

# Inference

February 3, 2024

```
[ ]: import matplotlib.pyplot as plt

# Dati per la tabella
data = [
    ["Distribution", "PDF/PMF", "CDF", "Expectation (Mean)", "Variance",
    ↪ "Correlation to Others"],
    ["Bernoulli", r"$p^k(1-p)^{1-k}$", "N/A", r"$p$", r"$p(1-p)$", "Binomial",
    ↪ "(n=1)"],
    ["Binomial", r"$\binom{n}{k}p^k(1-p)^{n-k}$", "N/A", r"$np$", r"$np(1-p)$",
    ↪ "Sum of Bernoullis"],
    ["Geometric", r"$p(1-p)^{k-1}$", r"$1-(1-p)^k$", r"$\frac{1}{p}$",
    ↪ r"$\frac{1-p}{p^2}$", "Negative Binomial (r=1)"],
    ["Hypergeometric", r"$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$",
    ↪ "Complicated", r"$\frac{nK}{N}$", r"$\frac{nK(N-K)(N-n)}{N^2(N-1)}$",
    ↪ "Differs from Binomial"],
    ["Negative Binomial", r"$\binom{k+r-1}{k}p^r(1-p)^k$", "Complicated",
    ↪ r"$\frac{r}{p}$", r"$\frac{r(1-p)}{p^2}$", "Sum of Geometrics"],
    ["Gamma", r"$\frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\theta^k}$",
    ↪ "Complicated", r"$k\theta$", r"$k\theta^2$", "Generalizes Exponential"],
    ["Poisson", r"$\frac{\lambda^k e^{-\lambda}}{k!}$",
    ↪ r"$\sum_{i=0}^{\infty} \frac{\lambda^i e^{-\lambda}}{i!}$", r"$\lambda$",
    ↪ r"$\lambda$", "Limit of Binomial (n large, p small)"],
    ["Exponential", r"$\lambda e^{-\lambda(x)}$", r"$1-e^{-\lambda(x)}$",
    ↪ r"$\frac{1}{\lambda}$", r"$\frac{1}{\lambda^2}$", "Gamma (k=1)"],
    ["Uniform (Discrete)", "Uniform", "N/A", r"$\frac{a+b}{2}$",
    ↪ r"$\frac{(b-a+1)^2-1}{12}$", "N/A"],
    ["Uniform (Continuous)", r"$\frac{1}{b-a}$", r"$\frac{x-a}{b-a}$",
    ↪ r"$\frac{a+b}{2}$", r"$\frac{(b-a)^2}{12}$", "N/A"],
    ["Normal",
    ↪ r"$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$",
    ↪ "Complicated", r"$\mu$", r"$\sigma^2$", "Limit of many distributions"],
    ["Beta", r"$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$",
    ↪ "Complicated", r"$\frac{\alpha}{\alpha+\beta}$",
    ↪ r"$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$", "N/A"],
    ["Chi-squared", r"$\frac{x^{k/2-1}e^{-x/2}}{2^{k/2}\Gamma(k/2)}$",
    ↪ "Complicated", r"$k$", r"$2k$", "Special case of Gamma"],
```

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    ["T-student", r"$\frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}}\Gamma(\nu/2)\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$", "Complicated", "0",
    (for $ \nu>1$)", r"$\frac{\nu}{\nu-2}$ (for $ \nu>2$)", "N/A"]
]

# Funzione per dividere il testo in più righe se necessario
def wrap_text(text, width=20):
    """Divide il testo in righe più corte."""
    words = text.split()
    wrapped_lines = []
    current_line = []
    current_length = 0
    for word in words:
        if current_length + len(word) > width:
            wrapped_lines.append(' '.join(current_line))
            current_line = [word]
            current_length = len(word)
        else:
            current_line.append(word)
            current_length += len(word) + 1 # +1 per lo spazio
    wrapped_lines.append(' '.join(current_line)) # Aggiungi l'ultima riga
    return '\n'.join(wrapped_lines)

# Applica wrap_text a ciascun elemento dei dati se necessario
for i in range(1, len(data)):
    for j in range(len(data[i])):
        data[i][j] = wrap_text(data[i][j], width=15) # Regola 'width' come
        necessario

headers = data.pop(0)

fig, ax = plt.subplots(figsize=(14, 10))
ax.axis('off')
ax.axis('tight')

table = ax.table(cellText=data, colLabels=headers, cellLoc='center',
    loc='center')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 3.3) # Adjusta scala per adattarsi al testo

plt.show()

```

Distribution	PDF/PMF	CDF	Expectation (Mean)	Variance	Correlation to Others
Bernoulli	$p^k(1-p)^{1-k}$	N/A	$p$	$p(1-p)$	Binomial (n=1)
Binomial	$\binom{n}{k}p^k(1-p)^{n-k}$	N/A	$np$	$np(1-p)$	Sum of Bernoullis
Geometric	$p(1-p)^{k-1}$	$1 - (1-p)^k$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	Negative Binomial (r=1)
Hypergeometric	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$	Complicated	$\frac{nK}{N}$	$\frac{nK(N-K)(N-n)}{N^2(N-1)}$	Differs from Binomial
Negative Binomial	$\binom{k+r-1}{k}p^r(1-p)^k$	Complicated	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	Sum of Geometrics
Gamma	$\frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\theta^k}$	Complicated	$k\theta$	$k\theta^2$	Generalizes Exponential
Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}$	$\sum_{i=0}^k \frac{\lambda^i e^{-\lambda}}{i!}$	$\lambda$	$\lambda$	Limit of Binomial (n large, p small)
Exponential	$(\lambda)e^{-\lambda(x)}$	$1 - e^{-\lambda(x)}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	Gamma (k=1)
Uniform (Discrete)	Uniform	N/A	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	N/A
Uniform (Continuous)	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	N/A
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$	Complicated	$\mu$	$\sigma^2$	Limit of many distributions
Beta	$\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$	Complicated	$\frac{a}{a+b}$	$\frac{a\beta}{(a+\beta)^2(a+\beta+1)}$	N/A
Chi-squared	$\frac{x^{k/2-1}e^{-x/2}}{2^{k/2}\Gamma(k/2)}$	Complicated	$k$	$2k$	Special case of Gamma
T-student	$\frac{\Gamma((v+1)/2)}{\sqrt{v\pi}\Gamma(v/2)}\left(1+\frac{x^2}{v}\right)^{-\frac{v+1}{2}}$	Complicated	0 (for $u > 1$ )	$\frac{v}{v-2}$ (for $v > 2$ )	N/A

```
[ ]: import matplotlib.pyplot as plt

# Preparazione dei dati per la tabella
headers = ["Distribution", "Expectation (Mean)", "Variance of Sample Mean"]
rows = [
    ["Bernoulli", r"$p$", r"$\frac{p(1-p)}{n}$"],
    ["Binomial", r"$np$", r"$p(1-p)$"],
    ["Geometric", r"$\frac{1}{p}$", r"$\frac{1-p}{np^2}$"],
    ["Hypergeometric", r"$\frac{nK}{N}$", r"$\frac{K(N-K)(N-n)}{N^2(N-1)}$"],
    ["Negative Binomial", r"$\frac{r}{p}$", r"$\frac{r(1-p)}{p^2}$"],
    ["Gamma", r"$k\theta$", r"$\frac{k\theta^2}{n}$"],
    ["Poisson", r"$\lambda$", r"$\frac{\lambda}{n}$"],
    ["Exponential", r"$\frac{1}{\lambda}$", r"$\frac{1}{\lambda^2 n}$"],
    ["Uniform (Discrete)", r"$\frac{a+b}{2}$", r"$\frac{(b-a+1)^2-1}{12n}$"],
    ["Uniform (Continuous)", r"$\frac{a+b}{2}$", r"$\frac{(b-a)^2}{12n}$"],
    ["Normal", r"$\mu$", r"$\frac{\sigma^2}{n}$"],
    ["Beta", r"$\frac{\alpha}{\alpha+\beta}$", r"$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)n}$"],
    ["Chi-squared", r"$k$", r"$\frac{2k}{n}$"],
```

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    ["T-student", r"0 (for  $\nu > 1$ )", r" $\frac{\nu}{\nu-2}$  (for large  $\nu$ ,"
    ↪  $\nu > 2$ )"]
]

# Aumenta figsize per più spazio verticale, ad es. (larghezza, altezza)
fig, ax = plt.subplots(figsize=(12, 10)) # Aumento l'altezza qui
ax.axis('off')

# Creazione della tabella
table = ax.table(cellText=rows, colLabels=headers, cellLoc='center',
    ↪ loc='center')

# Adjusta la scala della tabella per più spazio
# Il primo valore è per la larghezza, il secondo per l'altezza delle celle
table.scale(1, 2.5) # Aumenta il secondo valore per più spazio verticale

# Imposta manualmente la dimensione del font se necessario per adattarsi allo
    ↪ spazio extra
table.auto_set_font_size(False)
table.set_fontsize(10)

plt.title('Overview of Sample Distributions', fontsize=16, pad=20)

plt.show()

```

## Overview of Sample Distributions

Distribution	Expectation (Mean)	Variance of Sample Mean
Bernoulli	$p$	$\frac{p(1-p)}{n}$
Binomial	$np$	$p(1-p)$
Geometric	$\frac{1}{p}$	$\frac{1-p}{np^2}$
Hypergeometric	$\frac{nK}{N}$	$\frac{K(N-K)(N-n)}{N^2(N-1)}$
Negative Binomial	$\frac{r}{p}$	$\frac{r(1-p)}{np^2}$
Gamma	$k\theta$	$\frac{k\theta^2}{n}$
Poisson	$\lambda$	$\frac{\lambda}{n}$
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2 n}$
Uniform (Discrete)	$\frac{a+b}{2}$	$\frac{(b-a+1)^2 - 1}{12n}$
Uniform (Continuous)	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12n}$
Normal	$\mu$	$\frac{\sigma^2}{n}$
Beta	$\frac{\alpha}{\alpha + \beta}$	$\frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)n}$
Chi-squared	$k$	$\frac{2k}{n}$
T-student	0 (for $\nu > 1$ )	$\frac{\nu}{\nu-2}$ (for large $n$ , $\nu > 2$ )

```
[ ]: import matplotlib.pyplot as plt

plt.rc('text', usetex=True)
# Ensure matplotlib is configured to use LaTeX for text rendering
plt.rc('text.latex', preamble=r'\usepackage{amsmath}')

# Combined data for various estimation methods, with adjusted Delta Method row
data_combined = [
    ["Method", "Description", "Formula"],
    ["Method of Moments", "Equates population moments to sample moments.", "\u2192 r\"$\mu'_k = \frac{1}{n} \sum_{i=1}^n x_i^k$"],
    ["MLE", "Maximizes the likelihood function.", r"$\hat{\theta} = \arg\max_{\theta} L(\theta | x)$"],
    ["Least Squares", "Minimizes the sum of squared differences between \u2192 observed and estimated values.", r"$\hat{\theta} = \arg\min \sum_{i=1}^n (y_i - f(x_i, \theta))^2$"],
```

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["Delta Method", "Used for finding variance of a function of an estimator.
↪", r"$\text{Var}(g(\hat{\theta})) \approx [g'(\hat{\theta})]^2 \text{Var}(\hat{\theta})$"],
["Jackknife", "Estimates bias and variance by systematically re-sampling.",
↪ "See specific formula based on estimator."],
["Bootstrap", "Estimates the distribution of an estimator by resampling
↪ with replacement.", "See specific formula based on estimator."]
]

fig, ax = plt.subplots(figsize=(16, 10)) # Adjusted for content
ax.axis('off')
table = ax.table(cellText=data_combined[1:], colLabels=data_combined[0],
↪ cellLoc='center', loc='center')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 3) # Adjust scaling for visibility
plt.title("Estimation Methods and Their Formulas", fontsize=16, pad=20)
plt.tight_layout()
plt.show()

```

Estimation Methods and Their Formulas

Method	Description	Formula
Method of Moments	Equates population moments to sample moments.	$\mu'_k = \frac{1}{n} \sum_{i=1}^n x_i^k$
MLE	Maximizes the likelihood function.	$\hat{\theta} = \arg \max L(\theta x)$
Least Squares	Minimizes the sum of squared differences between observed and estimated values.	$\hat{\theta} = \arg \min \sum_{i=1}^n (y_i - f(x_i, \theta))^2$
Delta Method	Used for finding variance of a function of an estimator.	$\text{Var}(g(\hat{\theta})) \approx [g'(\hat{\theta})]^2 \text{Var}(\hat{\theta})$
Jackknife	Estimates bias and variance by systematically re-sampling.	See specific formula based on estimator.
Bootstrap	Estimates the distribution of an estimator by resampling with replacement.	See specific formula based on estimator.

```

[ ]: ["Sample Mean", "Average of sample values", r"$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$"],
["Sample Variance", "Measure of spread in the sample", r"$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$"],

```

```

["Bias", "Difference between the estimator's expected value and the true_
↪parameter value", r"$\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$"],
["Consistency", "An estimator's convergence in probability to the parameter as_
↪sample size increases", r"$\hat{\theta}_n \xrightarrow{p} \theta$ as $n \to_
↪\infty$"],
["Efficiency", "An unbiased estimator with the smallest variance among all_
↪unbiased estimators", r"$\text{Var}(\hat{\theta}_{\text{eff}}) \leq_
↪\text{Var}(\hat{\theta})$ for any unbiased $\hat{\theta}$"],
["Sufficiency", "An estimator that captures all information about the parameter_
↪from the sample", r"A statistic $T(X)$ is sufficient for $\theta$ if the_
↪conditional distribution of $X$ given $T(X)$ does not depend on $\theta$"],
["Point Estimator", "A single value that serves as a best guess or best_
↪estimate of a population parameter", r"A statistic $\hat{\theta}$ used to_
↪estimate $\theta$"],
import matplotlib.pyplot as plt

plt.rc('text', usetex=True)
# Ensure matplotlib is configured to use LaTeX for text rendering
plt.rc('text.latex', preamble=r'\usepackage{amsmath}')

# Combined data for various estimation methods, with adjusted Delta Method row
data_combined = [
    ["Sample Mean", "Average of sample values", r"$\bar{x} =_
↪\frac{1}{n}\sum_{i=1}^n x_i$"],
    ["Sample Variance", "Measure of spread in the sample", r"$s^2 =_
↪\frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})^2$"],
    ["Bias", "Difference between the estimator's expected value and the true_
↪parameter value", r"$\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$"],
    ["Consistency", "An estimator's convergence in probability to the parameter_
↪as sample size increases", r"$\hat{\theta}_n \xrightarrow{p} \theta$ as $n_
↪\to \infty$"],
    ["Efficiency", "An unbiased estimator with the smallest variance among all_
↪unbiased estimators", r"$\text{Var}(\hat{\theta}_{\text{eff}}) \leq_
↪\text{Var}(\hat{\theta})$ for any unbiased $\hat{\theta}$"],
    ["Sufficiency", "An estimator that captures all information about the_
↪parameter from the sample", r"A statistic $T(X)$ is sufficient for $\theta_
↪if the conditional distribution of $X$ given $T(X)$ does not depend on_
↪$\theta$"],
    ["Point Estimator", "A single value that serves as a best guess or best_
↪estimate of a population parameter", r"A statistic $\hat{\theta}$ used to_
↪estimate $\theta$"],
]

fig, ax = plt.subplots(figsize=(21, 10)) # Adjusted for content
ax.axis('off')

```

```

table = ax.table(cellText=data_combined[1:], colLabels=data_combined[0],
    cellLoc='center', loc='center')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 3) # Adjust scaling for visibility
plt.title("Estimation Methods and Their Formulas", fontsize=16, pad=20)
plt.tight_layout()
plt.show()

```

Estimation Methods and Their Formulas

Sample Mean	Average of sample values	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
Sample Variance	Measure of spread in the sample	$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$
Bias	Difference between the estimator's expected value and the true parameter value	$\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$
Consistency	An estimator's convergence in probability to the parameter as sample size increases	$\hat{\theta}_n \xrightarrow{p} \theta \text{ as } n \rightarrow \infty$
Efficiency	An unbiased estimator with the smallest variance among all unbiased estimators	$\text{Var}(\hat{\theta}_{\text{eff}}) \leq \text{Var}(\hat{\theta})$ for any unbiased $\hat{\theta}$
Sufficiency	An estimator that captures all information about the parameter from the sample	A statistic $T(X)$ is sufficient for $\theta$ if the conditional distribution of $X$ given $T(X)$ does not depend on $\theta$
Point Estimator	A single value that serves as a best guess or best estimate of a population parameter	A statistic $\hat{\theta}$ used to estimate $\theta$

```

[ ]: ["Sample Mean", "Average of sample values", r"$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$"],
    ["Sample Variance", "Measure of spread in the sample", r"$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$"],
    ["Bias", "Difference between the estimator's expected value and the true parameter value", r"$\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$"],
    ["Consistency", "An estimator's convergence in probability to the parameter as sample size increases", r"$\hat{\theta}_n \xrightarrow{p} \theta$ as $n \to \infty$"],
    ["Efficiency", "An unbiased estimator with the smallest variance among all unbiased estimators", r"$\text{Var}(\hat{\theta}_{\text{eff}}) \leq \text{Var}(\hat{\theta})$ for any unbiased $\hat{\theta}$"],
    ["Sufficiency", "An estimator that captures all information about the parameter from the sample", r"A statistic $T(X)$ is sufficient for $\theta$ if the conditional distribution of $X$ given $T(X)$ does not depend on $\theta$"],
    ["Point Estimator", "A single value that serves as a best guess or best estimate of a population parameter", r"A statistic $\hat{\theta}$ used to estimate $\theta$"],
import matplotlib.pyplot as plt

```



```

plt.rc('text', usetex=True)
# Ensure matplotlib is configured to use LaTeX for text rendering
plt.rc('text.latex', preamble=r'\usepackage{amsmath}')

# Combined data for various estimation methods, with adjusted Delta Method row
data_combined = [
    ["Confidence Intervals for Population Mean with Known Variance", "Range",
     "likely to contain the population mean", r"$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$"],
    ["Confidence Intervals for Population Mean with Unknown Variance", "Range",
     "likely to contain the population mean", r"$\bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}$"],
    ["Confidence Intervals for Population Proportion", "Range",
     "likely to contain the population proportion", r"$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$"],
    ["Analytical Methods Example", "Using formulas to derive estimators or",
     "properties", r"For example, the sample mean $\bar{x}$ as an estimator for",
     "$\mu$"],
]

fig, ax = plt.subplots(figsize=(16, 10)) # Adjusted for content
ax.axis('off')
table = ax.table(cellText=data_combined[1:], colLabels=data_combined[0],
                 cellLoc='center', loc='center')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 3) # Adjust scaling for visibility
plt.title("Estimation Methods and Their Formulas", fontsize=16, pad=20)
plt.tight_layout()
plt.show()

```

Confidence Intervals for Population Mean with Known Variance	Range likely to contain the population mean	$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$
Confidence Intervals for Population Mean with Unknown Variance	Range likely to contain the population mean	$\bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}$
Confidence Intervals for Population Proportion	Range likely to contain the population proportion	$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Analytical Methods Example	Using formulas to derive estimators or properties	For example, the sample mean $\bar{x}$ as an estimator for $\mu$

```
[ ]: ["Sample Mean", "Average of sample values", r"$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$"],
["Sample Variance", "Measure of spread in the sample", r"$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$"],
["Bias", "Difference between the estimator's expected value and the true parameter value", r"$\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$"],
["Consistency", "An estimator's convergence in probability to the parameter as sample size increases", r"$\hat{\theta}_n \xrightarrow{p} \theta$ as $n \to \infty$"],
["Efficiency", "An unbiased estimator with the smallest variance among all unbiased estimators", r"$\text{Var}(\hat{\theta}_{\text{eff}}) \leq \text{Var}(\hat{\theta})$ for any unbiased $\hat{\theta}$"],
["Sufficiency", "An estimator that captures all information about the parameter from the sample", r"A statistic $T(X)$ is sufficient for $\theta$ if the conditional distribution of $X$ given $T(X)$ does not depend on $\theta$"],
["Point Estimator", "A single value that serves as a best guess or best estimate of a population parameter", r"A statistic $\hat{\theta}$ used to estimate $\theta$"],
import matplotlib.pyplot as plt

plt.rc('text', usetex=True)
# Ensure matplotlib is configured to use LaTeX for text rendering
plt.rc('text.latex', preamble=r'\usepackage{amsmath}')
```

```

# Combined data for various estimation methods, with adjusted Delta Method row
data_combined = [
    ["Regularity: Identifiability", "Different parameter values lead to␣
    ↪different probability distributions", r"If  $\theta_1 \neq \theta_2$ , then␣
    ↪ $f(x|\theta_1) \neq f(x|\theta_2)$  for at least one  $x$ "],
    ["Regularity: Differentiability", "The likelihood function is␣
    ↪differentiable as a function of the parameter", r"The score function␣
    ↪ $U(\theta) = \frac{\partial}{\partial \theta} \log L(\theta)$  exists"],
    ["Regularity: Support Does Not Depend on Parameters", "The set of possible␣
    ↪observations does not depend on the parameter", r"The support of␣
    ↪ $f(x|\theta)$  is the same for all  $\theta$ "],
    ["Regularity: Existence of Moments", "Sufficient moments of the estimator␣
    ↪exist", r" $E|\hat{\theta}^k| < \infty$  for some  $k \geq 1$ "],
    ["Regularity: Independence and Identically Distributed (i.i.d.)", "Samples␣
    ↪are drawn independently from the same distribution", r"Observations  $X_1,$ ␣
    ↪ $X_2, \dots, X_n$  are i.i.d."],
    ["Regularity: Parameter Space is Open", "The parameter space is an open␣
    ↪subset of the Euclidean space", r" $\theta \subseteq \mathbb{R}^k$ "],
]

fig, ax = plt.subplots(figsize=(16, 10)) # Adjusted for content
ax.axis('off')
table = ax.table(cellText=data_combined[1:], colLabels=data_combined[0],␣
    ↪cellLoc='center', loc='center')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 3) # Adjust scaling for visibility
plt.title("Estimation Methods and Their Formulas", fontsize=16, pad=20)
plt.tight_layout()
plt.show()

```

# Estimation Methods and Their Formulas

Regularity: Identifiability	Different parameter values lead to different probability distributions	If $\theta_1 \neq \theta_2$ , then $f(x \theta_1) \neq f(x \theta_2)$ for at least one $x$
Regularity: Differentiability	The likelihood function is differentiable as a function of the parameter	The score function $U(\theta) = \frac{d}{d\theta} \log L(\theta)$ exists
Regularity: Support Does Not Depend on Parameters	The set of possible observations does not depend on the parameter	The support of $f(x \theta)$ is the same for all $\theta$
Regularity: Existence of Moments	Sufficient moments of the estimator exist	$E \hat{\theta} ^k < \infty$ for some $k \geq 1$
Regularity: Independence and Identically Distributed (i.i.d.)	Samples are drawn independently from the same distribution	Observations $X_1, X_2, \dots, X_n$ are i.i.d.
Regularity: Parameter Space is Open	The parameter space is an open subset of the Euclidean space	$\theta \subseteq \mathbb{R}^k$