Theoretical Mechanics Homework 4

Leonid Novikov, B22-RO-01

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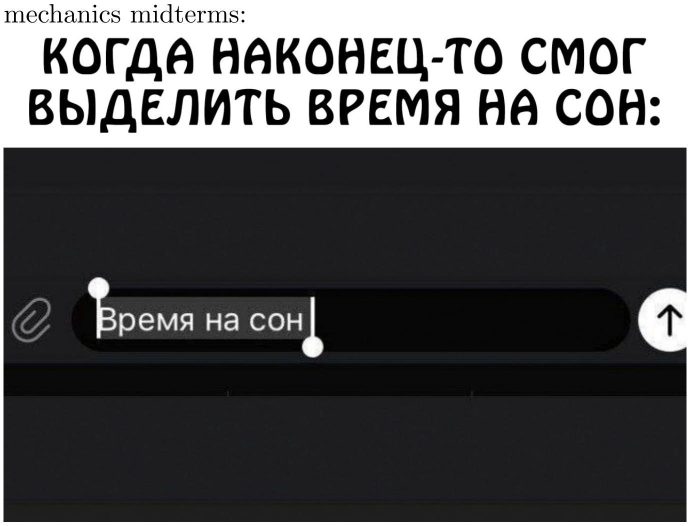
1 MEME

Me, while doing all the homeworks (i'm slow) and preparing for theoretical mechanics midterms:

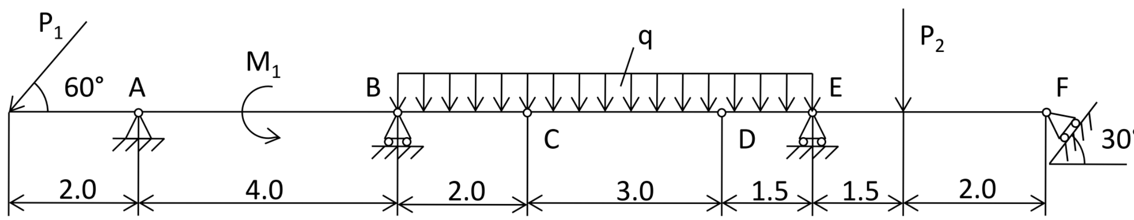


2 LINKS

Link back to GitHub

3 Task 1

3.1 Task description

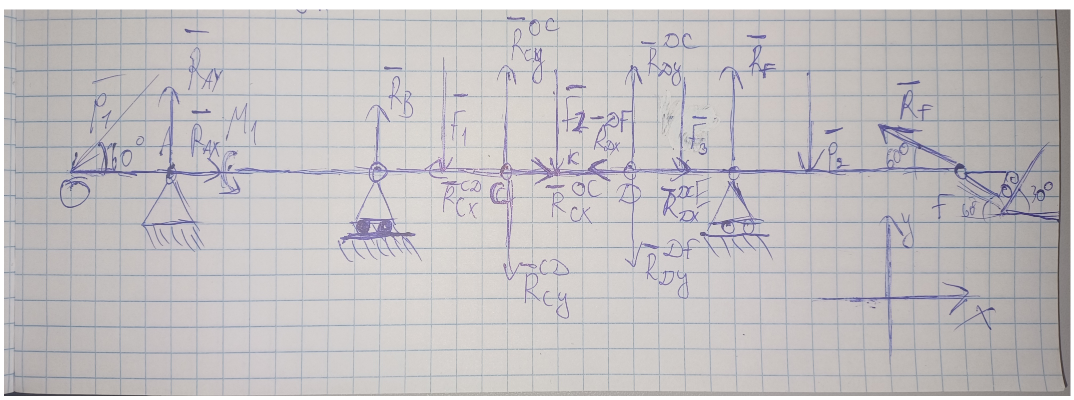


Given:

1. $P_1 = 6, P_2 = 10, M_1 = 30, q = 1.5$

Find all reaction forces on $A,\ B,\ C,\ D,\ E,\ F$

3.2 Solution



Research object: 3 rigid bodies: rods $OC,\ CD,\ DF$
Force analysis:

Points C and D are in equilibrium, it means that reaction forces that are applied on them are compensated, hence, we get:
$\vec{R}^{CD}_{CX} = -\vec{R}^{OC}_{CX}; \vec{R}^{CD}_{CY} = -\vec{R}^{OC}_{CY}$
$\vec{R}^{DC}_{DX} = -\vec{R}^{DF}_{DX}; \vec{R}^{DC}_{DY} = -\vec{R}^{DF}_{DY}$
Let's clarify that:
$R^{OC}_{CX} = R_{CX}$ and $R^{OC}_{CY} = R_{CY}$
$R^{DC}_{DY} = R_{DY}$ and $R^{DC}_{DX} = R_{DX}$

3 equations for rod OC:
$O_x : -P_1 \cdot \cos 90° + R^{OC}_{CX} + R_{AX} = 0$
$O_y : -P_1 \cdot \sin 60° + R^{OC}_{CY} + R_B - F_1 = 0$
$M(a) : 2 \cdot P_1 \sin 60° + M_1 + 4 \cdot R_B - 5 \cdot F_1 + 6 \cdot R^{OC}_{CY} = 0$

3 equations for rod CD:
$O_x : -R_{CX}^{CD} + R_{DX}^{DC} = 0$
$O_y : -R_{CY}^{CD} + R_{DY}^{DC} = 0$
$M(C) : -1.5 \cdot F_2 + 3 \cdot R_{DY}^{DC} = 0$

3 equations for rod DF:
$O_x : -R_{DX}^{DF} - R_F \cdot \cos 60^\circ = 0$
$O_y : -R_{DY}^{DF} - F_3 + R_E - P_2 + R_F \cdot \sin 60^\circ = 0$
$M(D) : -0.75 \cdot F_3 + 1.5 \cdot R_E - 3 \cdot P_2 + 5 \cdot R_F \cdot \sin 60^\circ = 0$

After feeding everything to sympy, it gave me:
$R_{AX} = 4.639$
$R_{AY} = 13.419$
$R_B = -2.973$
$R_{CX} = -1.639$
$R_{CY} = -2.25$
$R_{DX} = -1.639$
$R_{DY} = 2.25$
$R_E = 11.660$
$R_F = 3.278$

3.3 Answer

$R_{AX} = 4.639$
$R_{AY} = 13.419$
$R_B = -2.973$
$R_{CX} = -1.639$
$R_{CY} = -2.25$
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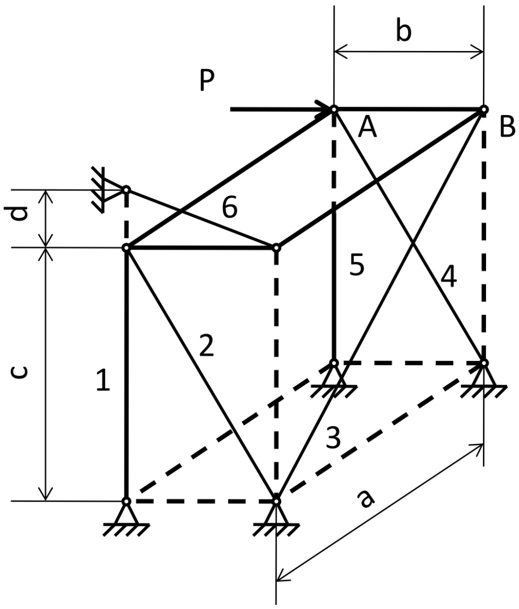
4 Task 2

4.1 Task description



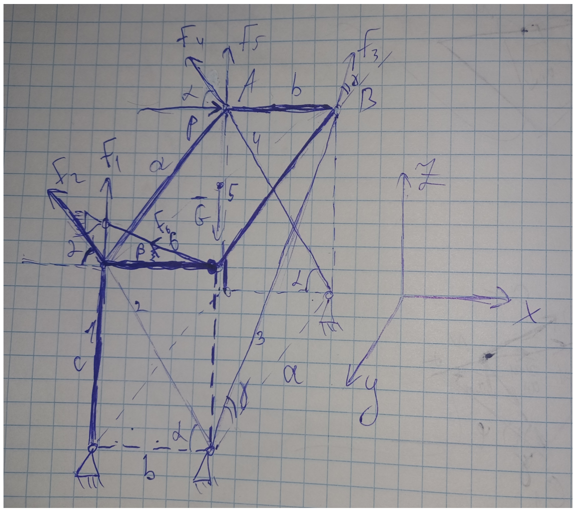
Given:

1. $G = 10,\ P = 20;$

2. $a = 8.5,\ b = 2.5,\ c = 3.5,\ d = 2$

Find reaction forces in rods $1 - 6$

4.2 Solution



Object of study: 1 rigid body.

Force analysis:

Firstly, I need to find angles α, β, γ.

$\alpha = \arctan \frac{c}{b}$;
$\beta = \arctan \frac{d}{c}$;
$\gamma = \arctan \frac{c}{a}$.

Then I can write down projection of forces on O_x, O_y, O_z:

$O_x : -F_2 \cdot \cos\alpha - F_r \cdot \cos\alpha - F_6 \cdot \cos\beta + P = 0$
$O_y : -F_3 \cdot \cos\gamma = 0$
$O_z : F_1 + F_2 \cdot \sin\alpha + F_3 \cdot \sin\gamma + F_4 \cdot \sin\alpha + F_5 + F_6 \sin\cdot\beta - G = 0$

And I can write equations of momentums that rotate the plate around x, y, and z axes in point A:

$M_A^x : -F_2 \cdot a \cdot \sin\alpha - F_1 \cdot a - F_6 \cdot a \cdot \sin\beta + G \cdot \frac{a}{2} = 0$
$M_A^y : b \cdot F_3 \cdot \sin\gamma + b \cdot F_6 \cdot \sin\beta - G\frac{b}{2} = 0$
$M_A^z : b \cdot F_3 \cdot \cos\gamma - a \cdot F_2 \cdot \cos\alpha - a \cdot F_6 \cdot \cos\beta = 0.$

After feeding those 6 equations to sympy, I got this answer:

$F_1 = 8.750$;
$F_2 = -10.752$;
$F_3 = 0.000$;
$F_4 = 34.409$;
$F_5 = -23.000$;
$F_6 = 8.003$

4.3 Answer

$F_1 = 8.750;$
$F_2 = -10.752;$
$F_3 = 0.000;$
$F_4 = 34.409;$
$F_5 = -23.000;$
$F_6 = 8.003$