Theoretical Mechanics Homework 3

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1 MEME

Me on Tuesday be like:

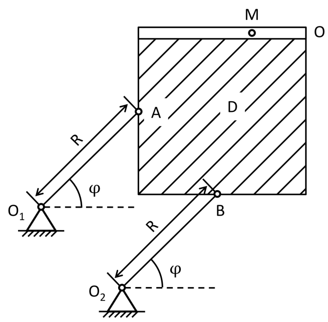
Now to the useful links.

2 LINKS

Link back to GitHub
Link to the Google Colab with the code
Now to the main topic (on next page).

3 Task 1

3.1 Task Description.



Given:

1. $OM = s_r(t) = f_3(t) = 2t^3 + 3t$
2. $\phi(t) = f_2(t) = \frac{1}{24}\pi t^2$
3. $t_1 = 2, R = 15$

Find $a_{cor}, a_{abs}, v_{abs}$ of a point M.

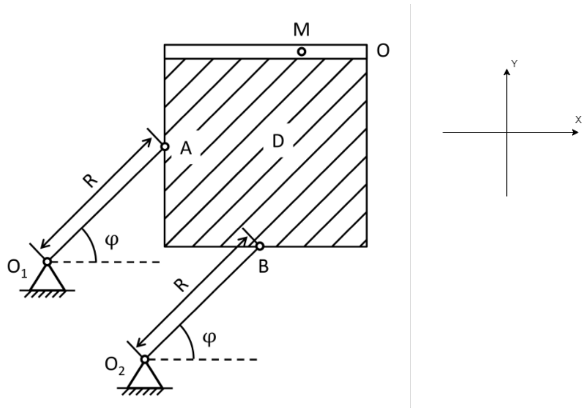
3.2 Solution

3.2.1 Finding necessary variables

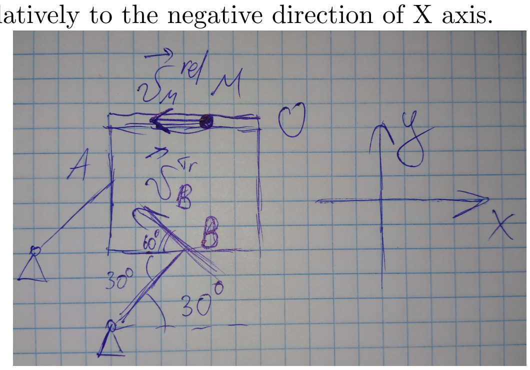
Firstly, I need to find $v_M^{rel}, v_B^{tr}, \omega_B^{tr}$
$v_M^{rel} = \frac{dOM}{dt} = \frac{d2t^3+3t}{dt} = 6t^2 + 3$
$\omega_B^{tr} = \frac{d\phi(t)}{dt} = \frac{d}{dt}\left(\frac{\pi t^2}{24}\right) = \frac{\pi t}{12}, \epsilon = \frac{d\omega_B^{tr}}{dt} = \frac{d}{dt}\left(\frac{\pi t}{12}\right) = \frac{\pi}{12}$
$v_B^{tr} = \omega_B^{tr} R = \frac{\pi t R}{12} = \frac{5\pi}{2}$

3.2.2 Finding absolute velocity of point M

Then I need to denote axes on the picture. Let Axis X be headed towards $\vec{MO}$ and Axis Y be perpendicular to Axis X and headed upwards corresponding to the picture.



After finding that $\phi(t=2) = 30°$, I can tell that v_B^{tr} is inclined by $60°$ relatively to the negative direction of X axis.

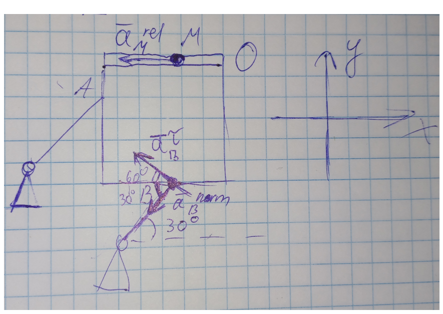


After that, I can project all the velocities, applied to point M and get projections of those velocities on X and Y axes.

$$\begin{cases} Ox : -v_M^{rel} - v_B^{tr} \cos 60° = v_x \\ Oy : v_B^{tr} \sin 60° = v_y \end{cases}$$

Then, we can compute $v_{abs} = \sqrt{v_x^2 + v_y^2} \approx 31.67\ m/sec$

3.2.3 Finding absolute acceleration



Now I need to calculate $\vec{a}_{abs} = \vec{a}_M^{rel} + \vec{a}_b^{tr} + \vec{a}_{cor}$.

As the body does not rotate around its own axis and performs only translatory motion, then $a_{cor} = 0$.

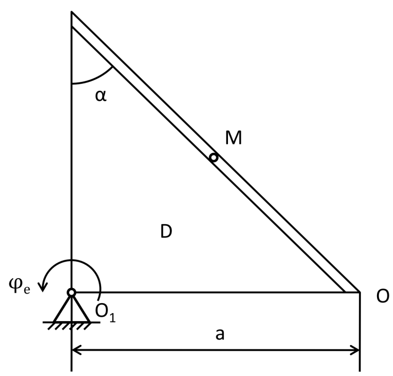
$a_M^{rel} = \dot{v}_M^{rel} = 12t$;
$\vec{a}_B^{tr} = \vec{a}_B^{\tau} + \vec{a}_B^{n}$
$a_B^{\tau} = \epsilon R = \frac{5\pi}{4}$
$a_B^{n} = \omega_B^2 R = \frac{5\pi^2}{12}$
$a_x : a_M^{rel} + a_B^{\tau} \cos 60^\circ + a_B^{n} \cos 30^\circ$
$a_y : a_B^{\tau} \sin 60^\circ - a_B^{n} \sin 30^\circ$
$a_{abs} = \sqrt{a_x^2 + a_y^2} \approx 29.55\ m/sec.$

3.2.4 Answers

$a_{abs} = 29.55\ m/sec, a_{cor} = 0, v_{abs} = 31.67\ m/sec$

4 Task 2.

4.1 Task description.

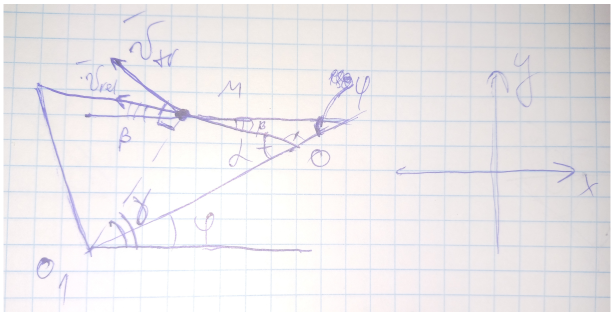


Given:

1. $\phi_e = f_1(t) = 0.2t^3 + t$
2. $OM = s_r = f_2(t) = 5\sqrt{2}(t^2 + t)$
3. $a = 60, \alpha = 45^\circ$

Find $a_{abs}^M, a_{tr}^M, a_{rel}^M, v_{abs}^M, v_{tr}^M, v_{rel}^M, t_{O_2}^M$

4.2 Solution.



4.2.1 Finding time of collision

To find the point of time, when a point M will reach the end of the tube can be found through this equation:
$OO_2 = OM(t)$, where $OO_2 = 60\sqrt{2}$
From this, we get $t^2 + t = 12; => t_{1,2} = 3, -4$

As the time is, apparently, a non-negative value, then I obtain $t^M_{O_2} = 3$

4.2.2 Finding velocities

Firstly, let's find $\vec{v^M_{abs}} = \vec{v}^M_{rel} + \vec{v}^M_{tr}$
$v^M_{rel} = \dot{OM}(t) = 5\sqrt{2}(2t+1)$
$v^M_{tr} = R\omega$, where $R = O_1M, \omega = \dot{\phi_e}(t) = 0.6t^2 + 1$

Then I need to project those velocities on X-axis and Y-axis. For this, I need to find angles β and γ.
$\beta = 180° - (180° - \alpha) - \phi = 45° - \phi$
$\gamma = \angle OO_1M + \phi$

Using sine theorem for triangle $\triangle OO_1M$, we get $\frac{OM}{\sin \angle OO_1M} = \frac{O_1M}{\sin \alpha} =>$
$=> \angle OO_1M = \arcsin\left(\frac{OM}{O_1M} \cdot \sin(\alpha)\right)$

To find O_1M, we can apply cosine theorem:
$O_1M = \sqrt{OM^2 + OO_1^2 - 2 \cdot OM \cdot OO_1 \cdot \cos\alpha} = \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)} =$
$= 5 \cdot \sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)};$
Then, $\angle OO_1M = \arcsin \frac{(t^2+t)}{\sqrt{144+2(t^2+t)^2-24(t^2+t)}}$ and $v^M_{tr} = (3t^2+5)\cdot\sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)}$
From which we obtain $\gamma = \arcsin \frac{(t^2+t)}{\sqrt{144+2(t^2+t)^2-24(t^2+t)}} + \phi$.

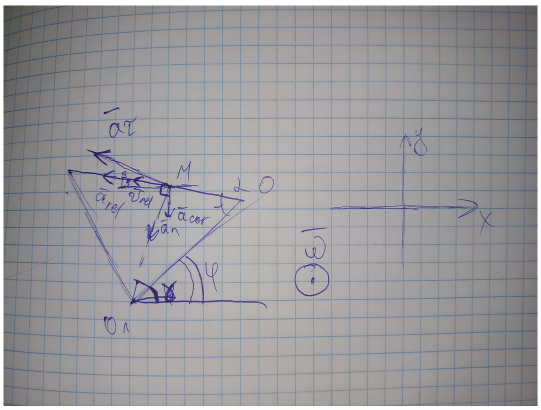
This way, I can find $\widehat{\vec{v}^M_{tr}; O_x} = 90° - \gamma$

Now we are ready to project $\vec{v}^M_{rel}$ and $\vec{v}^M_{tr}$ on O_x and O_y:
$O_x : v_x = -v^M_{tr} \cdot \cos(90° - \gamma) - v^M_{rel} \cdot \cos(\beta)$
$O_y : v_y = v^M_{tr} \cdot \sin(90° - \gamma) + v^M_{rel} \cdot \sin(\beta)$

$$\vec{v}^M_{abs} = \begin{bmatrix} -v^M_{tr} \cdot \cos(90° - \gamma) - v^M_{rel} \cdot \cos(\beta) \\ v^M_{tr} \cdot \sin(90° - \gamma) + v^M_{rel} \cdot \sin(\beta) \end{bmatrix}$$

$v_{abs}^M = \sqrt{v_x^2 + v_y^2}$.
I will not calculate it any further, as the formula with respect to t is obnoxiously long.

4.2.3 Finding accelerations



To find $\vec{a}_{abs}^M$, we need to find $\vec{a}_{cor}^M, \vec{a}_\tau^M, \vec{a}_n^M$ and $\vec{a}_{rel}^M$

$\vec{a}_{cor}^M = 2 \cdot \vec{\omega} \times \vec{v}_{rel}^M$

$||\vec{a}_{cor}^M|| = 2 \cdot \omega \cdot v_{rel}^M = 10\sqrt{2}(0.6t^2+1) \cdot (2t+1)$

$a_\tau^M = \epsilon \cdot R = 6t \cdot \sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)}$

$a_n^M = \frac{(v_{tr}^M)^2}{R} = \frac{0.6t^2+1}{5 \cdot \sqrt{144+2(t^2+t)^2-24(t^2+t)}}$

$a_{tr}^M = \sqrt{(a_\tau^M)^2 + (a_n^M)^2} = \sqrt{36t^2 \cdot (144 + 2(t^2+t)^2 - 24(t^2+t)) + \frac{(0.6t^2+1)^2}{25 \cdot (144+2(t^2+t)^2-24(t^2+t))}}$

$a_{rel}^M = \ddot{OM}(t) = 10\sqrt{2}$

Now I need to find angles between O_x and $\vec{a}_{cor}^M, \vec{a}_\tau^M, \vec{a}_n^M, \vec{a}_{rel}^M$

$\widehat{\vec{a}_{cor}^M; O_x} = 90° - \beta$

$\widehat{\vec{a}_n^M; O_x} = \gamma$

$\widehat{\vec{a}_\tau^M; O_x} = \widehat{\vec{v}_{tr}^M; O_x} = 90° - \gamma$

$\widehat{\vec{a}_{rel}^M; O_x} = \beta$

Now I am ready to project all those accelerations on X and Y axes.

$O_x : a_x = -a_{cor}^M \cdot \cos(90° - \beta) - a_n^M \cdot \cos(\gamma) - a_\tau^M \cdot \cos(90° - \gamma) - a_{rel}^M \cdot \cos(\beta)$

$O_y : a_y = -a_{cor}^M \cdot \sin(90° - \beta) - a_n^M \cdot \sin(\gamma) + a_\tau^M \cdot \sin(90° - \gamma) + a_{rel}^M \cdot \sin(\beta)$

$$\vec{a}_{abs}^M = \begin{bmatrix} -a_{cor}^M \cdot \cos(90° - \beta) - a_n^M \cdot \cos(\gamma) - a_\tau^M \cdot \cos(90° - \gamma) - a_{rel}^M \cdot \cos(\beta) \\ -a_{cor}^M \cdot \sin(90° - \beta) - a_n^M \cdot \sin(\gamma) + a_\tau^M \cdot \sin(90° - \gamma) + a_{rel}^M \cdot \sin(\beta) \end{bmatrix}$$

$a_{abs}^M = \sqrt{a_x^2 + a_y^2}$.
I will not calculate it any further, as the formula with respect to t is obnoxiously long.

4.2.4 Answers

$t^M_{O_2} = 3$

$\vec{a}^M_{abs} = \begin{bmatrix} -a^M_{cor} \cdot \cos(90^\circ - \beta) - a^M_n \cdot \cos(\gamma) - a^M_\tau \cdot \cos(90^\circ - \gamma) - a^M_{rel} \cdot \cos(\beta) \\ -a^M_{cor} \cdot \sin(90^\circ - \beta) - a^M_n \cdot \sin(\gamma) + a^M_\tau \cdot \sin(90^\circ - \gamma) + a^M_{rel} \cdot \sin(\beta) \end{bmatrix}$

$a^M_{abs} = \sqrt{a_x^2 + a_y^2}$

$a^M_{tr} = \sqrt{36t^2 \cdot (144 + 2(t^2+t)^2 - 24(t^2+t)) + \frac{(0.6t^2+1)^2}{25 \cdot (144+2(t^2+t)^2-24(t^2+t))}}$

$a^M_{rel} = 10\sqrt{2}$

$a^M_{cor} = 10\sqrt{2}(0.6t^2+1) \cdot (2t+1)$

$\vec{v}^M_{abs} = \begin{bmatrix} -v^M_{tr} \cdot \cos(90^\circ - \gamma) - v^M_{rel} \cdot \cos(\beta) \\ v^M_{tr} \cdot \sin(90^\circ - \gamma) + v^M_{rel} \cdot \sin(\beta) \end{bmatrix}$

$v^M_{abs} = \sqrt{v_x^2 + v_y^2}$

$v^M_{rel} = 5\sqrt{2}(2t+1)$

$v^M_{tr} = (3t^2+5) \cdot \sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)}$.

Where $\beta = 45^\circ - \phi, \gamma = \arcsin \frac{(t^2+t)}{\sqrt{144+2(t^2+t)^2-24(t^2+t)}} + \phi$