432 Class 06 Slides

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Today's Agenda

- Predicting a Binary outcome
 - using a linear probability model
 - using logistic regression and glm

Setup

```
library(here); library(magrittr)
library(janitor); library(knitr)
library(patchwork); library(broom)
library(simputation); library(naniar)
library(rsample); library(yardstick)
library(tidyverse)
theme_set(theme_bw())
```

Regression on a Binary Outcome

Linear Probability Model (a linear model)

```
lm(event ~ predictor1 + predictor2 + ..., data = tibblename)
```

• Pr(event) is linear in the predictors

Logistic Regression Model (generalized linear model)

- Logistic Regression forces a prediction in (0, 1)
- log(odds(event)) is linear in the predictors

The logistic regression model

$$logit(event) = log\left(\frac{Pr(event)}{1 - Pr(event)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k$$

$$odds(event) = \frac{Pr(event)}{1 - Pr(event)}$$

$$Pr(event) = \frac{odds(event)}{odds(event) + 1}$$

$$Pr(event) = \frac{exp(logit(event))}{1 + exp(logit(event))}$$

BRFSS and SMART (Creating smart3)

smart3 Variables, by Type

Variable	Туре	Description
landline	Binary (1/0)	survey conducted by landline? (vs. cell)
healthplan	Binary $(1/0)$	subject has health insurance?
age_imp	Quantitative	age (imputed from groups - see Notes)
fruit_day	Quantitative	mean servings of fruit / day
drinks_wk	Quantitative	mean alcoholic drinks / week
bmi	Quantitative	body-mass index (in kg/m^2)
physhealth	Count (0-30)	of last 30 days, # in poor physical health
dm_status	Categorical	diabetes status (4 levels, we'll collapse to 2)
activity	Categorical	physical activity level (4 levels, we'll re-level)
smoker	Categorical	smoking status (4 levels, we'll collapse to 3)
genhealth	Categorical	self-reported overall health (5 levels)

Collapsing Two Factors, Re-leveling another

```
smart3 <- smart3 %>% type.convert() %>%
    mutate(SEQNO = as.character(SEQNO)) %>%
    mutate(dm status =
           fct_collapse(factor(dm_status),
                        Yes = "Diabetes",
                        No = c("No-Diabetes",
                                "Pre-Diabetes",
                                "Pregnancy-Induced"))) %>%
    mutate(smoker =
           fct_collapse(factor(smoker),
                        Current = c("Current not daily",
                                     "Current daily"))) %>%
    mutate(activity =
             fct relevel(factor(activity),
                         "Highly Active", "Active",
                         "Insufficiently Active",
                         "Inactive"))
```

"Simple" Imputation Strategy, with Shadow

```
smart3 sh <- smart3 %>% bind shadow()
set.seed(2021432)
smart3 sh <- smart3 sh %>%
    data.frame() %>%
        impute_rhd(dm_status + smoker ~ 1) %>%
        impute_rhd(healthplan + activity ~ 1) %>%
        impute_rlm(age_imp + fruit_day + drinks_wk + bmi ~
                     mmsa + landline + healthplan) %>%
        impute_knn(physhealth ~ bmi) %>%
        impute cart(genhealth ~ activity + physhealth +
                      mmsa + healthplan) %>%
    tibble()
```

Saving the smart3 and smart3_sh tibbles to .Rds

```
saveRDS(smart3, "data/smart3.Rds")
saveRDS(smart3_sh, "data/smart3_sh.Rds")
```

Create binary outcome variable

bmilt30	n	mean(bmi)	min(bmi)	max(bmi)
0	2343	35.84	30.0	75.52
1	5069	25.26	13.3	29.99

Predicting BMI < 30 using diabetes status (a factor)

```
mod_DM <- smart3_sh %$%
  glm(bmilt30 ~ dm_status,
     family = binomial(link = logit))

tidy(mod_DM) %>% select(term, estimate) %>%
  kable(digits = 3)
```

term	estimate
(Intercept)	0.947
dm_statusYes	-1.053

Equation: logit(BMI < 30) = 0.947 - 1.053 (dm_statusYes)

How can we interpret this result?

Interpreting the mod_DM Equation

- Harry has diabetes.
 - His predicted logit(BMI < 30) is 0.947 1.053(1) = -0.106
- Sally does not have diabetes.
 - Her predicted logit (BMI < 30) is 0.947 1.053(0) = 0.947

Now, logit(BMI < 30) = log(odds(BMI < 30)), so exponentiate to get the odds...

- Harry has predicted odds (BMI < 30) = exp(-0.106) = 0.899
- Sally has predicted odds (BMI < 30) = exp(0.947) = 2.578

Can we convert these odds into something more intuitive?

Converting Odds to Probabilities

- Harry has predicted odds (BMI < 30) = exp(-0.106) = 0.899
- Sally has predicted odds (BMI < 30) = exp(0.947) = 2.578

$$odds(BMI < 30) = \frac{Pr(BMI < 30)}{1 - Pr(BMI < 30)}$$

and

$$Pr(BMI < 30) = \frac{odds(BMI < 30)}{odds(BMI < 30) + 1}$$

- So Harry's predicted Pr(BMI < 30) = 0.899 / 1.899 = 0.47
- Sally's predicted Pr(BMI < 30) = 2.578 / 3.578 = 0.72
- odds range from 0 to ∞ , and log(odds) range from $-\infty$ to ∞ .
- ullet odds > 1 if probability > 0.5. If odds = 1, then probability = 0.5.

What about the odds ratio?

 $logit(BMI < 30) = 0.947 - 1.053 (dm_status = Yes)$

- Harry, with diabetes, has odds(BMI < 30) = 0.899
- Sally, without diabetes, has odds(BMI < 30) = 2.578

Odds Ratio for BMI < 30 associated with having diabetes (vs. not) =

$$\frac{0.899}{2.578} = 0.349$$

• Our model estimates that a subject with diabetes has 34.9% of the odds of a subject without diabetes of having BMI < 30.

Can we calculate the odds ratio from the equation's coefficients?

• Yes, $\exp(-1.053) = 0.349$.

Tidy with exponentiation

term	estimate	conf.low	conf.high
(Intercept)	2.578	2.462	2.700
dm_statusYes	0.349	0.313	0.389

- ullet The odds ratio for BMI < 30 among subjects with diabetes as compared to those without diabetes is 0.349
- The odds of BMI < 30 are 34.9% as large for subjects with diabetes as they are for subjects without diabetes, according to this model.
- A 90% uncertainty interval for the odds ratio estimate includes (0.313, 0.389).

Interpreting these summaries

Connecting the Odds Ratio and Log Odds Ratio to probability statements. . .

- If the probabilities were the same (for diabetes and non-diabetes subjects) of having BMI < 30, then the odds would also be the same, and so the odds ratio would be 1.
- If the probabilities of BMI < 30 were the same and thus the odds were the same, then the log odds ratio would be log(1) = 0.

```
logit(BMI < 30) = 0.947 - 1.053 (dm_status = Yes)
```

- If the log odds of a coefficient (like diabetes = Yes) are negative, then what does that imply?
- What if we flipped the order of the levels for diabetes so our model was about diabetes = No?

Flipping the model changes slope and intercept!

```
mod_DM_no <- smart3_sh %$%
  glm(bmilt30 ~ (dm_status == "No"),
     family = binomial(link = logit))

tidy(mod_DM_no) %>% select(term, estimate) %>%
  kable(digits = 3)
```

term	estimate
(Intercept)	-0.106
dm_status == "No"TRUE	1.053

```
Old: logit(BMI < 30) = 0.947 - 1.053 (dm_status = Yes) New: logit(BMI < 30) = -0.106 + 1.053 (dm_status = No)
```

Predictions from the two models?

```
DMYes: logit(BMI < 30) = 0.947 - 1.053 (dm_status = Yes)
DMNo: logit(BMI < 30) = -0.106 + 1.053 (dm_status = No)</pre>
```

Harry lives with diabetes. Sally does not.

Using the DMYes model:

- logit(Harry's BMI < 30) = 0.947 1.053 = -0.106
- logit(Sally's BMI < 30) = 0.947

Using the DMNo model:

- logit(Harry's BMI < 30) = -0.106
- logit(Sally's BMI < 30) = -0.106 + 1.053 = 0.947

Two Predictor Model

We'll fit the model with and without an interaction term. . .

- How will we interpret the model coefficients?
- ② Can we compare the models based on in-sample performance?
- (later) How can we assess predictive quality (with a holdout sample)?

Coefficients in the Model without Interaction

tidy(mod_FDmain, conf.int = TRUE, conf.level = 0.95) %>%
 select(term, estimate, std.error, conf.low, conf.high) %>%
 kable(dig = 3)

term	estimate	std.error	conf.low	conf.high
(Intercept)	0.762	0.042	0.679	0.845
fruit_day	0.142	0.025	0.094	0.192
dm_statusYes	-1.051	0.067	-1.182	-0.920

ID	fruit	dm	logit(BMI < 30)	odds(BMI < 30)	Pr(BMI < 30)
Art	1	Yes	-0.147	0.863	0.463
Bob	0	Yes	-0.289	0.749	0.428
Cal	1	No	0.904	2.469	0.712
Don	0	No	0.762	2.143	0.682

Exponentiating fruit_day coefficient to get OR

```
tidy(mod_FDmain, exponentiate = TRUE) %>%
select(term, estimate) %>% kable(dig = 2)
```

term	estimate
(Intercept)	2.14
fruit_day	1.15
dm_statusYes	0.35

ID	fruit	dm	logit(BMI < 30)	odds(BMI < 30)	Pr(BMI < 30)
Art	1	Yes	-0.147	0.863	0.463
Bob	0	Yes	-0.289	0.749	0.428

- Art's odds = 0.863. Bob's odds = 0.749
- Art/Bob odds ratio = 0.863/0.749 = 1.15

Exponentiating dm_status coefficient to get OR

```
tidy(mod_FDmain, exponentiate = TRUE) %>%
select(term, estimate) %>% kable(dig = 2)
```

term	estimate
(Intercept)	2.14
fruit_day	1.15
dm_statusYes	0.35

ID	fruit	dm	logit(BMI < 30)	odds(BMI < 30)	Pr(BMI < 30)
Bob	0	Yes	-0.289	0.749	0.428
Don	0	No	0.762	2.143	0.682

- Bob's odds = 0.749, Don's odds (2.143)
- Bob/Don odds ratio = 0.749/2.143 = 0.35

Same result when fruit_day = 1?

```
tidy(mod_FDmain, exponentiate = TRUE) %>%
select(term, estimate) %>% kable(dig = 2)
```

term	estimate
(Intercept)	2.14
fruit_day	1.15
dm_statusYes	0.35

ID	fruit	dm	logit(BMI < 30)	odds(BMI < 30)	Pr(BMI < 30)
Art	1	Yes	-0.147	0.863	0.463
Cal	1	No	0.904	2.469	0.712

- Art's odds = 0.863, Cal's odds (2.469)
- Art/Cal odds ratio = 0.863/2.469 = 0.35 as well.

Coefficients in the Model WITH Interaction

```
tidy(mod_FDint, conf.int = TRUE, conf.level = 0.95) %>%
  select(term, estimate, std.error, conf.low, conf.high) %>%
  kable(dig = 3)
```

term	estimate	std.error	conf.low	conf.high
(Intercept)	0.762	0.045	0.673	0.851
fruit_day	0.142	0.028	0.087	0.198
dm_statusYes	-1.053	0.104	-1.258	-0.850
fruit_day:dm_statusYes	0.001	0.062	-0.119	0.125

- Again, these estimates are on the logit scale
- Does the interaction look like it has a large effect?

Resulting Predictions from mod_FDint

ID	fruit	dm	logit(BMI < 30)	odds(BMI < 30)	Pr(BMI < 30)
Art	1	Yes	-0.148	0.862	0.463
Bob	0	Yes	-0.291	0.748	0.428
Cal	1	No	0.904	2.469	0.712
Don	0	No	0.762	2.143	0.682

Predicted logits (also rounded to 3 decimal places) from mod_FDmain were:

• Art: -0.147, Bob: -0.289, Cal: 0.904, Don: 0.762

That's a small enough difference to leave the (rounded) probability estimates unchanged.

Split the Sample and Compare Models?

Let's split our sample

• What does strata = bmilt30 do?

Impact of using strata = bmilt30 in split

0 702 0.3159316 1 1520 0.6840684

Now, we'll build and compare three models...

```
mod_2 <- glm(bmilt30 ~ fruit_day + dm_status,</pre>
              data = sm3 train, family = binomial)
mod 4 <- glm(bmilt30 ~ fruit_day + dm_status +</pre>
                genhealth + age imp,
              data = sm3 train, family = binomial)
mod_6 <- glm(bmilt30 ~ fruit_day + dm_status +</pre>
                genhealth + age imp + smoker + physhealth,
    data = sm3 train, family = binomial)
```

Tidied Coefficients for Model 2

term	est	se	conf.low	conf.high
(Intercept)	0.732	0.051	0.633	0.832
fruit_day	0.164	0.030	0.105	0.224
dm_statusYes	-1.045	0.080	-1.202	-0.888

Tidied Coefficients for Model 6

term	est	se	conf.low	conf.high
(Intercept)	1.020	0.140	0.747	1.297
fruit_day	0.129	0.030	0.070	0.189
dm_statusYes	-0.903	0.086	-1.072	-0.734
genhealth2_VeryGood	-0.620	0.117	-0.853	-0.394
genhealth3_Good	-1.072	0.116	-1.303	-0.848
genhealth4_Fair	-1.217	0.129	-1.473	-0.967
genhealth5_Poor	-1.338	0.158	-1.650	-1.029
age_imp	0.010	0.002	0.007	0.014

Tidied Coefficients for Model 6

term	est	se	conf.low	conf.high
(Intercept)	1.284	0.153	0.987	1.587
fruit_day	0.138	0.031	0.079	0.200
dm_statusYes	-0.890	0.087	-1.060	-0.720
genhealth2_VeryGood	-0.628	0.117	-0.861	-0.401
genhealth3_Good	-1.077	0.117	-1.310	-0.850
genhealth4_Fair	-1.171	0.138	-1.444	-0.902
genhealth5_Poor	-1.187	0.183	-1.547	-0.829
age_imp	0.013	0.002	0.009	0.016
smokerFormer	-0.547	0.096	-0.737	-0.358
smokerNever	-0.390	0.089	-0.565	-0.217
physhealth	-0.011	0.004	-0.019	-0.003

Summary Statistics for In-Sample Fit

```
bind_rows(glance(mod_2), glance(mod_4), glance(mod_6)) %>%
  mutate(preds = c("2", "4", "6")) %>%
  select(preds, nobs, deviance, df.residual, AIC, BIC) %>%
  kable(digits = 1)
```

preds	nobs	deviance	df.residual	AIC	BIC
2	5190	6276.3	5187	6282.3	6301.9
4	5190	6109.7	5182	6125.7	6178.1
6	5190	6070.3	5179	6092.3	6164.4

Likelihood Ratio Test comparing Models 2 and 4

```
anova(mod 2, mod 4, test = "LRT")
Analysis of Deviance Table
Model 1: bmilt30 ~ fruit_day + dm_status
Model 2: bmilt30 ~ fruit day + dm status + genhealth + age im
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
    5187 6276.3
2 5182 6109.7 5 166.63 < 2.2e-16 ***
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Could also consider
```

- Rao's efficient score test (test = "Rao")
- Pearson's chi-square test (test = "Chisq").

Likelihood Ratio Test comparing Models 4 and 6

```
anova(mod_4, mod_6, test = "LRT")
Analysis of Deviance Table
Model 1: bmilt30 ~ fruit_day + dm_status + genhealth + age_im
Model 2: bmilt30 ~ fruit_day + dm_status + genhealth + age_im
   physhealth
  Resid. Df Resid. Dev Df Deviance Pr(>Chi)
      5182 6109.7
2 5179 6070.3 3 39.321 1.484e-08 ***
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Mallows' Cp statistic?

```
anova(mod_2, mod_4, mod_6, test = "Cp")
Analysis of Deviance Table
Model 1: bmilt30 ~ fruit day + dm status
Model 2: bmilt30 ~ fruit_day + dm_status + genhealth + age_im
Model 3: bmilt30 ~ fruit day + dm status + genhealth + age im
   physhealth
  Resid. Df Resid. Dev Df Deviance
                                     Ср
      5187 6276.3
                                 6282.3
2
      5182 6109.7 5 166.628 6125.7
3
      5179 6070.3 3 39.321 6092.3
```

• Same as what glance provided for AIC in this case.

Build mod_2 Confusion Matrix with 0.5 cutoff

• We'll review the confusion matrix on the next slide...

Confusion Matrix for mod_2 with 0.5 cutoff

confuse2

```
bmilt30_act
bmilt30_pre TRUE FALSE
TRUE 3291 1333
FALSE 258 308
```

- Total # of Predictions = 3291 + 1333 + 258 + 308 = 5190
- 3291 + 308 = 3599 accurate predictions (69.3% accuracy)
- Sensitivity = 3291 / (3291 + 258) = 0.927
 - \bullet if subject actually has BMI $<30~\text{mod}_2$ with this 0.5 cutoff decision rule correctly predicts BMI <30~92.7% of the time.
- Specificity = 308 / (308 + 1333) = 0.188
 - if subject actually doesn't have BMI < 30 mod_2 with this 0.5 cutoff decision rule correctly predicts BMI >= 30 18.8% of the time.

Get confusion matrix more easily?

1 1333 3291

Accuracy and Kappa Results for mod_2

```
metrics(data = mod2_aug, truth = obs, estimate = pred)
```

- Kappa = a correlation statistic from -1 to +1, with complete agreement +1 and complete disagreement -1.
- Kappa measures the inter-rater reliability of our predicted and true classifications.

Confusion Matrix for mod_4 with 0.5 cutoff

Prediction 0 1

0 309 224

1 1332 3325

- 3325 + 309 = 3634 accurate predictions (70.0% accuracy)
- Sensitivity = 3325 / (3325 + 224) = 0.937
- Specificity = 309 / (309 + 1332) = 0.188

Confusion Matrix for mod_6 with 0.5 cutoff

```
Truth
Prediction 0 1
0 312 218
1 1329 3331

• 3331 + 312 = 3643 accurate predictions (70.2% accuracy)
• Sensitivity = 3331 / (3331 + 218) = 0.939
• Specificity = 312 / (312 + 1329) = 0.190
```

metrics for models 2, 4 and 6

```
bind_cols(
metrics(data = mod2_aug, truth = obs, estimate = pred) %>%
  select(.metric, mod2 = .estimate),
metrics(data = mod4 aug, truth = obs, estimate = pred) %>%
  select(mod4 = .estimate).
metrics(data = mod6 aug, truth = obs, estimate = pred) %>%
  select(mod6 = .estimate).
# A tibble: 2 x 4
  .metric mod2 mod4 mod6
  <chr> <dbl> <dbl> <dbl>
1 accuracy 0.693 0.700 0.702
2 kap 0.140 0.153 0.157
```

Holdout Sample?

• do the same thing for models 4 and 6...

metrics for test sample: models 2, 4 and 6

```
bind cols(
metrics(data = mod2_aug_test,
        truth = obs, estimate = pred) %>%
  select(.metric, mod2 = .estimate),
metrics(data = mod4_aug_test,
        truth = obs, estimate = pred) %>%
  select(mod4 = .estimate),
metrics(data = mod6_aug_test,
        truth = obs, estimate = pred) %>%
  select(mod6 = .estimate)
# A tibble: 2 \times 4
  .metric mod2 mod4 mod6
  <chr> <dbl> <dbl> <dbl>
1 accuracy 0.686 0.694 0.694
```

2 kap 0.118 0.136 0.136

What's next?

- ROC curve analysis
- Using 1rm and ols from the rms package to fit and evaluate logistic and linear regressions, respectively