

Theoretical Mechanics Homework 3

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1 MEME

Me on Tuesday be like:

если экзамены не сдастся, то



Now to the useful links.

2 LINKS

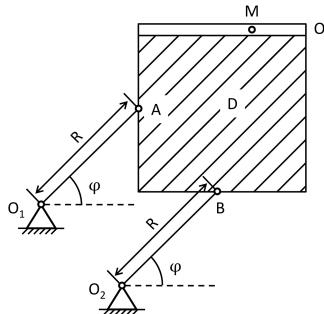
Link back to GitHub

Link to the Google Colab with the code

Now to the main topic (on next page).

3 Task 1

3.1 Task Description.



Given:

- $OM = s_r(t) = f_3(t) = 2t^3 + 3t$
 - $\phi(t) = f_2(t) = \frac{1}{24}\pi t^2$
 - $t_1 = 2, R = 15$

Find $a_{cor}, a_{abs}, v_{abs}$ of a point M .

3.2 Solution

3.2.1 Finding necessary variables

Firstly, I need to find $v_M^{rel}, v_B^{tr}, \omega_B^{tr}$

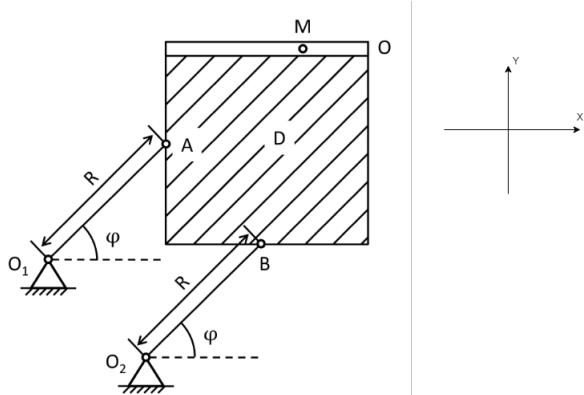
$$v_M^{rel} = \frac{dOM}{dt} = \frac{d2t^3 + 3t}{dt} = 6t^2 + 3$$

$$\omega_B^{tr} = \frac{d\phi(t)}{dt} = \frac{d}{dt}\left(\frac{\pi t^2}{24}\right) = \frac{\pi t}{12}, \epsilon = \frac{d\omega_B^{tr}}{dt} = \frac{d}{dt}\left(\frac{\pi t}{12}\right) = \frac{\pi}{12}$$

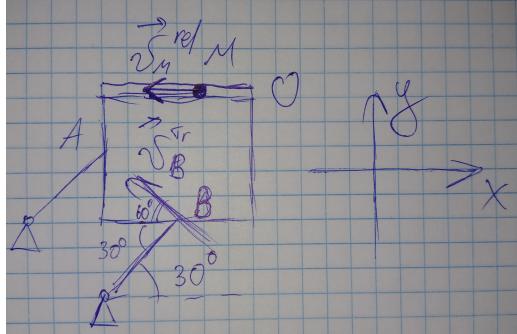
$$v_B^{tr} = \omega_B^{tr} R = \frac{\pi t R}{12} = \frac{5\pi}{2}$$

3.2.2 Finding absolute velocity of point M

Then I need to denote axes on the picture. Let Axis X be headed towards \vec{MO} and Axis Y be perpendicular to Axis X and headed upwards corresponding to the picture.



After finding that $\phi(t = 2) = 30^\circ$, I can tell that v_B^{tr} is inclined by 60° relatively to the negative direction of X axis.

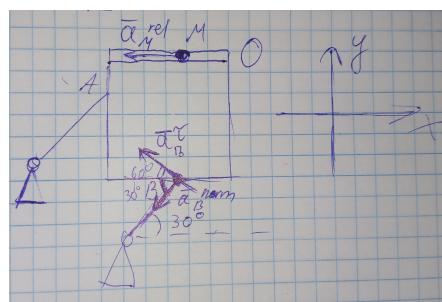


After that, I can project all the velocities, applied to point M and get projections of those velocities on X and Y axes.

$$\begin{cases} Ox : -v_M^{rel} - v_B^{tr} \cos 60^\circ = v_x \\ Oy : v_B^{tr} \sin 60^\circ = v_y \end{cases}$$

Then, we can compute $v_{abs} = \sqrt{v_x^2 + v_y^2} \approx 31.67 \text{ m/sec}$

3.2.3 Finding absolute acceleration



Now I need to calculate $\vec{a}_{abs} = \vec{a}_M^{rel} + \vec{a}_B^{tr} + \vec{a}_{cor}$.

As the body does not rotate around its own axis and performs only translatory motion, then $a_{cor} = 0$.

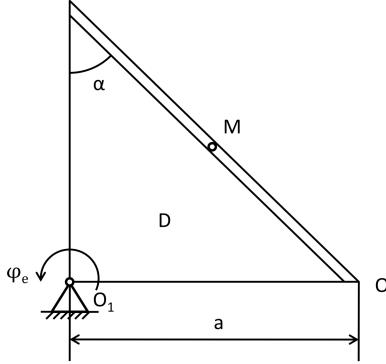
$$\begin{aligned} a_M^{rel} &= \dot{v}_M^{rel} = 12t; \\ \vec{a}_B^{tr} &= \vec{a}_B^\tau + \vec{a}_B^n \\ a_B^\tau &= \epsilon R = \frac{5\pi}{4} \\ a_B^n &= \omega_B^2 R = \frac{5\pi^2}{12} \\ a_x : a_M^{rel} + a_B^\tau \cos 60^\circ + a_B^n \cos 30^\circ \\ a_y : a_B^\tau \sin 60^\circ - a_B^n \sin 30^\circ \\ a_{abs} &= \sqrt{a_x^2 + a_y^2} \approx 29.55 \text{ m/sec.} \end{aligned}$$

3.2.4 Answers

$$a_{abs} = 29.55 \text{ m/sec}, a_{cor} = 0, v_{abs} = 31.67 \text{ m/sec}$$

4 Task 2.

4.1 Task description.

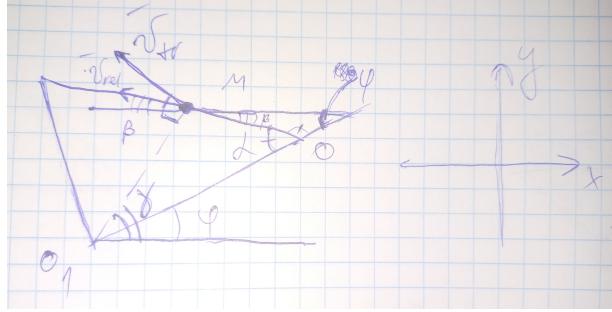


Given:

1. $\phi_e = f_1(t) = 0.2t^3 + t$
2. $OM = s_r = f_2(t) = 5\sqrt{2}(t^2 + t)$
3. $a = 60, \alpha = 45^\circ$

Find $a_{abs}^M, a_{tr}^M, a_{rel}^M, v_{abs}^M, v_{tr}^M, v_{rel}^M$

4.2 Solution.



4.2.1 Finding velocities

Firstly, let's find $\vec{v}_{abs}^M = \vec{v}_{rel}^M + \vec{v}_{tr}^M$

$$v_{rel}^M = \dot{OM}(t) = 5\sqrt{2}(2t+1)$$

$$v_{tr}^M = R\omega, \text{ where } R = O_1M, \omega = \dot{\phi}_e(t) = 0.6t^2 + 1$$

Then I need to project those velocities on X-axis and Y-axis. For this, I need to find angles β and γ .

$$\beta = 180^\circ - (180^\circ - \alpha) - \phi = 45^\circ - \phi$$

$$\gamma = \angle O_1OM + \phi$$

Using sine theorem for triangle $\triangle O_1OM$, we get $\frac{OM}{\sin \angle O_1OM} = \frac{O_1M}{\sin \alpha} \Rightarrow \angle O_1OM = \arcsin \left(\frac{OM}{O_1M} \cdot \sin(\alpha) \right)$

To find O_1M , we can apply cosine theorem:

$$O_1M = \sqrt{OM^2 + O_1O^2 - 2 \cdot OM \cdot O_1O \cdot \cos \alpha} = \sqrt{3600 + 50(t^2+t)^2 - 600(t^2+t)} = \\ = 5 \cdot \sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)}$$

$$\text{Then, } \angle O_1OM = \arcsin \frac{(t^2+t)}{\sqrt{144+2(t^2+t)^2-24(t^2+t)}} \text{ and } v_{tr}^M = (3t^2+5) \cdot \sqrt{144 + 2(t^2+t)^2 - 24(t^2+t)}$$

$$\text{From which we obtain } \gamma = \arcsin \frac{(t^2+t)}{\sqrt{144+2(t^2+t)^2-24(t^2+t)}} + \phi.$$

This way, I can find $\widehat{\vec{v}_{tr}^M; O_x} = 90^\circ - \gamma$

Now we are ready to project \vec{v}_{rel}^M and \vec{v}_{tr}^M on O_x and O_y :

$$O_x : v_x = -v_{tr}^M \cdot \cos(90^\circ - \gamma) - v_{rel}^M \cdot \cos(\beta)$$

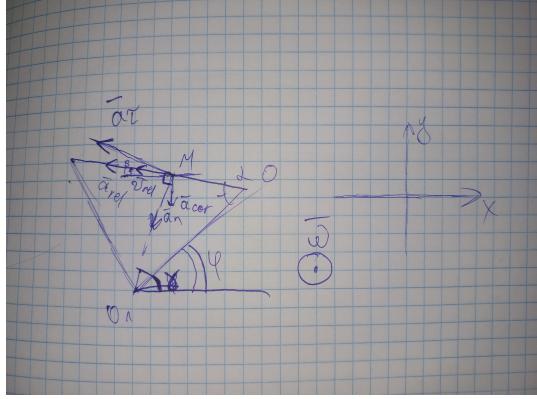
$$O_y : v_y = v_{tr}^M \cdot \sin(90^\circ - \gamma) + v_{rel}^M \cdot \sin(\beta)$$

$$\vec{v}_{abs}^M = \begin{bmatrix} -v_{tr}^M \cdot \cos(90^\circ - \gamma) - v_{rel}^M \cdot \cos(\beta) \\ v_{tr}^M \cdot \sin(90^\circ - \gamma) + v_{rel}^M \cdot \sin(\beta) \end{bmatrix}$$

$$v_{abs}^M = \sqrt{v_x^2 + v_y^2}.$$

I will not calculate it any further, as the formula with respect to t is obnoxiously long.

4.2.2 Finding accelerations



To find \vec{a}_{abs}^M , we need to find \vec{a}_{cor}^M , \vec{a}_τ^M , \vec{a}_n^M and \vec{a}_{rel}^M

$$\vec{a}_{cor}^M = 2 \cdot \vec{\omega} \times \vec{v}_{rel}^M$$

$$\|\vec{a}_{cor}^M\| = 2 \cdot \omega \cdot v_{rel}^M = 10\sqrt{2}(0.6t^2 + 1) \cdot (2t + 1)$$

$$a_\tau^M = \epsilon \cdot R = 6t \cdot \sqrt{144 + 2(t^2 + t)^2 - 24(t^2 + t)}$$

$$a_n^M = \frac{(v_{tr}^M)^2}{R} = \frac{0.6t^2 + 1}{5 \cdot \sqrt{144 + 2(t^2 + t)^2 - 24(t^2 + t)}}$$

$$a_{tr}^M = \sqrt{(a_\tau^M)^2 + (a_n^M)^2} = \sqrt{36t^2 \cdot (144 + 2(t^2 + t)^2 - 24(t^2 + t)) + \frac{(0.6t^2 + 1)^2}{25 \cdot (144 + 2(t^2 + t)^2 - 24(t^2 + t))}}$$

$$a_{rel}^M = \ddot{OM}(t) = 10\sqrt{2}$$

Now I need to find angles between O_x and \vec{a}_{cor}^M , \vec{a}_τ^M , \vec{a}_n^M , \vec{a}_{rel}^M

$$\widehat{\vec{a}_{cor}^M; O_x} = 90^\circ - \beta$$

$$\widehat{\vec{a}_n^M; O_x} = \gamma$$

$$\widehat{\vec{a}_\tau^M; O_x} = \widehat{\vec{v}_{tr}^M; O_x} = 90^\circ - \gamma$$

$$\widehat{\vec{a}_{rel}^M; O_x} = \beta$$

Now I am ready to project all those accelerations on X and Y axes.

$$O_x : a_x = -a_{cor}^M \cdot \cos(90^\circ - \beta) - a_n^M \cdot \cos(\gamma) - a_\tau^M \cdot \cos(90^\circ - \gamma) - a_{rel}^M \cdot \cos(\beta)$$

$$O_y : a_y = -a_{cor}^M \cdot \sin(90^\circ - \beta) - a_n^M \cdot \sin(\gamma) + a_\tau^M \cdot \sin(90^\circ - \gamma) + a_{rel}^M \cdot \sin(\beta)$$

$$\vec{a}_{abs}^M = \begin{bmatrix} -a_{cor}^M \cdot \cos(90^\circ - \beta) - a_n^M \cdot \cos(\gamma) - a_\tau^M \cdot \cos(90^\circ - \gamma) - a_{rel}^M \cdot \cos(\beta) \\ -a_{cor}^M \cdot \sin(90^\circ - \beta) - a_n^M \cdot \sin(\gamma) + a_\tau^M \cdot \sin(90^\circ - \gamma) + a_{rel}^M \cdot \sin(\beta) \end{bmatrix}$$

$$a_{abs}^M = \sqrt{a_x^2 + a_y^2}.$$

I will not calculate it any further, as the formula with respect to t is obnoxiously long.

4.2.3 Answers

$$\vec{a}_{abs}^M = \begin{bmatrix} -a_{cor}^M \cdot \cos(90^\circ - \beta) - a_n^M \cdot \cos(\gamma) - a_\tau^M \cdot \cos(90^\circ - \gamma) - a_{rel}^M \cdot \cos(\beta) \\ -a_{cor}^M \cdot \sin(90^\circ - \beta) - a_n^M \cdot \sin(\gamma) + a_\tau^M \cdot \sin(90^\circ - \gamma) + a_{rel}^M \cdot \sin(\beta) \end{bmatrix}$$

$$a_{abs}^M = \sqrt{a_x^2 + a_y^2}$$

$$a_{tr}^M = \sqrt{36t^2 \cdot (144 + 2(t^2 + t)^2 - 24(t^2 + t)) + \frac{(0.6t^2 + 1)^2}{25 \cdot (144 + 2(t^2 + t)^2 - 24(t^2 + t))}}$$

$$\begin{aligned}
a_{rel}^M &= 10\sqrt{2} \\
a_{cor}^M &= 10\sqrt{2}(0.6t^2 + 1) \cdot (2t + 1) \\
\vec{v}_{abs}^M &= \begin{bmatrix} -v_{tr}^M \cdot \cos(90^\circ - \gamma) - v_{rel}^M \cdot \cos(\beta) \\ v_{tr}^M \cdot \sin(90^\circ - \gamma) + v_{rel}^M \cdot \sin(\beta) \end{bmatrix} \\
v_{abs}^M &= \sqrt{v_x^2 + v_y^2} \\
v_{rel}^M &= 5\sqrt{2}(2t + 1) \\
v_{tr}^M &= (3t^2 + 5) \cdot \sqrt{144 + 2(t^2 + t)^2 - 24(t^2 + t)} \\
\text{Where } \beta &= 45^\circ - \phi, \gamma = \arcsin \frac{(t^2 + t)}{\sqrt{144 + 2(t^2 + t)^2 - 24(t^2 + t)}} + \phi
\end{aligned}$$