# Theoretical Mechanics Homework 8

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## 1 MEME

My friends asking me to go touch grass after not going out for two weeks straight:


## 2 LINKS

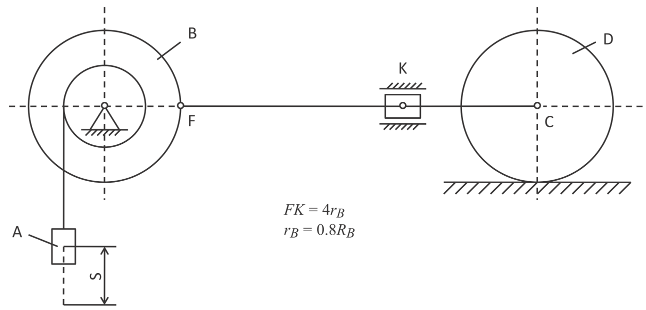
Link back to GitHub
Link to source code

## 3 Task 1

### 3.1 Task Description

Given:

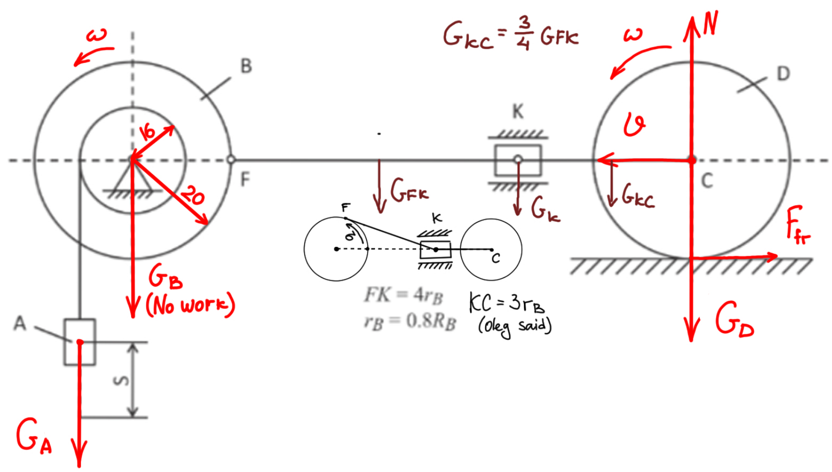
- $m_A = 1kg,\ m_B = 3kg,\ m_D = 20kg$




- $R_B = R_D = 20cm,\ i_B = 18cm$
- $\psi = 0.6(cm)$

Find $v_A(s)$

## 3.2 Solution




Force analysis:
$\vec{G}_A,\ \vec{G}_B,\ \vec{F}_{fr},\ \vec{N},\ \vec{G}_K$(second part), $\vec{G}_{FK}$(second part), $\vec{G}_{KC}$(second part)

Kinematic analysis:

$$\omega_b = \frac{v_a}{r_B},\ v_C = v_a \cdot \frac{R_B}{r_B} \cdot \frac{\sqrt{FK^2 - \sin\left(\frac{s \cdot 180}{\pi \cdot r_B}\right)^2 \cdot R_B^2}}{FK},\ \omega_D = \frac{v_C}{R_D}$$

According to Euler-Lagrange approach:

$$\sum T_k = \sum A$$

Deriving Kinetic energies for all the bodies (and 2nd part bodies too)

$$T_A = \frac{m_A \cdot v_A^2}{2}$$
$$T_B = \frac{m_B \cdot i_B^2 \cdot \omega_B^2}{2}$$
$$T_D = \frac{m_D \cdot v_c^2}{2} + \frac{m_D \cdot R_D^2 \cdot \omega_D^2}{4} = \frac{3m_D \cdot v_C^2}{4}$$
$$T_K = \frac{m_K \cdot v_C^2}{2}$$
$$T_{FK} = \frac{m_{FK} \cdot v_C^2}{2}$$
$$T_{KC} = \frac{m_{FK} \cdot v_C^2}{2}$$

Setting Y-axis in the same direction as N and applying 2nd law of Newton to the body D we get:

$$0 = N - m_D \cdot g;$$
$$N = m_D \cdot g$$

Because $F_{fr} = \psi \cdot N$, we get $F_{fr} = \psi \cdot m_D \cdot g$;
Deriving work done by the system:

$$A_{G_A} = m_A \cdot g \cdot s$$
$$A_{G_B} = 0$$
$$A_{F_{fr}} = m_D \cdot g \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin\left(\frac{s \cdot 180}{\pi \cdot r_B}\right)^2 \cdot R_B^2}}{FK}$$
$$A_{G_N} = 0$$
$$A_{G_K} = 0$$
$$A_{G_{FK}} = m_{FK} \cdot g \cdot \frac{\sin\left(\frac{s \cdot 180}{\pi \cdot r}\right) \cdot R_B}{2}$$
$$A_{G_{KC}} = 0$$

After substituting all of this into the theorem of change of kinetic energy in the system, we can derive $v_a$ (pls spare me, there is a shit ton of everything and I do not want to show all the calculations because it will take 20 or more minutes of just retyping it here):

With neglection of masses of piston and rods:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A)}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + \frac{3}{2} \cdot m_D \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2}}}$$

Without neglection of masses of piston and rods:skull:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A) + m_{FK} \cdot g \cdot \sin(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + (\frac{3}{2} \cdot m_D + m_K + m_{KC}) \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2} + \frac{m_{FK} R_B^2}{r_B^2}}}$$

I will not substitute any number into this formula to preserve at least a little bit of what's left of my sanity.

## 3.3 Answers

With neglection of masses of piston and rods:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A)}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + \frac{3}{2} \cdot m_D \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2}}}$$

Without neglection of masses of piston and rods:skull:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A) + m_{FK} \cdot g \cdot \sin(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + (\frac{3}{2} \cdot m_D + m_K + m_{KC}) \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2} + \frac{m_{FK} R_B^2}{r_B^2}}}$$