

Derivation of the scaling factor for ADAM

We have (1)

$$S^{[k+1]} = \rho * S^{[k]} + (1 - \rho)g \quad (2)$$

$$S^{[0]} = 0 \quad (3)$$

By (1) and (2), we then will have:

$$S^{[1]} = (1 - \rho)g$$

$$S^{[2]} = \rho * S^{[1]} + (1 - \rho)g$$

$$= \rho * [(1 - \rho)g] + (1 - \rho)g$$

$$= (1 + \rho)(1 - \rho)g:$$

$$S^{[k+1]} = (1 + \dots + \rho^k)(1 - \rho)g$$

$$\Rightarrow E(S^{[k+1]}) = E((1 - \rho^{k+1})g)$$

$$= (1 - \rho^{k+1})E(g)$$