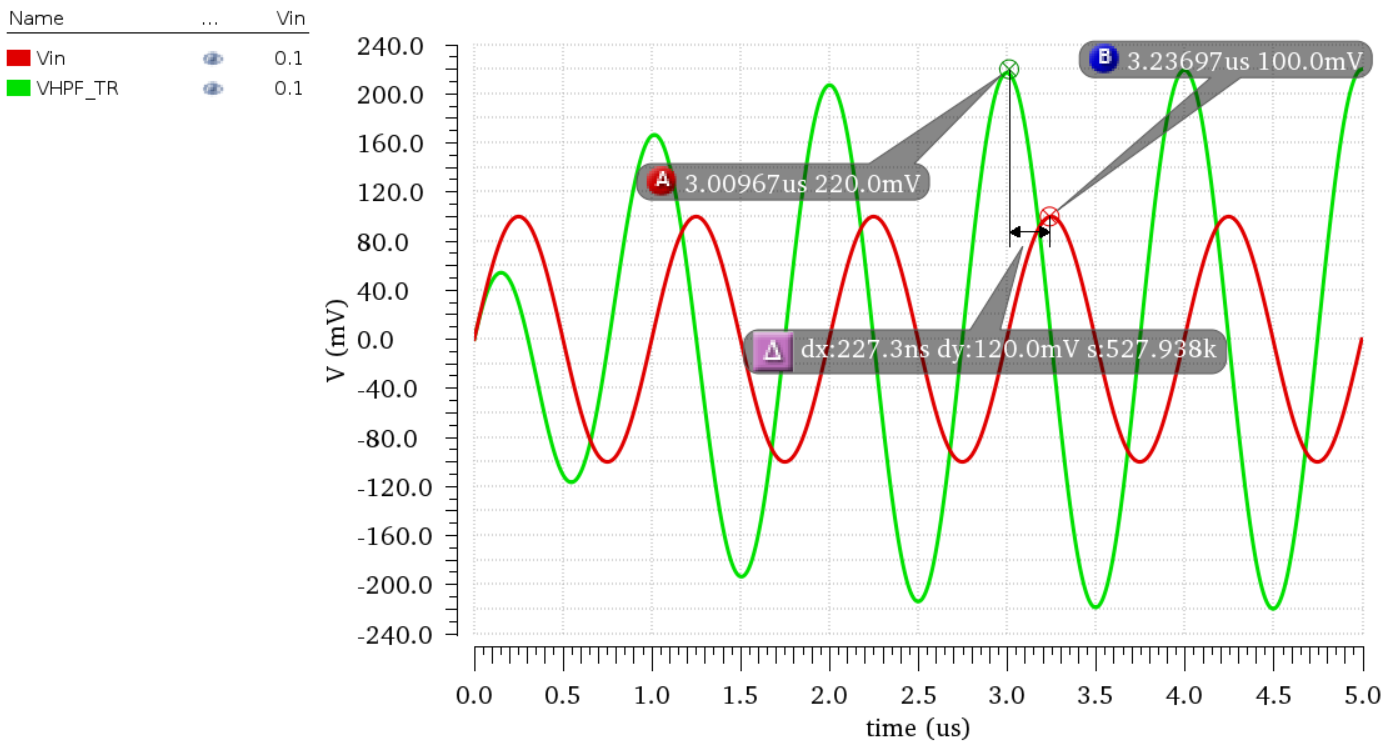
HPF Response

HPF Transfer Function for Sine input Waveform in Transient Response

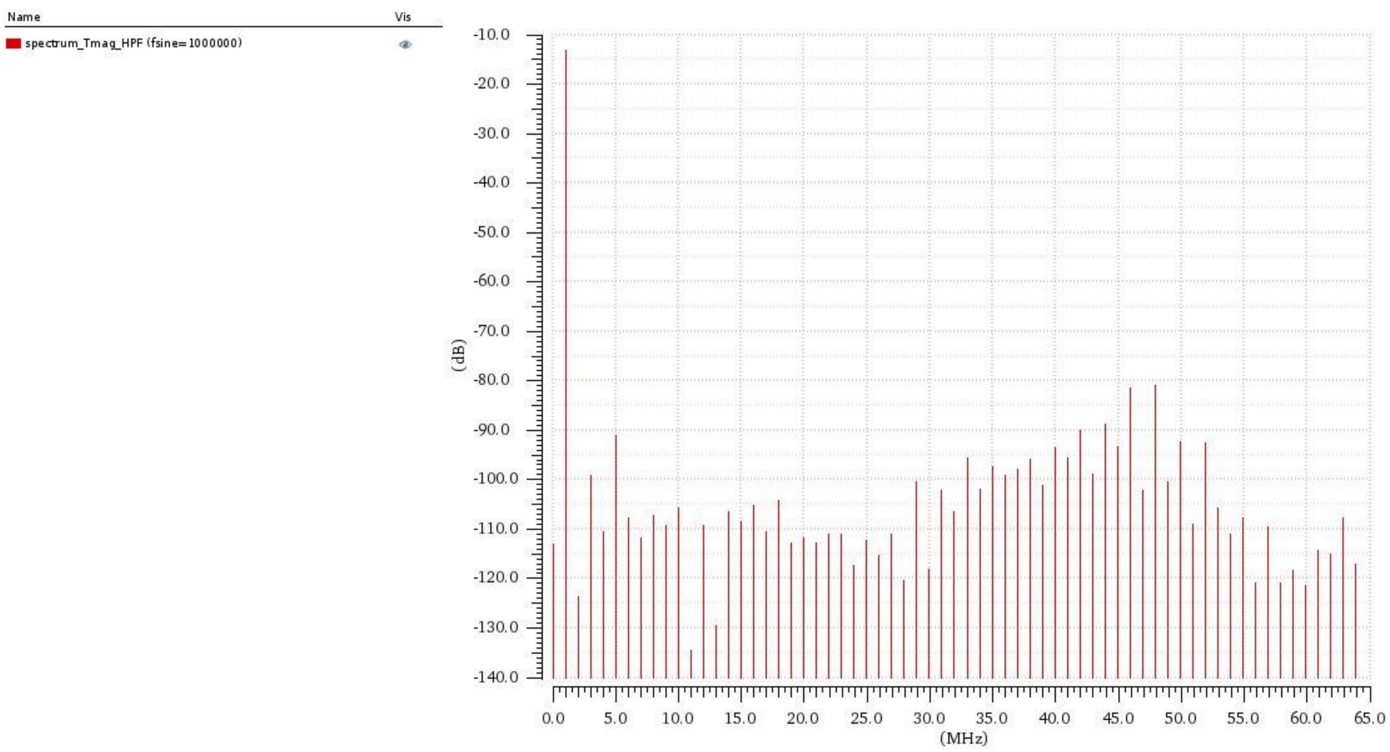


HPF in Transient Response Show the Phase difference

$$\Delta\theta = \frac{\Delta T\ (Between\ \ I/O\ peaks)}{T\ (period)} \times 2\pi = \frac{0.2273\ \mu\ s}{1\ \mu\ s} \times 360° \rightarrow\approx \frac{1}{4}\ cycle \approx +90°$$



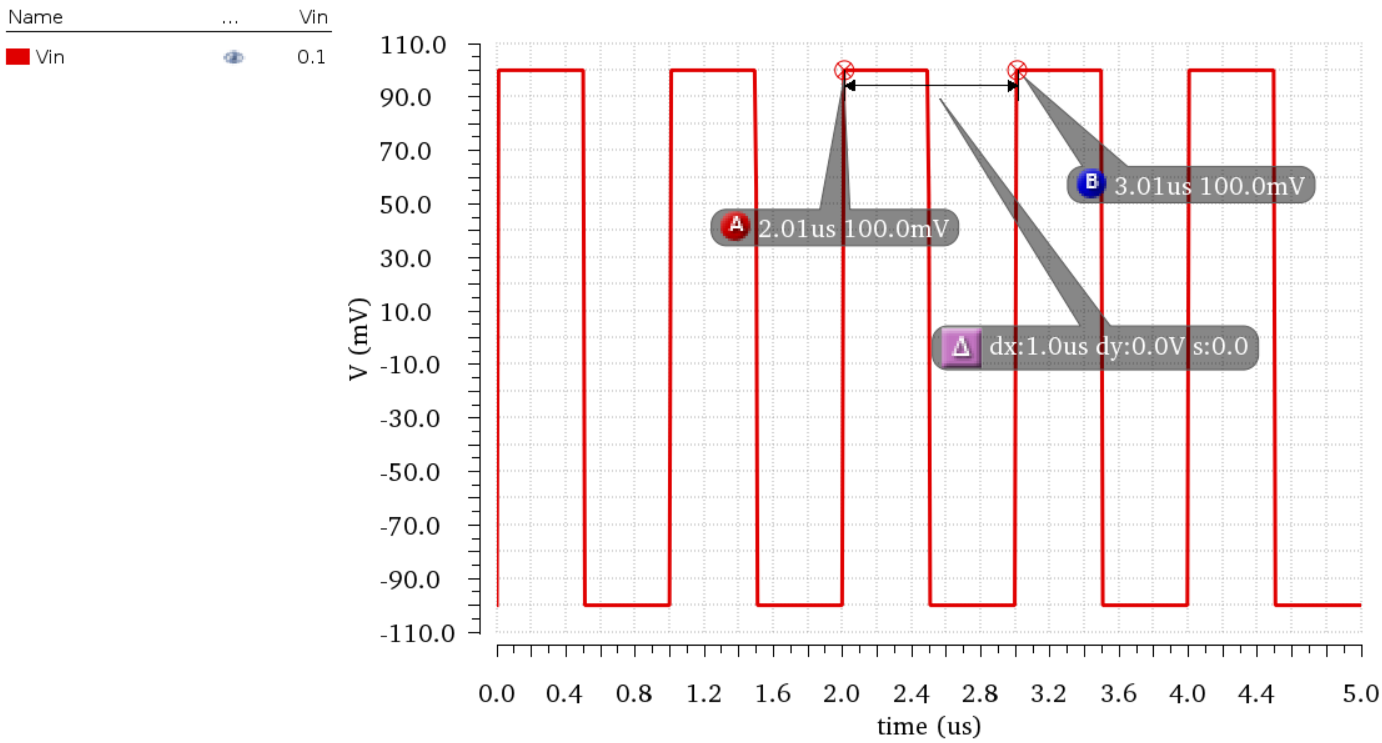
HPF DFT Analysis



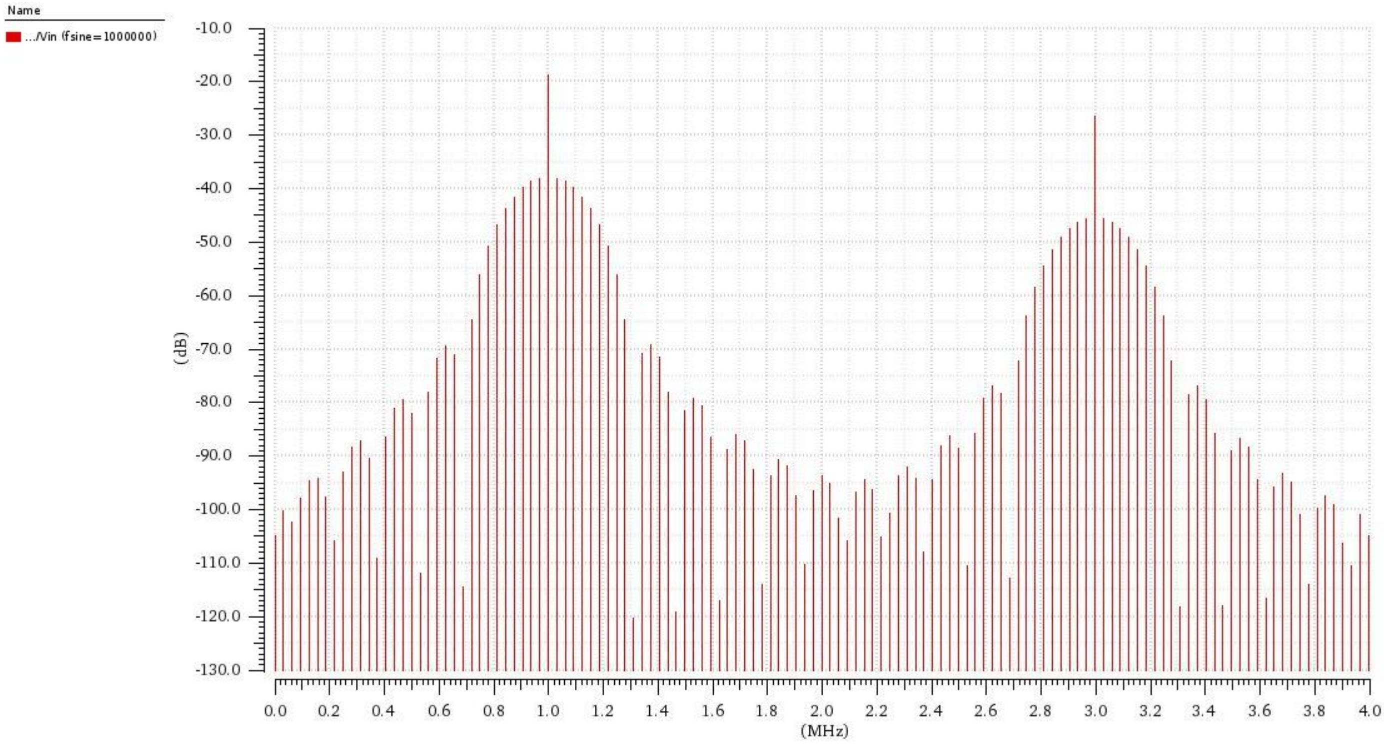
P.O.C	Transient Simulation	AC Simulation
Amplitudes	$V_{in} = 100\ mV$ $V_{LPF} = 100 \times 2.19283\ mV = 219.3\ mV$ $V_{HPF} = 100 \times 2.19433\ mV = 219.4\ mV$	$\lvert H_{LPF}(\omega_o)\rvert = H_o Q = Q$ $\lvert H_{HPF}(\omega_o)\rvert = H_o Q = Q$ $Q = 2.198$ $V_{LPF} = V_{in} \times Q = 219.8\ mV$ $V_{HPF} = V_{in} \times Q = 219.9\ mV$
Delay Phase Shift	V_{HPF} and V_{LPF} are 180° Out of Phase V_{in} peak @ $t = 3.23697\ \mu s$ V_{LPF} peak @ $t = 3.51461\ \mu s$ V_{HPF} peak @ $t = 3.00967\ \mu s$ $T_{period} = 1\ \mu s \rightarrow \lambda = 360°$ $\Delta T_{peak}(LPF - HPF) = 0.5\ \mu s = \frac{\lambda}{2} = 180°$ $\Delta T_{peak}(V_{in} - V_{out}) = 0.25\ \mu s = \frac{\lambda}{4} = 90°$	@ ω_o Cut off Freq. $\theta_{HPF} = 90°$ $\theta_{LPF} = -90°$ $V_{HPF} = V_{in}\ \lvert Q\rvert \angle 90°$ $V_{LPF} = V_{in}\ \lvert Q\rvert \angle -90°$
Comments	V_{HPF} Leads V_{in} by 90° V_{LPF} Lags V_{in} by 90° V_{HPF} Leads V_{LPF} by 180°	Between V_{LPF} and V_{HPF} 180° Phase Difference

Transient Response Square Wave

$$V_{source}\ Pulse\ Wave\ \left(V_{one} = 100\ mV\ , V_{zero} = -100\ mV\ , T = \frac{1}{f_o} = 1\ \mu S\right)$$

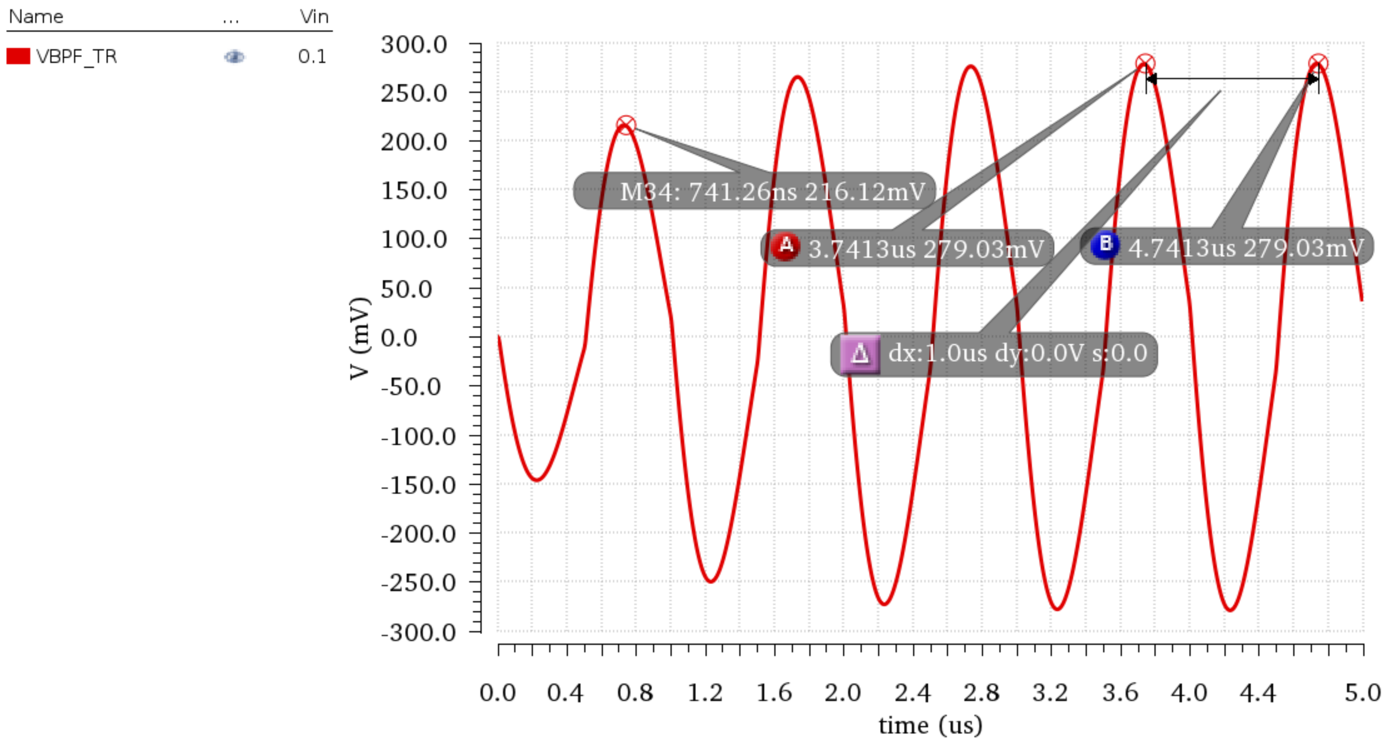


DFT Analysis For input Square Wave



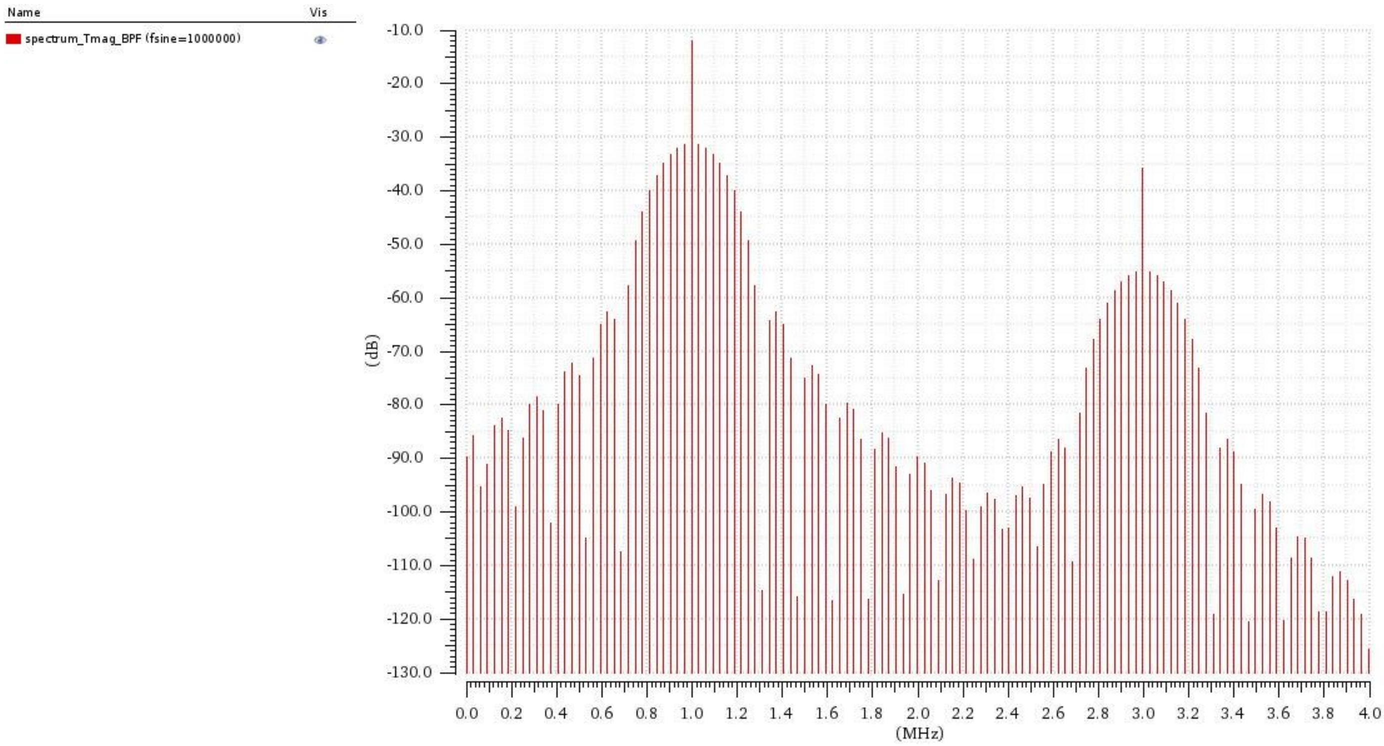
BPF RESPONSE

BPF Transfer Function for Square input Waveform in Transient Response



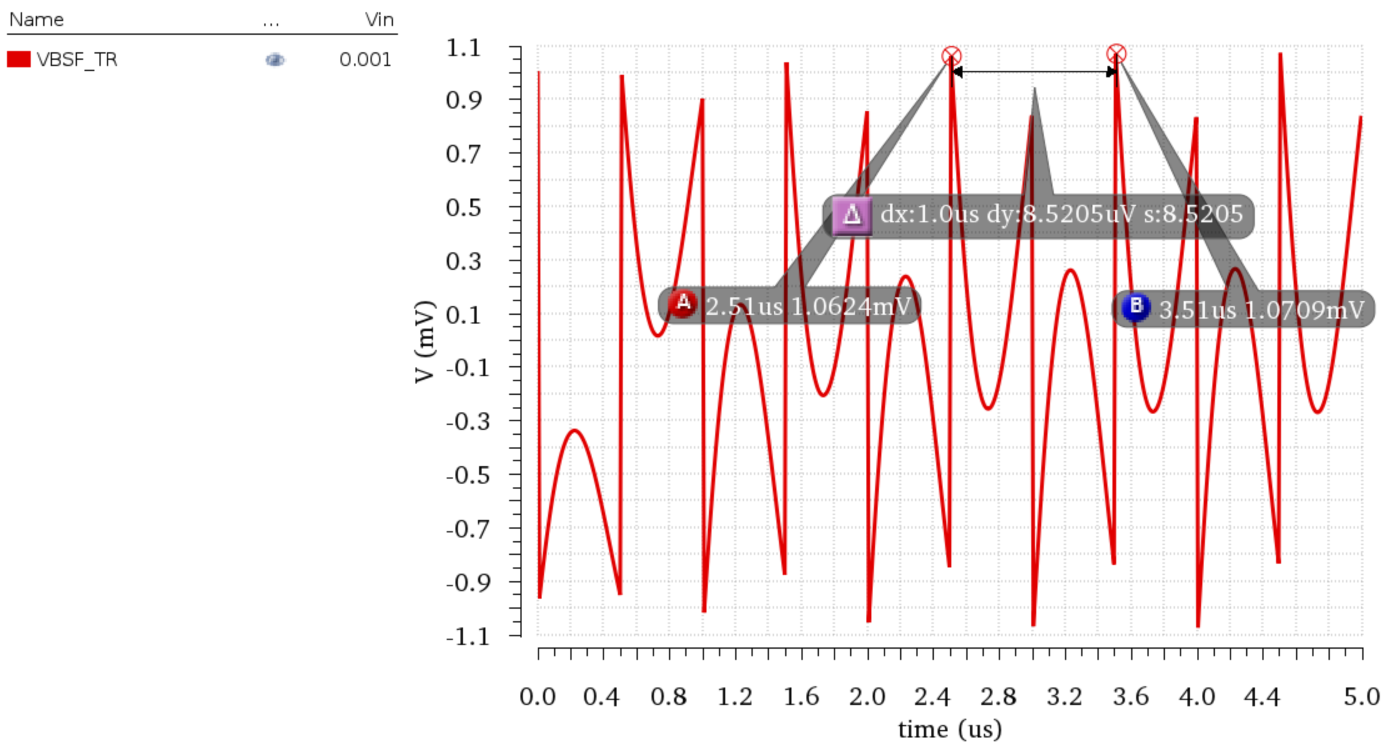
BPF DFT Analysis

(Highest Components is at $f = f_o$ While Other Harmonics get Rejected)

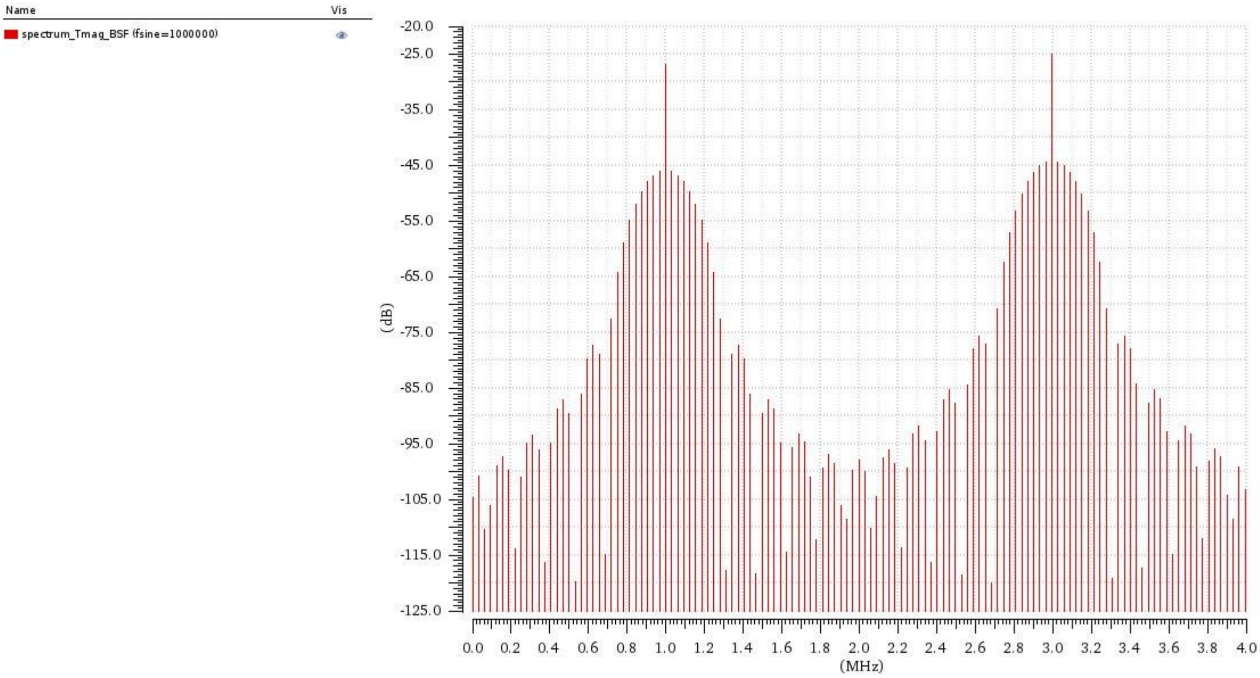


BSF Response

BSF Transfer Function for Square input Waveform in Transient Response



BSF DFT Analysis (All Components Seems Rejected Except at $f = f_o$)



COMMENTS

Analysis of Square Pulse Use Fourier Series Coefficient:

$$a_o = \frac{Area}{T} \text{ , } b_n = 0 \ (even) \text{ , } a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin(n\omega_o t)\,dt \ = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) \ (n: odd)$$

$$V_{in}(t) = 2a_o\left[\frac{1}{2} + \sum \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_o t)\right] = 2a_o\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_o t) - \frac{1}{3}\cos(3\omega_o t) + \frac{1}{5}\cos(5\omega_o t) - \ldots\right)\right]$$

$$V_{out}(t) = 2a_o\left[\frac{1}{2}H(0) + \frac{2}{\pi}\left(\cos(\omega_o t)\,H(\omega_o) - \frac{1}{3}\cos(3\omega_o t)\,H(3\omega_o) + \frac{1}{5}\cos(5\omega_o t)\,H(5\omega_o) - \ldots\right)\right]$$

These Frequency Components are (Pass , Amplified , Rejected)

Frequency	BPF	BSF
@ $\boldsymbol{\omega = 0}$	$H_{BPF}(0) = 0$ (Rejected)	$H_{BSF}(0) = 1$ (Pass)
@ $\boldsymbol{\omega = \omega_o}$	$H_{BPF}(\omega_o) = Q$ (Pass and amplified)	$H_{BSF}(\omega_o) = 0$ (Rejected and Blocked)
@ $\boldsymbol{\omega = 3\omega_o}$	$H_{BPF}(3\omega_o)$ (Rejected by Some dB)	$H_{BSF}(3\omega_o) = 1$ (Pass)
@ $\boldsymbol{\omega = 5\omega_o}$, …	All Higher Harmonics are Rejected	All Higher Harmonics Pass

Notes:

- **BPF Signal:** Slightly Distorted Sine Wave as BPF amplified Component at $f = f_o$ And Reject the Other Components.
- **BSF Signal:** Rectangular Pulse Subtract a Sine Wave From it as BSF Reject the Component at $f = f_o$ and Pass all Other Components.

The END Thank You