Theoretical Mechanics Homework 3

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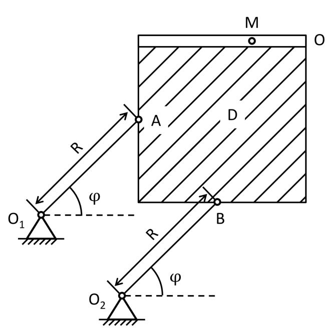
1 Task 1

1.1 To start off, a meme:

Me on Tuesday be like:

Now to the actual task.

1.2 Task Description.



Given:

1. $OM = s_r(t) = f_3(t) = 2t^3 + 3t$
2. $\phi(t) = f_2(t) = \frac{1}{24}\pi t^2$
3. $t_1 = 2, R = 15$

Find $a_{cor}, a_{abs}, v_{abs}$ of a point M.

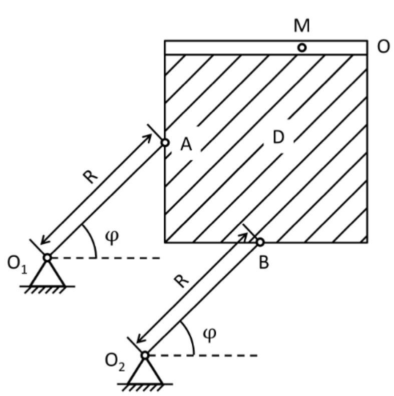
1.3 Solution

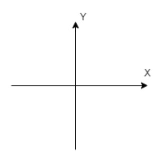
1.3.1 Finding necessary variables

Firstly, I need to find $v_M^{rel}, v_B^{tr}, \omega_B^{tr}$
$v_M^{rel} = \frac{dOM}{dt} = \frac{d2t^3+3t}{dt} = 6t^2 + 3$
$\omega_B^{tr} = \frac{d\phi(t)}{dt} = \frac{d}{dt}\left(\frac{\pi t^2}{24}\right) = \frac{\pi t}{12}, \epsilon = \frac{d\omega_B^{tr}}{dt} = \frac{d}{dt}\left(\frac{\pi t}{12}\right) = \frac{\pi}{12}$
$v_B^{tr} = \omega_B^{tr} R = \frac{\pi t R}{12} = \frac{5\pi}{2}$

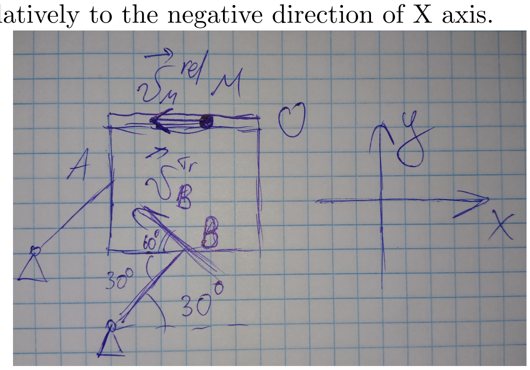
1.3.2 Finding absolute velocity of point M

Then I need to denote axes on the picture. Let Axis X be headed towards $\vec{MO}$ and Axis Y be perpendicular to Axis X and headed upwards corresponding to the picture.





After finding that $\phi(t = 2) = 30°$, I can tell that v_B^{tr} is inclined by $60°$ relatively to the negative direction of X axis.

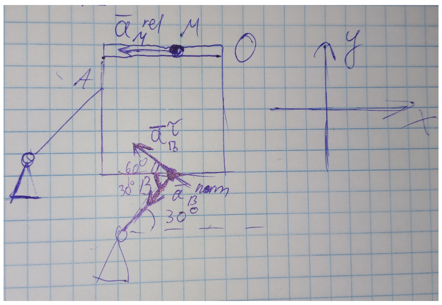


After that, I can project all the velocities, applied to point M and get projections of those velocities on X and Y axes.

$$\begin{cases} Ox: -v_M^{rel} - v_B^{tr}\cos 60° = v_x \\ Oy: v_B^{tr}\sin 60° = v_y \end{cases}$$

Then, we can compute $v_{abs} = \sqrt{v_x^2 + v_y^2} \approx 31.67\ m/sec$

1.3.3 Finding absolute acceleration



Now I need to calculate $\vec{a}_{abs} = \vec{a}_M^{rel} + \vec{a}_b^{tr} + \vec{a}_{cor}$.

As the body does not rotate around its own axis and performs only translatory motion, then $a_{cor} = 0$.

$a_M^{rel} = \dot{v}_M^{rel} = 12t$;
$\vec{a}_B^{tr} = \vec{a}_B^{\tau} + \vec{a}_B^{n}$
$a_B^{\tau} = \epsilon R = \frac{5\pi}{4}$
$a_B^{n} = \omega_B^2 R = \frac{5\pi^2}{12}$
$a_x : a_M^{rel} + a_B^{\tau}\cos 60° + a_B^{n}\cos 30°$
$a_y : a_B^{\tau}\sin 60° - a_B^{n}\sin 30°$
$a_{abs} = \sqrt{a_x^2 + a_y^2} \approx 29.55\ m/sec.$

1.3.4 Answers

$a_{abs} = 29.55\ m/sec, a_{cor} = 0, v_{abs} = 31.67\ m/sec$