Theoretical Mechanics Homework 7

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1 MEME

Bulichev on his way to hold the Rage: Knights sport while being somewhere out of innopolis:

2 LINKS

Link back to GitHub
Link to Google Colab source code for task 2 of the HW

3 Task 1

3.1 Task description

Given:

1. $AB = BC = 2l$
2. D and E - centres of gravity
3. ρ - radius of gyration
4. h - distance between point B and the floor

Find:

1. v_1, when B hits the floor
2. v_2, when B is halfway to the floor

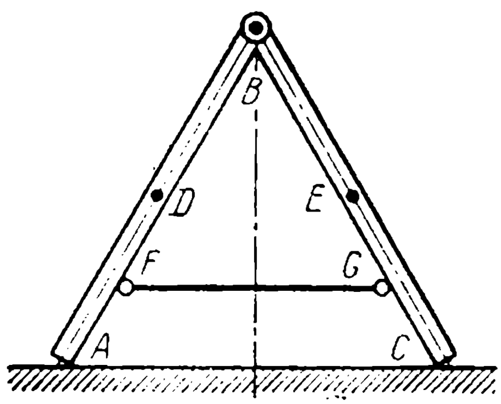
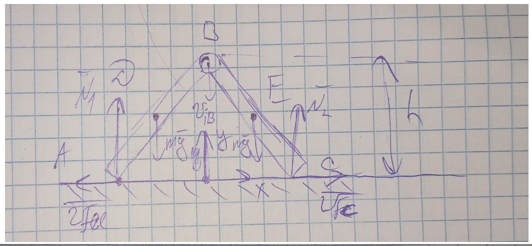
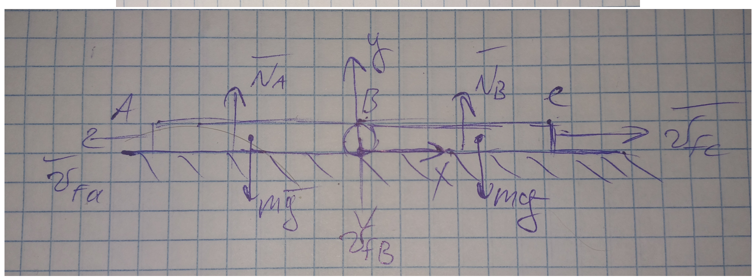


Figure 1: Caption

3.2 Solution

3.2.1 part 1





	$initial$	$final$
x_b	0	0
y_b	h	0
x_a	$-\sqrt{4l^2 - h^2}$	$-2l$
y_a	0	0
x_c	$\sqrt{4l^2 - h^2}$	$2l$
y_c	0	0

Force analysis:

$$m\vec{g}, \vec{N}_A, \vec{N}_B$$

Kinetic energies of rods AB and BC are:

$$T_{AB} = \frac{1}{2} J \omega_{AB}^2$$

$$T_{BC} = \frac{1}{2} J \omega_{BC}^2$$

Using Hyugens-Steiner theorem, I can find the inertia of a rod (And I assume they are the same, as it wasn't mentioned in the task description):

$$J = m_{AB} l^2 + m_{AB} \rho^2$$

At the end of motion, ICoV of AB will be situated at a point A, and ICoV of BC - at point C, hence:

$$\begin{cases} v_B = \omega_{AB} * 2l \\ v_B = \omega_{BC} * 2l \end{cases}$$

$=> \omega_{AB} = \omega_{BC} = \frac{v_B}{2l}$

Work done by applied forces to the system:

$$A = m_{AB} g \frac{h}{2} + m_{AB} g \frac{h}{2} \tag{1}$$

From this, we can apply theorem of change of kinetic energy in the system

$$T_{AB} + T_{BC} = A$$

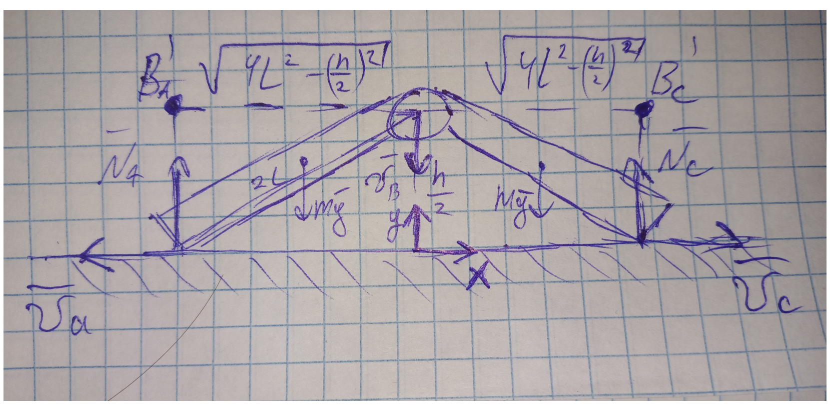
$$\frac{J}{2} \cdot (\frac{v_B}{2l})^2 + \frac{J}{2} \cdot (\frac{v_B}{2l})^2 = m_{AB} g h$$

$$v_B^2 = 4l^2 \cdot \frac{m_{AB} g h}{J} = 4l^2 \cdot \frac{m_{AB} g h}{m_{AB} l^2 + m_{AB} \rho^2}$$

$$v_B = 2l \cdot \sqrt{\frac{gh}{l^2 + \rho^2}}$$

3.2.2 part 2

	$initial$	$final$
x_b	0	0
y_b	h	$\frac{h}{2}$
x_a	$-\sqrt{4l^2 - h^2}$	$-\sqrt{4l^2 - (\frac{h}{2})^2}$
y_a	0	0
x_c	$\sqrt{4l^2 - h^2}$	$\sqrt{4l^2 - (\frac{h}{2})^2}$
y_c	0	0



ICoV's of AB and BC rods are denoted as B'_A and B'_C correspondingly. We can calculate v_B:

$$\begin{cases} v_B = \omega_{AB} \cdot \sqrt{4l^2 - (\frac{h}{2})^2} \\ v_B = \omega_{BC} \cdot \sqrt{4l^2 - (\frac{h}{2})^2} \end{cases}$$

From which, we get $\omega_{AB} = \omega_{BC} = \frac{v_B}{\sqrt{4l^2 - (\frac{h}{2})^2}}$

Amount of work done also changes:

$$A = mg\frac{h}{4} + mg\frac{h}{4} = mg\frac{h}{2}$$

Substituting this in theorem of change of kinetic energy in the system, we acquire:

$$T_{AB} + T_{AC} = A$$
$$mg\frac{h}{2} = \frac{J}{2} \cdot (\frac{v_B}{\sqrt{4l^2 - (\frac{h}{2})^2}})^2 + \frac{J}{2} \cdot (\frac{v_B}{\sqrt{4l^2 - (\frac{h}{2})^2}})^2$$
$$v_B = \frac{1}{2} \cdot \sqrt{\frac{gh(16l^2 - h^2)}{2(l^2 + \rho^2)}}$$

3.3 Answers:

3.3.1 part 1

$v_B = 2l \cdot \sqrt{\frac{gh}{l^2+\rho^2}}$

3.3.2 part 2

$v_b = \frac{1}{2} \cdot \sqrt{\frac{gh(16l^2-h^2)}{2(l^2+\rho^2)}}$

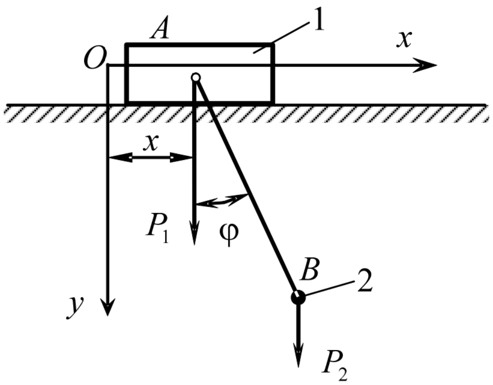
4 Task 2

4.1 Task description

Initial conditions:

1. $x = 0,\ \phi = 10°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\ kg,\ m_2 = 1\ kg,\ l = 1\ m.$

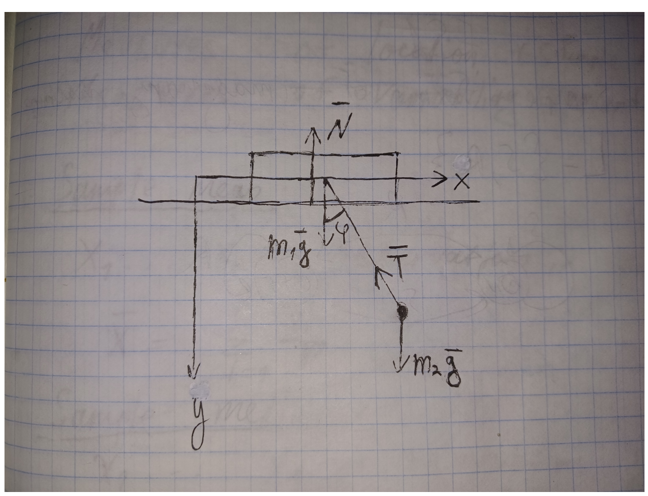


The task is to derive equations, characterizing the motion of the system

4.2 solution

	initial	final	initial	final	initial	final
x	0	?	0.5	?	0.5	?
ϕ	$10°$	?	$45°$	?	$-135°$	?
$\dot{x}$	0	?	0	?	0	?
$\dot{\phi}$	0	?	0	?	0	?
t	0	?	0	?	0	?

- R.O - system of 2 bodies
- body A - translatory motion
- body B - curvilinear motion



Force analysis:

$m_1\vec{g},\ m_2\vec{g},\ \vec{N},\ \vec{T}$

Using Newton-Euler approach to solve the task, I divide the system into two bodies.

First body (cart) equations:

$$\begin{cases} O_x : m_1\ddot{x} = T \cdot \sin(\phi) \\ O_y : 0 = m_1 g - N + t\cos(\phi) \end{cases}$$

Second body (pendulum) equations:

$$\begin{cases} O_x : m_2(-l\sin(\phi)\dot{\phi}^2 + l\cos(\phi)\ddot{\phi} + \ddot{x}) = -T\sin(\phi) \\ O_y : m_2(-l\sin(\phi)\ddot{\phi} - l\cos(\phi)\dot{\phi}^2) = -T\cos(\phi) + m_2 g \end{cases}$$

After sacrificing this eldritch horror of a system to some python algorithm, that was written by some madlad, that can actually solve this thing, I've got:

$$\ddot{\phi} = -\frac{g \cdot m_1 \sin(\phi(t))}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2} - \frac{g \cdot m_2 \sin(\phi(t))}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2} - \frac{l \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t)) \cdot (\dot{\phi})^2}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2}$$

$$\ddot{x} = \frac{g \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t))}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2} + \frac{l \cdot m_2 \sin(\phi(t))^3 \cdot (\dot{\phi})^2}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2} + \frac{l \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t))^2 \cdot (\dot{\phi})^2}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2}$$

As for the plot and everything, just look into the github, you can find it in the "links" section.

4.3 Answer

$$\ddot{\phi} = -\frac{g \cdot m_1 \sin(\phi(t))}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2} - \frac{g \cdot m_2 \sin(\phi(t))}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2} - \frac{l \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t)) \cdot (\dot{\phi})^2}{l \cdot m_1 \sin(\phi(t))^2 + l \cdot m_1 \cos(\phi(t))^2 + l \cdot m_2 \sin(\phi(t))^2}$$

$$\ddot{x} = \frac{g \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t))}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2} + \frac{l \cdot m_2 \sin(\phi(t))^3 \cdot (\dot{\phi})^2}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2} + \frac{l \cdot m_2 \sin(\phi(t)) \cdot \cos(\phi(t))^2 \cdot (\dot{\phi})^2}{m_1 \sin(\phi(t))^2 + m_1 \cos(\phi(t))^2 + m_2 \sin(\phi(t))^2}$$