Differentiation Assignment

November 18, 2023

Questions

1. If $\sin y = x \sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

2. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point (3,22).

3. If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   If $y = (\tan^{-1} x)^2$, then show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2.$

4. Show that of all the rectangles inscribed in a given fixed circle, the square has maximum area.

   Find the intervals in which the function f given by $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

5. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is equal to

   (a) $\frac{x-1}{y-1}$

   (b) $\frac{x-1}{y+1}$

   (c) $\frac{y-1}{x+1}$

   (d) $\frac{y+1}{x-1}$