Transistor (TRA) Protocol

Version 1.0

beginners laboratory course Part II

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1

Introduction and theoretical background

1.1 Introduction

The goal of this report is to understand and study the behaviour of bipolar transistors in grounded emitter circuits.

1.2 Circuits for the emitter grounded transistor

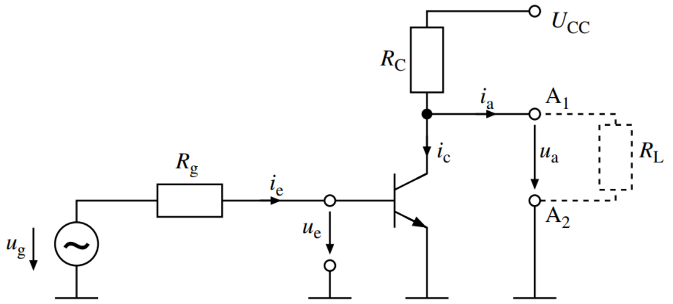


Figure 1.1: The simplest circuit for operating an emitter grounded transistor[1]. Note the in-/output voltage u_e / u_a, and the optional load resistance R_L.

Although the transistor could be used in three different configurations based on the grounded terminal, we restrict ourselves to the emitter grounded configuration, whose most basic circuit that can be run without a short circuit can be seen in figure 1.1.

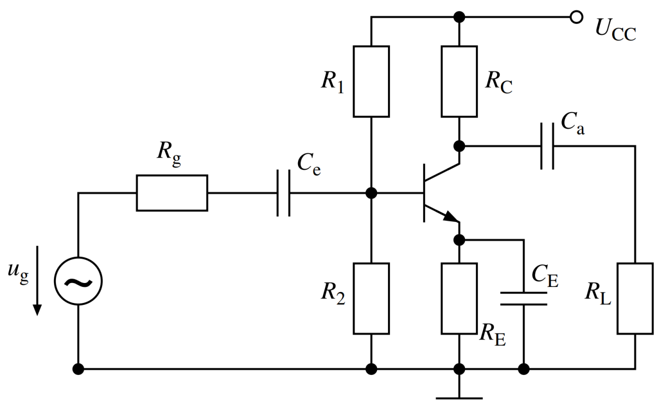


Figure 1.2: Emitter grounded transistor circuit improved with negative-feedback-mechanism and optional capacity C_E. Also note that in-/output voltage are now induced via capacity[1].

Due to the physical properties of bipolar transistors, they need to be operated around their working point to guarantee linear amplification. To stabilize that, the setup can be improved by adding a resistor between emitter terminal and ground (see figure 1.2), such that by a negative feedback mechanism, the collector current is less affected by the signals.

1.2.1 Mathematical modelling of two-port networks

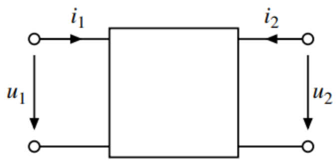


Figure 1.3: The structure of a general two-port network.[1].

For Two-port networks like in figure 3.5, the two currents can be written as a function of their corresponding voltages using y-parameters.

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{1.1}$$

These two linear equations can be rewritten using h- Parameters to create a function for the voltage $u_1(i_1)$ and one for the current $i_2(u_2)$.

$$\begin{pmatrix} u_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ u_2 \end{pmatrix} \tag{1.2}$$

This formalism can now be used to describe transistors in first order approximation for small signal voltages, leading to the expressions

$$\begin{pmatrix} i_b \\ i_c \end{pmatrix} = \begin{pmatrix} \frac{1}{r_{BE}} & 0 \\ S & \frac{1}{r_{CE}} \end{pmatrix} \begin{pmatrix} u_{BE} \\ u_{CE} \end{pmatrix} \tag{1.3}$$

$$\begin{pmatrix} u_{BE} \\ i_c \end{pmatrix} = \begin{pmatrix} r_{BE} & 0 \\ S \cdot r_{BE} & \frac{1}{r_{CE}} \end{pmatrix} \begin{pmatrix} i_b \\ u_{CE} \end{pmatrix} \tag{1.4}$$

where i_b / i_c is the current flowing into basis/collector and $u_{\mathrm{BE}} / u_{\mathrm{CE}}$ is the voltage between basis-emitter/collector-emitter. Equation 1.3, which is written in y-notation like equation 1.1, was converted to h-notation like equation 1.2 to generate equation 1.4 using the formulas derived in section 3.1.2. The entries of these matrices are the differential characteristic quantities

$$\frac{1}{r_{BE}} = \left.\frac{\partial I_B}{\partial U_{BE}}\right|_{U_{CE}} \tag{1.5}$$

$$\frac{1}{r_{CE}} = \left.\frac{\partial I_C}{\partial U_{CE}}\right|_{U_{BE}} \tag{1.6}$$

$$S = \left.\frac{\partial I_C}{\partial U_{BE}}\right|_{U_{CE}} \tag{1.7}$$

which can be interpreted as the differential resistances $r_{\mathrm{BE}} / r_{\mathrm{CE}}$ between basis-emitter/collector-emitter, and as the steepness S.

Using physical formulas for npn transistors, one further finds the relation

$$I_B \propto \exp\left(\frac{qU_{BE}}{k_B T}\right), \tag{1.8}$$

which connects the basis current I_B to the voltage between basis and emitter U_BE using the temperature of the transistor $k_\mathrm{B} T$ and the elementary charge q.

1.3 Calculating the amplification of voltage

In case a negative-feedback-stabilisation of the working point is used, i.e. there is resitance between the emitter and the ground, one can calculate the amplification of voltage $A = \frac{U_\mathrm{a}}{U_\mathrm{e}}$ by

$$|A| = \frac{\beta(R_\mathrm{C}\|R_\mathrm{L})}{r_\mathrm{BE} + (1+\beta)R_\mathrm{R}} \approx \begin{cases} \frac{R_\mathrm{C}\|R_\mathrm{L}}{R_\mathrm{E}} & \text{with } R_\mathrm{L} \\ \frac{R_\mathrm{C}}{R_\mathrm{E}} & \text{without } R_\mathrm{L} \end{cases} \tag{1.9}$$

where the approximation is valid for big β, which is the amplification of currents. In case the negative-feedback is short-circuited, for example using a capacity like C_E in figure 1.2, one has to know the differential parameters S and r_CE and finds

$$A = -S \cdot \begin{cases} (R_\mathrm{C}\|r_\mathrm{CE}\|R_\mathrm{L}) & \text{with } R_\mathrm{L} \\ (R_\mathrm{C}\|r_\mathrm{CE}) & \text{without } R_\mathrm{L} \end{cases} \tag{1.10}$$

where the equation depends on whether a load resitance R_L is used like in figure 1.2.

2
Experimental setup and procedure

2.1 Setup and procedure for the emitter grounded transistor

The experiment as seen in figure 2.1 was setup after graphic 1.2 and the Instructions [1].

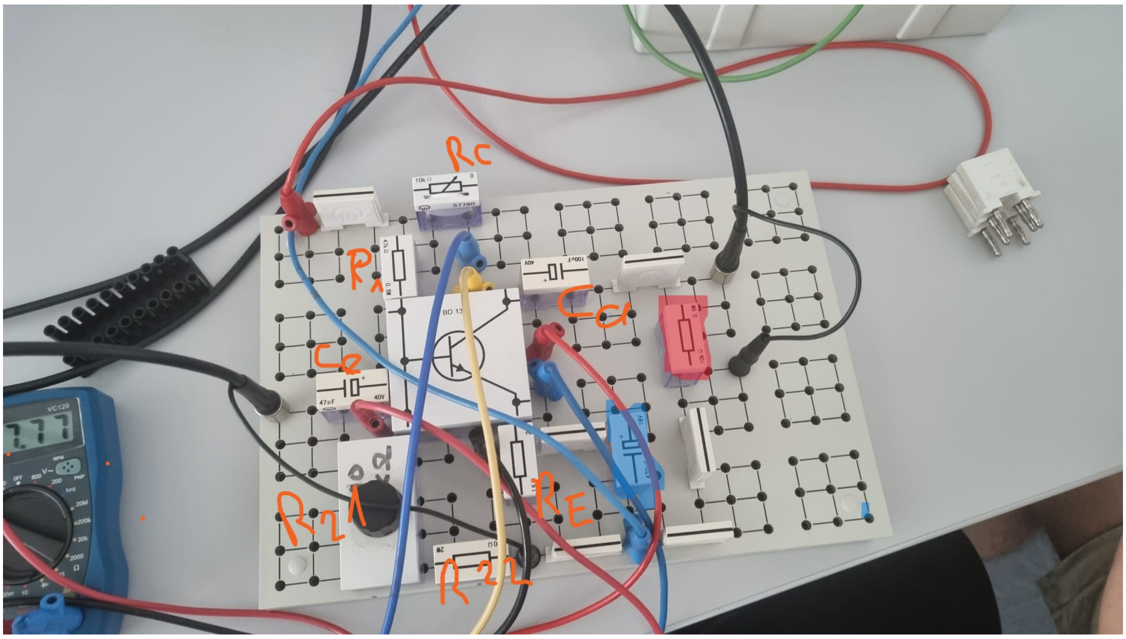


Figure 2.1: Experiental setup with Resistor marked with R and capacitors marked with C, of which you can find the values in table 2.1. In blue you can see another Capacitor C_E, which has a value of 470 μF. In Red you can see another Resistor R_L, which has a value of 10 kΩ. R_C is a potentiometer, which is describeable as a variable Resistor, with values ranging from 0 to 10 kΩ

components	R_1	R_{22}	R_E	C_e	C_a
values	47 kΩ	100 Ω	1 kΩ	47 μF	100 μF

Table 2.1: Setup values of constant components of the Experiment

2.1.1 Setting the operating point

To set the operating point, the resistance R_C is set to 10 kΩ. The input signal U_e is set to be a sinusoidal signal with a frequency of approx. 5.5 kHz and an amplitude of approx. 10 mV. The potentiometer R_{21} is now set so, that the amplitude of the output signal U_a

is maximised, but not yet distorted. Doing this, we get an output voltage of $U_a = 4.48\,\mathrm{V}$. This will be the operating point used for the rest of this report.
To measure the resistance caused by the potentiometer in this configuration, it has to be build out of the circuit. If this is not done, one gets unstable measurements depending on the direction of the current (the measurements are $120\,\mathrm{k}\Omega$ and $-24.4\,\mathrm{k}\Omega$, which differ greatly). Therefore, the potentiometer is build out and measured giving a stable and reasonable resistance of $R_{21} = 6.6\,\mathrm{k}\Omega$.

Next measurements where taken, with the same setup forU_{BE}, U_{CE} and I_C as seen in Table 2.2 with a mean value of U_{BE} of 551 mV

	$10\,\mathrm{k}\Omega$	$5\,\mathrm{k}\Omega$	$1\,\mathrm{k}\Omega$
U_{BE}	551 mV	551 mV	551 mV
U_{CE}	2.98 V	5.64 V	7.76 V
I_{C}	535 µA	536 µA	536 µA

Table 2.2: Table containing the values of U_{BE}, U_{CE} and I_C for three different resistances measured in the experiment.

2.1.2 Measuremnets for the amplification of the emitter circuit

The frequency generatorthe frequency generator was set to a frequency of 5.5 kHz. With that there were three measuring sequences done, each by variing the potentiometer R_C varied from $1\,\mathrm{k}\Omega$ up to $10\,\mathrm{k}\Omega$. The first sequence was with the experimental setup from figure 2.1 without Resistor R_L. The second sequence was then done without neither the Resistor R_L or the Capacitor C_E. The last row of measurements was then done with the whole setup, like shown in the figure 2.1.

2.1.3 Frequency response

The next measurement where done in order to determine the phase angle between the input and output signal for some frequencies. For that the setup from figure 2.1 was used again, with R_C fixed at the value of $10\,\mathrm{k}\Omega$. The frequency was then changed in the range from 6 Hz up to 250 kHz. Measurements were taken from the amplitudes of the input and output signals for every change of frequency.

2.2 Experimental setup for calculating the characteristic output curve of the transistor

To record the output characteristic curve $I_C = f(U_{CE})$ for a transistor, we used the circuit setup depicted in Figure 2.2. Three multi-meters were employed to measure the

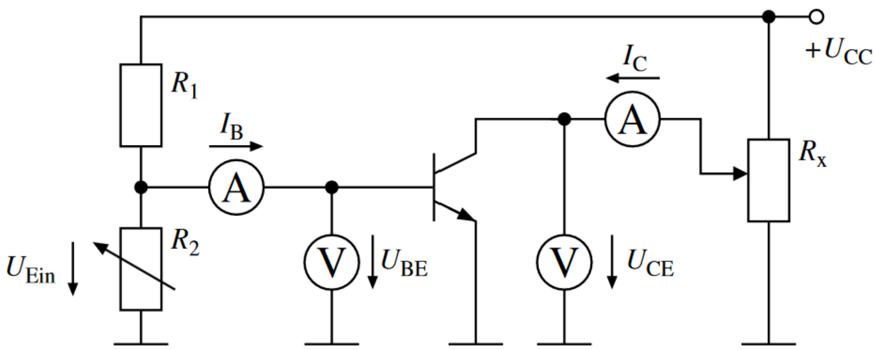


Figure 2.2: Experiental setup for calculating the characteristic output curve of the transistor with Resistors marked with R, multi-meters measuring Voltage marked with V and multi-meters measuring current marked with A. The value for R_1 is $1\,k\Omega$. R_1 is a Potentiometer ranging up to $220\,\Omega$ and R_X is a Potentiometer ranging up to $1\,k\Omega$

collector current I_C, the base-emitter voltage U_{BE}, and the collector-emitter voltage U_{CE}. First, we set the base-emitter voltage U_{BE} to the (average) value determined earlier and kept it constant throughout the experiment. We varied the collector-emitter voltage U_{CE} from 0 to 10 V, recording the corresponding collector current I_C. The measurements were taken both in an increasing order (from low to high U_{CE}) and in a decreasing order (from high to low U_{CE}) to observe any hysteresis effects. Special attention was given to the range of very low U_{CE} values, where the collector current I_C changes rapidly. Multiple measurements were taken in this region to capture the detailed behavior of the transistor.

3
Results and Discussion

3.1 Preliminary considerations

3.1.1 Thoughts on why the differential quantities can't just be measured using a multimeter

Naivly, one could try to just measure the differential quantities of a transistor using a multimeter. Since we have seen that the multimeter can only measure resistances for resistors that are not currently under voltage, that would be only possible for a transistor, that is not plugged into a circuit.

But the differential quantities used to describe transistors in small signal approximation are mathematical auxiliary quantities rather than physical properties. They describe proportionality factors in a mathematical model, where the transistor is approximated to behave linearly. Since that model itself is only valid around the working point, it cannot be applied to a transistor that has been plugged off, hence the multimeter-measurements would not result in any relevant data.

3.1.2 Converting formulas from y- to h-notation

To convert eq. 1.3 to eq. 1.4 one uses the following relations, also taken from [1].

$$h_{11} = \frac{1}{y_{11}} \quad h_{12} = -\frac{y_{12}}{y_{11}} \quad h_{21} = \frac{y_{21}}{y_{11}} \quad h_{22} = \frac{y_{11}y_{22} - y_{21}y_{12}}{y_{11}}.$$

The differential small-signal gain β is defined as the ratio between I_C and I_B, whereas the steepness was defined in eq. 1.7 as the derivative of I_C regarding U_{BE}. If one now uses eq. 1.5, which defines r_{BE} as the derivative of U_{BE} regarding I_B, one gets this relation between S and β

$$\beta = S \cdot r_{BE} \tag{3.1}$$

This is of course only valid for the linearisation of these quantities, which can only be applied for small signals, which we luckily have here. It may be noted that β is only well defined in this special case. For bigger signals one has to look at another more complicated expression for the gain called B. The relations between the currents, resistances and voltages in this more general case get to complicated to be discussed in such a small lab report as this one, therefore we limit ourselves to the linear case.

3.2 Calculating the small signal values for a Bipolar junction transistor

3.2.1 The characteristic input curve I_B vs. U_{BE}

To calculate the values that describe the network, one has to plot the characteristic curves and fit a tangent at the working point. U_{CE} was held constant at 5.64 V for this part.
For the input curve depicted in fig 3.1 the entire Data was fitted using eq. 1.8. The fit parameter λ which is multiplied by U_{BE} in the exponent is $\frac{q}{k_B T}$, where k_B is the Boltzmann-constant, T the working temperature and q the charge, in this case given by that of an electron. Using this relation we can also calculate the temperature at which the transistor is functioning using

$$T = \frac{e}{k_B \cdot \lambda}. \tag{3.2}$$

This results in T = (281 ± 3) K, which is around 8 °C. It is at least the right order of magnitude, but it has to be wrong by many times the computed uncertainty, as the transistor is expected to warm up relative to the surrounding room temperature instead of cooling down.

Now we can Taylor this theory function to the first order to get the tangent at the working point. We get a slope of (126 ± 42) μA/V, which can be interpreted as the inversion of r_{BE} as shown in eq. 1.5. This gives us a differential resistance of r_{BE} = (7.93 ± 2.64) kVA^{-1}

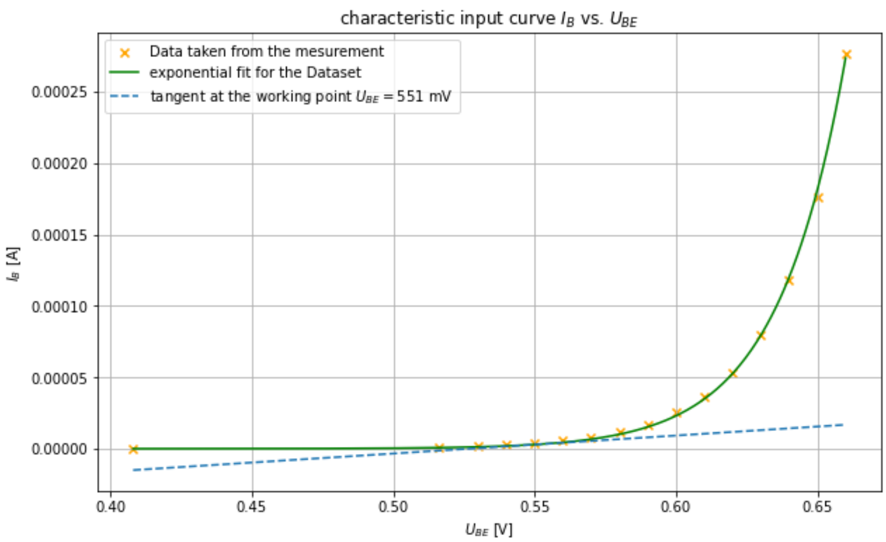


Figure 3.1: The Characteristic input curve plotting I_B against U_{BE}. The Dataset is plotted in orange and fittet by a exponential function in green. The tangent at the workingpoint = 551 mV is shown in a purple colour

3.2.2 The characteristic output curve I_C vs. U_{CE}

We can determine the differential resistance r_{CE} in a very similar fashion. First we plot the data, where in this case we measured two Datasets. Since here we do not have a theory Function, from which we can get the tangent, we simply look at the measurements near the working point, given by $I_C = 536\,\mu A$ in this case and fit a linear function through these points. As we can see in fig. 3.2 this gives us a very good approximation to a tangent. We get a slope of $(7.82 \pm 0.48)\,\mathrm{mAV^{-1}}$ for the purple tangent and a slope of $(6.44 \pm 0.48)\,\mathrm{mAV^{-1}}$ for the red one.

Now taking the weighted average we get $\frac{1}{r_{CE}} = (7.13 \pm 0.69)\,\mathrm{mAV^{-1}}$. Therefore we get $r_{CE} = (140 \pm 13)\,\mathrm{VA^{-1}}$

If we now use the equation

$$S = \lambda \cdot I_C \tag{3.3}$$

we get a value of $S = 0.022\,\mathrm{AV^{-1}}$. This value is almost the same for the resistances $R_C = 1, 5, 10\ \Omega$, since the current stays almost the same as we can see in 2.2.

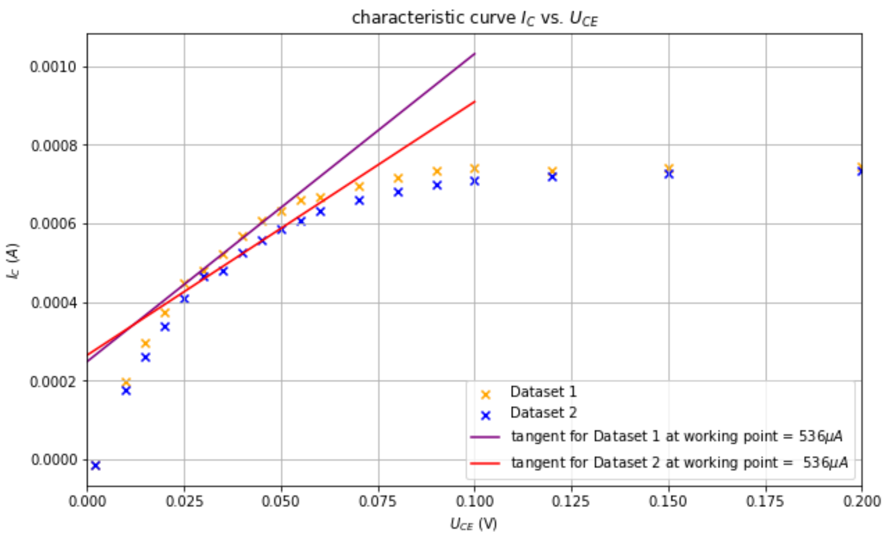
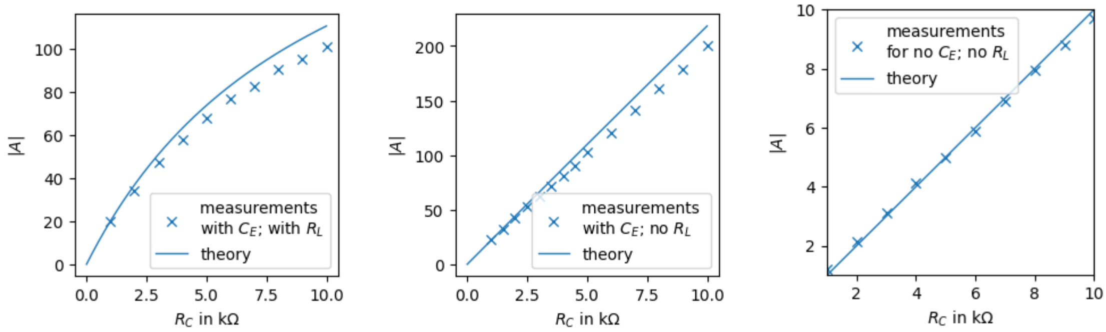


Figure 3.2: Two sersies of measurement for the characteristic output curve plotting I_C against U_{CE}, while U_{BE} is held constantly equal to the working-point-setting at 551 mV. Also, two tangent lines at the working point.

3.3 Results for the amplification of voltage

For the negative-feedback circuit from figure 1.2, the amplification of voltage $|A| = \frac{U_A}{U_E}$ dependent on the collector resistance R_C can be seen in figure 3.3 for three slightly differing configurations:



((**a**)) with C_E; with R_L ((**b**)) with C_E; no R_L ((**c**)) no C_E; no R_L

Figure 3.3: The measurements of the unitless absolute amplification of voltage $|A| = \frac{U_\mathrm{A}}{U_\mathrm{E}}$ against the resistance R_C, as well as the expected theory curves calculated using the differential quantities determined above. In figure 3.3(a), the circuit is the negative-feedback-circuit according to figure 1.2. For the circuit corresponding to figure 3.3(b), the load resistor R_L was removed, and for figure 3.3(*c*), C_E was additionally removed.

In figure 3.3(a), the exact circuit from 1.2 was used. Hence, the first case of equation 1.10 could be used to predict the amplification, which is shown as the theory curve in the plot. The needed values $R_\mathrm{L} = 10\,\mathrm{k\Omega}$ and $R_\mathrm{E} = 1\,\mathrm{k\Omega}$ were given in the instructions[1], and the value of r_CE can be neglected, since it is big as calculated above. For S, the calculated value of $S = 0.022\,\mathrm{AV}^{-1}$ was used.

For figure 3.3(b), R_L was removed, hence now the second case of equation 1.10 applies.

For figure 3.3(c), both R_L and C_E where removed. Thus, now the second case of equation 1.9 has to be used, where only $R_\mathrm{E} = 10\,\mathrm{k\Omega}$ needs to be known to determine $|A(R_\mathrm{C})|$.

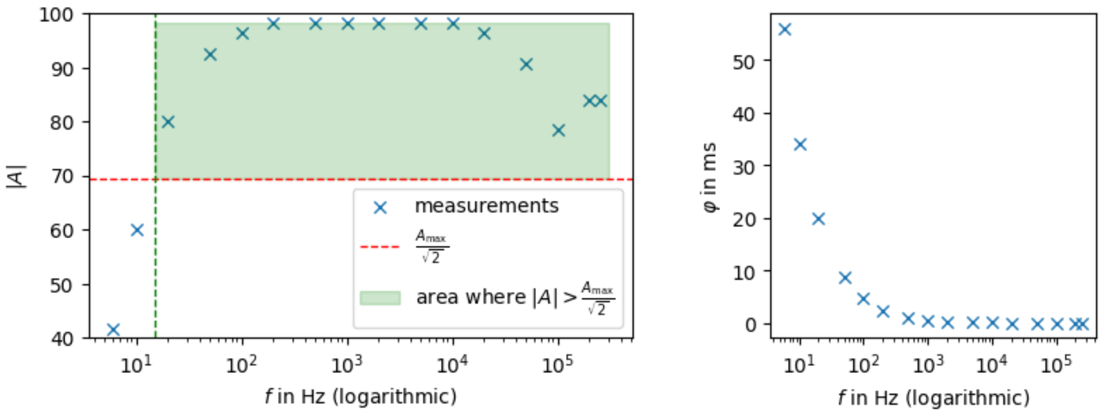
In summary, the (approx.) linear predictions for the amplification of voltages around the working point is predicted well by theory, since the measured data matches the theory curves in all three cases in terms of measurement accuracy.

3.4 Results for the frequency dependence

In figure 3.4, one can see how the amplification of voltage A and the difference in phase φ depends on the frequency f, where a wide range over more than 5 orders of magnitude is investigated.

For A in figure 3.4(a), we can approximately divide the plot into three areas of different behaviour:

In the first area ($\sim 0\,\mathrm{Hz}$ to $500\,\mathrm{Hz}$), we see that the amplification grows at first rapidly with f, but the slope decreasing until A reaches A_max. As shown in the plot, $|A| > \frac{A_\mathrm{max}}{\sqrt{2}}$



((**a**)) dimensionless amplification $|A|$ ((**b**)) phase difference

Figure 3.4: Absolute amplification of currents $|A| = \left|\frac{U_A}{U_E}\right|$ (3.4(a)) and difference of phase between incoming and outgoing current (3.4(b)) for a large range of frequencies, which is in logarithmical scale.

for about $f > 150\,\mathrm{Hz}$ and never falls under that threshhold again. In summary, this area can be described as highpass-behaviour.

In the second area ($\sim 1\,\mathrm{kHz}$ to $10\,\mathrm{kHz}$), we observe the usual behaviour of the transistor: The signal is amplified by a constant $|A| = A_{\mathrm{max}}$, independent of f. This is the area that all other experiments where conducted in.

In the third area ($> 20\,\mathrm{kHz}$), the amplification begins falling again, with increasingly steep slope. The last two measurements break that behaviour, but this irregularity is expected to be an error, possibly a short circuit was created by mistake at this point, but the cause is unsure. Ignoring the last two points, this area can be described as lowpass-behaviour.

In contrary to the amplification, which had to be studies in three different areas, the phase difference constantly falls towards zero with decreasingly steep slope, approximating $\varphi = 0\,\mathrm{ms}$ asymptotically. This behaviour is similar to the phase difference in a highpass.

3.5 Results for theoretical descriptions of high- and lowpass

In this section, a simple two-port circuit consisting of two impedances (see figure 3.5 should be studied to gain the formulars for high- and lowpass. The general two-port description from 1.1 becomes

$$\begin{pmatrix} I_E \\ I_A \end{pmatrix} = \begin{pmatrix} \frac{1}{Z_1} & -\frac{1}{Z_1} \\ -\frac{1}{Z_1} & \frac{1}{Z_1} + \frac{1}{Z_2} \end{pmatrix} \begin{pmatrix} U_E \\ U_A \end{pmatrix} \tag{3.4}$$

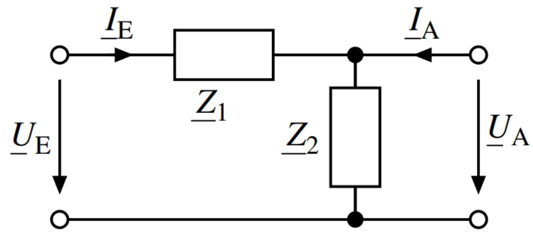


Figure 3.5: The simple two-port network, consisting of two impedances[1].

	highpass	lowpass
Z_1	$\frac{1}{iw\cdot C}$	R
Z_2	R	$\frac{1}{iw\cdot C}$
$H(i\omega)$	$\frac{1}{1+\frac{1}{iw\cdot RC}}$	$\frac{1}{1+i\omega\cdot RC}$
$\lvert H(\omega)\rvert$	$\frac{1}{\sqrt{1+\frac{1}{w^2\cdot R^2C^2}}}$	$\frac{1}{\sqrt{1+w^2\cdot R^2C^2}}$

Figure 3.6: The values of Z_1 and Z_2, as well as the result for the transmission curve $H(i\omega)$ and its absolute value $|H(\omega)|$ for low- and highpass respectively.

and by applying the inverted matrix to both sides one gets

$$\begin{pmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 \end{pmatrix} \begin{pmatrix} I_E \\ I_A \end{pmatrix} = \begin{pmatrix} U_E \\ U_A \end{pmatrix}. \tag{3.5}$$

Assuming $I_A = 0$, multiplying out the matrix and dividing the voltages by each other we find

$$H(Z_1, Z_2) = \frac{U_A}{U_E} = \frac{Z_2}{Z_1 + Z_2}. \tag{3.6}$$

Setting one Z to $Z = R$ (resistance) and the other one to $Z = \frac{1}{iw\cdot C}$ (capacity) one finds the two possibilities in the columns of table 3.6. From the asymptotics of the $|H(\omega)|$-values, one clearly sees which one has to be low- and which one highpass.

Bibliography

[1] TU München Physik Department. “Anfängerpraktikum 2 Anleitung Transformator.” (2024), [Online]. Available: https://academics.nat.tum.de/fileadmin/w00bzl/nat/studium/org/labs/ph-ap/ap2/TRA.pdf (visited on 08/13/2024).