Instructions

- The homework is due on **Friday 3/24 at 5pm ET**.
- No extension will be provided, unless for serious documented reasons.
- Start early!
- Study the material taught in class, and feel free to do so in small groups, but the solutions should be a product of your own work.
- This is not a multiple choice homework; reasoning, and mathematical proofs are required before giving your final answer.

1 $G(n, p)$ [50 points]

1. (5pts) In the limit as n goes to infinity, how does $(1 - \frac{1}{n})^{n \ln n}$ behave?
2. (5pts) How many labeled graphs on n nodes have exactly m edges, where $0 \leq m \leq \binom{n}{2}$
3. (10pts) Consider a graph G sampled from the $G(n, p)$ model. Prove that conditioned on G having m edges, it is equally likely among all graphs that have m edges.
4. (10pts) Suppose that $p = \frac{c}{n}$ where c is a constant. Prove that the number of vertices of degree k is asymptotically equal to $\frac{c^k e^{-c}}{k!} n$ for any fixed positive integer k.
5. (10pts) Consider generating the edges of a random graph by flipping two coins, one with probability p_1 of heads and the other with probability p_2 of heads. For each pair of nodes add an edge between them if either of the coins comes down heads. Show that this is equivalent to generating a graph G from $G(n, p)$ for an appropriate value of p. What is this value p?
6. (10pts) Consider G sampled from $G(n, 0.1)$. How does the Central limit theorem apply to the degree of any node in G? Specifically, within what range will the degree of a node lie with probability at least 99%?

2 Coding [50 points]

Check the Jupyter notebook on our Git repo.