Sept 29th

Probability v. Statistics
↳ know population and want to understand things about the sample
↳ unknown population but have access to sample, from which we characterize population

1. A coin toss $\Omega = \{H, T\}$

$\mathcal{F} = P(\Omega) = \{0; \{H\}, \{T\}, \{H, T\}\}$

$P: \mathcal{F} \rightarrow [0,1]$

$(\Omega, \mathcal{F}, P) \rightarrow$ probability space

where: $P(\Omega) = 1$

$P(A) \geq 0 \;, A \in \mathcal{F}$

$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} Pr(A_i)$ iff $A_i \cap A_j = \emptyset$

$P(\Omega) = P(A \cup A^c)$ $\quad A \cap A^c = \emptyset$

$P(\Omega) = P(A) + P(A^c)$

$1 = P(A) + P(A^c)$

$P(A^c) = 1 - P(A)$

$P(A) \leq 1 \longrightarrow P(A^c) \geq 0$

$P(\emptyset) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ ☆ reframe w/ omega as B now

$P(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)}$ ☆ Bayes rule

Independence $(A \perp\!\!\!\perp B)$
- $P(A|B) = P(A)$
- $P(A \cap B) = P(A) \cdot P(B)$

A random variable $X: \Omega \rightarrow \mathbb{R}$

$$X(w) = \begin{cases} 1 \text{ if } w = \{H\} \\ 0 \text{ if } w = \{T\} \end{cases}$$

$P_X(1) = P(X=1)$

$= P(\{w \mid X(w) = 1\})$

$= P(\{H\})$

Bernoulli trial

$P(\{H\}) = p \qquad P(\{T\}) = 1-p$

$X \sim \text{Bernoulli}(p)$

Geometric distribution

k tosses for first head (assumes iid)

$Pr_X(k) = (1-p)^{k-1} p$

$Y \sim \text{Geometric}(k, p)$

Binomial distribution

k heads in n coin flips (assume iid)

$Pr_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$

$Z \sim \text{Binomial}(n, p)$

Poisson distribution

X heads per hour

$P(k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$ where λ = rate

Exponential distribution

continuous

$P(X < t)$

$P(\text{time b/w consec. Hs}) = e^{-\mu t}$

$f(t) = \mu e^{-\mu t}$

probability density func

$\int f(t)\,dt$

cumulative density func