Theoretical Mechanics Homework 8

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1 MEME

My friends asking me to go touch grass after not going out for two weeks straight:

2 LINKS

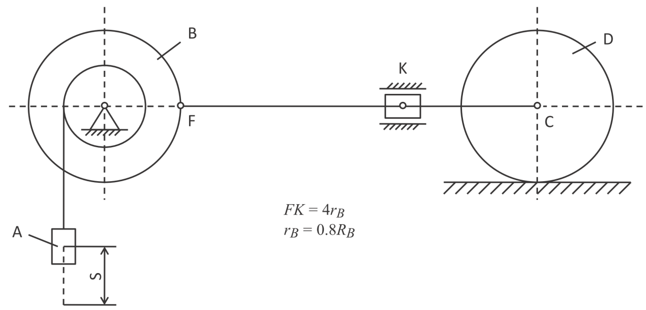
Link back to GitHub
Link to source code

3 Task 1

3.1 Task Description

Given:

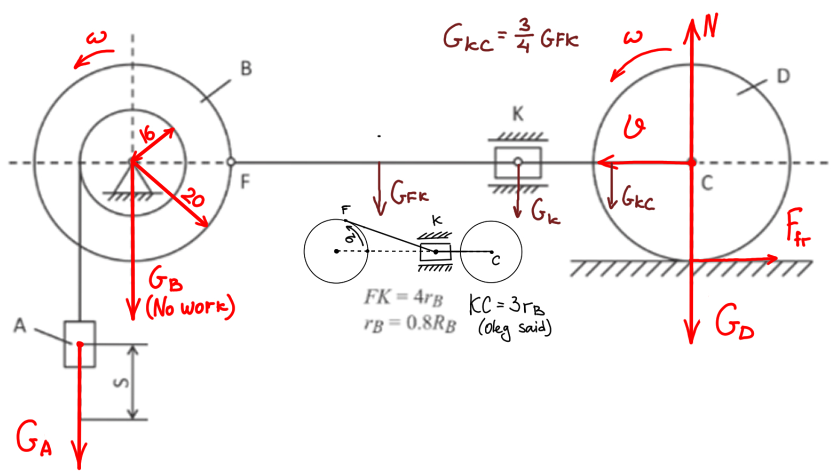
- $m_A = 1kg,\ m_B = 3kg,\ m_D = 20kg$



- $R_B = R_D = 20cm,\ i_B = 18cm$
- $\psi = 0.6(cm)$

Find $v_A(s)$

3.2 Solution



R.O. - one big system

Force analysis:

$$\vec{G}_A,\ \vec{G}_B,\ \vec{F}_{fr},\ \vec{N},\ \vec{G}_K(\text{second part}),\ \vec{G}_{FK}(\text{second part}),\ \vec{G}_{KC}(\text{second part})$$

Kinematic analysis:
$$\omega_b = \frac{v_a}{r_B},\ v_C = v_a \cdot \frac{R_B}{r_B} \cdot \frac{\sqrt{FK^2 - \sin\left(\frac{s\cdot 180}{\pi \cdot r_B}\right)^2 \cdot R_B^2}}{FK},\ \omega_D = \frac{v_C}{R_D}$$

According to theorem of change of kinetic energy in the system

$$\sum T_k = \sum A$$

Deriving Kinetic energies for all the bodies (and 2nd part bodies too)

$$T_A = \frac{m_A \cdot v_A^2}{2}$$
$$T_B = \frac{m_B \cdot i_B^2 \cdot \omega_B^2}{2}$$
$$T_D = \frac{m_D \cdot v_c^2}{2} + \frac{m_D \cdot R_D^2 \cdot \omega_D^2}{4} = \frac{3m_D \cdot v_C^2}{4}$$
$$T_K = \frac{m_K \cdot v_C^2}{2}$$
$$T_{FK} = \frac{m_{FK} \cdot v_C^2}{2}$$
$$T_{KC} = \frac{m_{FK} \cdot v_C^2}{2}$$

Setting Y-axis in the same direction as N and applying 2nd law of Newton to the body D we get:

$$0 = N - m_D \cdot g;$$
$$N = m_D \cdot g$$

Because $F_{fr} = \psi \cdot N$, we get $F_{fr} = \psi \cdot m_D \cdot g$;
Deriving work done by the system:

$$A_{G_A} = m_A \cdot g \cdot s$$
$$A_{G_B} = 0$$
$$A_{F_{fr}} = m_D \cdot g \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin\left(\frac{s\cdot 180}{\pi\cdot r_B}\right)^2 \cdot R_B^2}}{FK}$$
$$A_{G_N} = 0$$
$$A_{G_K} = 0$$
$$A_{G_{FK}} = m_{FK} \cdot g \cdot \frac{\sin\left(\frac{s\cdot 180}{\pi \cdot r}\right) \cdot R_B}{2}$$
$$A_{G_{KC}} = 0$$

After substituting all of this into the theorem of change of kinetic energy in the system, we can derive v_a (pls spare me, there is a shit ton of everything and I do not want to show all the calculations because it will take 20 or more minutes of just retyping it here):

With neglection of masses of piston and rods:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A)}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + \frac{3}{2} \cdot m_D \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2}}}$$

Without neglection of masses of piston and rods:skull:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A) + m_{FK} \cdot g \cdot \sin(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + (\frac{3}{2} \cdot m_D + m_K + m_{KC}) \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2} + \frac{m_{FK} R_B^2}{r_B^2}}}$$

I will not substitute any number into this formula to preserve at least a little bit of what's left of my sanity.

3.3 Answers

With neglection of masses of piston and rods:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A)}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + \frac{3}{2} \cdot m_D \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2}}}$$

Without neglection of masses of piston and rods:skull:

$$v_A = \sqrt{\frac{2 \cdot s \cdot g(m_D \cdot \psi \cdot \frac{\sqrt{FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_B}) \cdot R_B^2}}{FK} + m_A) + m_{FK} \cdot g \cdot \sin(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B}{m_A + \frac{m_b \cdot i_B^2}{r_B^2} + (\frac{3}{2} \cdot m_D + m_K + m_{KC}) \cdot \frac{R_B^2(FK^2 - \sin^2(\frac{s \cdot 180}{\pi \cdot r_b}) \cdot R_B^2)}{r_B^2 \cdot FK^2} + \frac{m_{FK} R_B^2}{r_B^2}}}$$

As for the explanation of why Yablonskii decided to restrict a piston and links. He did so, because he did not want the dynamics of piston and links to affect the dynamic of the body D, on which we already need to consider friction force. Kinematics of those links and piston would have mattered either way.

4 Task 2

4.1 Task Description

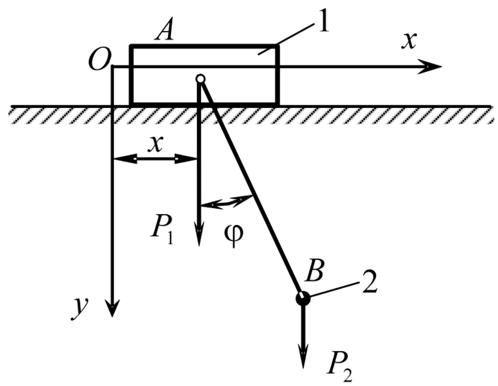
Initial conditions:

1. $x = 0,\ \phi = 10^\circ,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\ kg,\ m_2 = 1\ kg,\ l = 1\ m.$

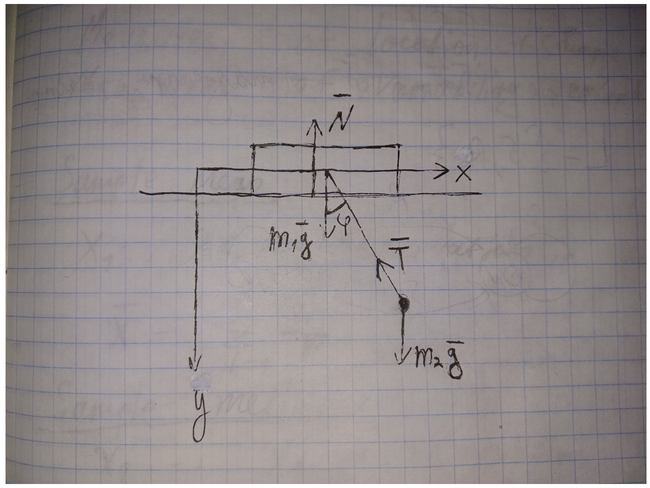


The task is to derive equations, characterizing the motion of the system

4.2 solution

	initial	final	initial	final	initial	final
x	0	?	0.5	?	0.5	?
ϕ	$10°$	?	$45°$	?	$-135°$	?
$\dot{x}$	0	?	0	?	0	?
$\dot{\phi}$	0	?	0	?	0	?
t	0	?	0	?	0	?

- R.O - system of 2 bodies
- body A - translatory motion
- body B - curvilinear motion



Force analysis:
$m_1\vec{g},\ m_2\vec{g},\ \vec{N},\ \vec{T}$

Kinematic Analysis:

body A:

$$\begin{cases} X = x_a; \dot{X} = \dot{x}_a; \ddot{X} = \ddot{x}_a; \\ Y = 0; \dot{Y} = 0; \ddot{Y} = 0 \end{cases}$$

body B:

$$\begin{cases} X = x_a + l \cdot \sin(\phi); \dot{X} = l \cdot \dot{\phi}\cos(\phi) + \dot{x}_a; \ddot{X} = -l\dot{\phi}^2\sin(\phi) + l \cdot \ddot{\phi}\cos(\phi) + \ddot{x}_a; \\ Y = l \cdot \cos(\phi); \dot{Y} = -l \cdot \dot{\phi} \cdot \sin(\phi); \ddot{Y} = -l \cdot \ddot{\phi} \cdot \sin(\phi) - l \cdot \dot{\phi}^2 \cdot \cos(\phi) \end{cases}$$

According to Euler-Lagrange approach:

$$L = \sum T - \sum P$$

Let's find kinetic energy of the system: $\begin{cases} T_a = \frac{m_a \cdot \dot{x}_a^2}{2}; \\ T_b = \frac{m_b((l \cdot \dot{(\phi)}\cos(\phi) + \dot{x}_a)^2 + (-l \cdot \sin(phi) \cdot \dot{\phi})^2)}{2} \end{cases}$

Let's find potential energy of the system:

$$P = -m_b \cdot l \cdot g \cdot \cos(\phi)$$

Now we can find Lagrangian:

$$L = \sum T - \sum P = \frac{m_b \cdot (l^2 \cdot \phi^2 + 2l \cdot \cos(\phi) \cdot (\dot{\phi} \cdot \dot{x}_a + g) + \dot{x}_a^2) + m_a \cdot \dot{x}_a^2}{2}$$

Let's calculate necessary partial derivatives:

$$\begin{cases} \frac{\partial L}{\partial \dot{x}_a} = m_b(l \cdot \dot{\phi}\cos(\phi) + \dot{x}_a) + m_a \cdot \dot{x}_a \\ \frac{\partial L}{\partial x_a} = 0 \\ \frac{\partial L}{\partial \dot{\phi}} = m_b \cdot l(l \cdot \dot{\phi} + \dot{x}_a\cos(\phi)) \\ \frac{\partial L}{\partial \phi} = -m_b \cdot l(\dot{\phi} \cdot \dot{x}_a \cdot \sin(\phi) + g \cdot \sin(\phi)) \end{cases}$$

And substituting all of this into the system:
$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_a}\right) - \frac{\partial L}{\partial x_a} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0 \end{cases}$$
We get: $\begin{cases} m_a \cdot \ddot{x}_a + m_b \cdot \ddot{x}_a + l \cdot \ddot{\phi} \cdot \cos(\phi) - l \cdot \phi \cdot \sin(\phi) = 0 \\ m_b l^2 \cdot \ddot{\phi} + m_b l \cdot \cos(\phi)\ddot{x}_a + m_b l \cdot \sin(\phi)(g + \dot{\phi} \cdot \dot{x}_a - \phi \cdot \dot{x}_a) = 0 \end{cases}$

After feeding it to python... It didn't digest it well, but what it did - it derived $\ddot{\phi}$ and $\ddot{x}_a$

$$\ddot{\phi} = \frac{g \cdot m_a \cdot \sin(\phi) + g \cdot m_b \cdot \sin(\phi) l_b \cdot \sin(\phi) \cdot \cos(\phi) \cdot \dot{\phi}^2}{-l \cdot m_a + l \cdot m_b \cdot \cos^2(phi) - l \cdot m_b}$$
$$\ddot{x}_a = -\frac{g \cdot m_b \sin(\phi) \cdot \cos(\phi) + l \cdot m_b \cdot \sin(\phi)\dot{\phi}^2}{-m_a + m_b \cdot \cos^2(\phi) - m_b}$$

As for the plot and everything, just look into the github, you can find it in the "links" section.

5 Answer

$$\ddot{\phi} = \frac{g \cdot m_a \cdot \sin(\phi) + g \cdot m_b \cdot \sin(\phi) l_b \cdot \sin(\phi) \cdot \cos(\phi) \cdot \dot{\phi}^2}{-l \cdot m_a + l \cdot m_b \cdot \cos^2(phi) - l \cdot m_b}$$
$$\ddot{x}_a = -\frac{g \cdot m_b \sin(\phi) \cdot \cos(\phi) + l \cdot m_b \cdot \sin(\phi)\dot{\phi}^2}{-m_a + m_b \cdot \cos^2(\phi) - m_b}$$