Important Theorems, Definitions, and Concepts

1 Theorems, Definitions, and Important Concepts

Theorem 1.1 (Fundamental Theorem of Calculus)**.** If f is continuous on $[a, b]$, then f has an antiderivative F on $[a, b]$ and

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Definition 1.1. A function $f : \mathbb{R} \to \mathbb{R}$ is called continuous at a point $a \in \mathbb{R}$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|x-a| < \delta$ implies $|f(x)-f(a)| < \epsilon$.

Lemma 1.1. Let f be a differentiable function. If f' is continuous on $[a, b]$, then f is uniformly continuous on $[a, b]$.

Proposition 1.1. If f and g are continuous on $[a, b]$, then the product fg is continuous on $[a, b]$.

Corollary 1.1. If f is continuous on $[a, b]$, then f is bounded on $[a, b]$.

Remark 1.1. Continuity is a local property; a function can be continuous at each point of an interval without being uniformly continuous on the interval.

Example 1.1. Consider the function $f(x) = x^2$. This function is continuous everywhere on $\mathbb{R}$.