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Homework

Spring PHS597
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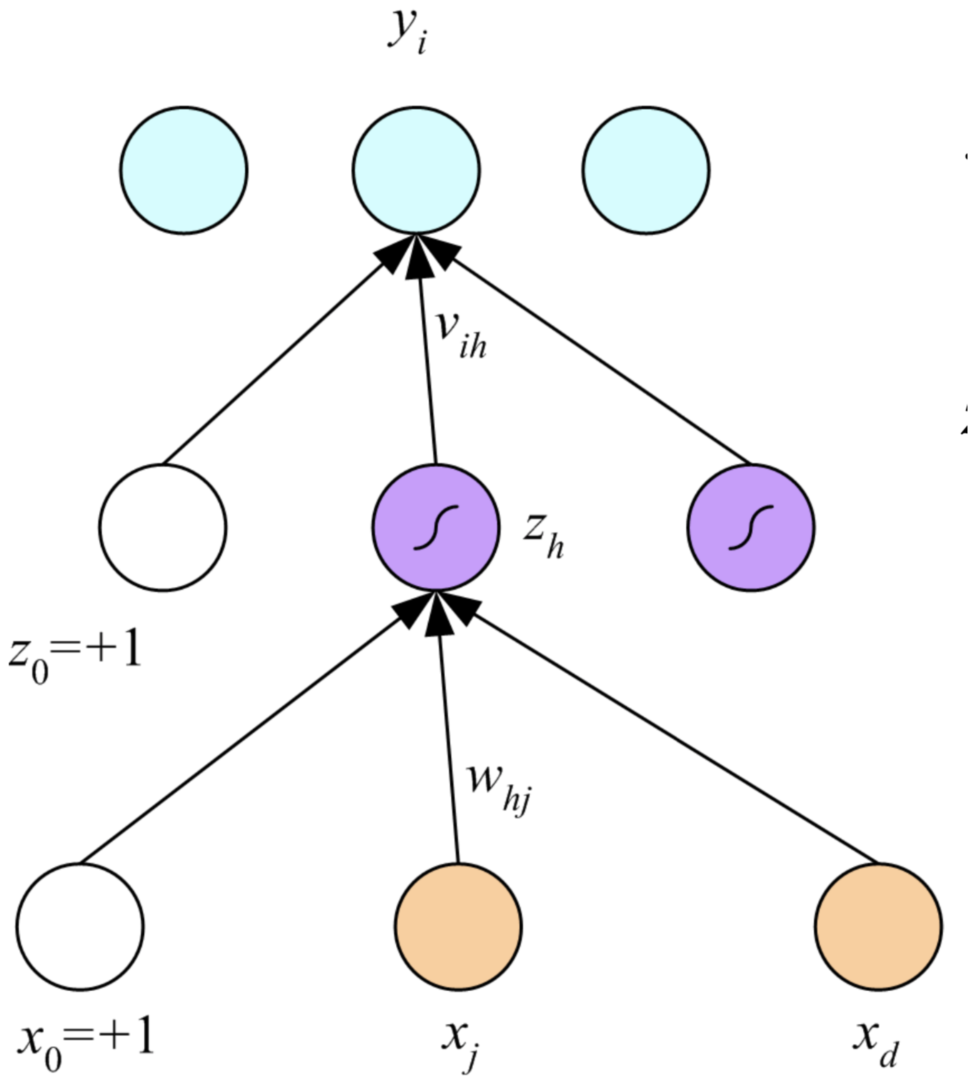
Problem Backpropagation

Derivation of the backpropagation formula for logistic regression:



$$y_i = \nu_i^T Z = \sum_{h=1}^{H} \nu_{ih} Z_h + \nu_{i0}$$
$$z_h = sigmoid(W_h^T X)$$
$$= \frac{1}{1 + exp[-(\sum_{j=1}^{d} \omega_{hj} x_j + \omega_{h0})]}$$
$$E^t(\omega|x^t, r^t) = -r^t logy^t - (1-r^t)log(1-y^t)$$
$$\frac{\partial E}{\partial \omega_{hj}} = \frac{E}{\partial y_i}\frac{\partial y_i}{\partial z_h}\frac{\partial z_h}{\partial \omega_{hj}}$$

Backward

$$E(W, \nu|X) = \frac{1}{2}\sum_t (r^t - y^t)^2$$
$$\Delta\nu_h = \eta \sum_t (r^t - y^t) z_h^t$$
$$\Delta\omega_{hj} = -\eta \frac{\partial E}{\partial \omega_{hj}}$$
$$= -\eta \sum_t \frac{E}{\partial y^t}\frac{\partial y^t}{\partial z_h^t}\frac{\partial z_h^t}{\partial \omega_{hj}}$$
$$= -\eta \sum_t -(r^t - y^t)\nu_h z_h^t(1 - z_h^t)x_j^t$$
$$= \eta \sum_t (r^t - y^t)\nu_h z_h^t(1 - z_h^t)x_j^t$$

In. multiple output setting

$$E(W, \nu|X) = \frac{1}{2}\sum_t \sum_i (r^t - y^t)^2$$
$$\Delta\nu_{ih} = \eta \sum_t (r_i^t - y_i^t) z_h^t$$
$$\Delta\omega_{hj} = -\eta \frac{\partial E}{\partial \omega_{hj}}$$
$$= -\eta \sum_t \sum_i \frac{E}{\partial y_i^t}\frac{\partial y_i^t}{\partial z_h^t}\frac{\partial z_h^t}{\partial \omega_{hj}}$$
$$= -\eta \sum_t \sum_i -(r_i^t - y_i^t)\nu_{ih} z_h^t(1 - z_h^t)x_j^t$$
$$= \eta \sum_t \sum_i (r_i^t - y_i^t)\nu_{ih} z_h^t(1 - z_h^t)x_j^t$$