Theoretical Mechanics Homework 5

Leonid Novikov, B22-RO-01

February 2024

1 MEME

Я, когда всегда(чел снизу):

2 LINKS

Link back to GitHub
Link to the colab source code

3 Task 1

3.1 Task description:

The legend shall speak that this situation was in WW2. There are two actors in this story: a sniper and an officer. Both knew about each other's existence. There was a river between them. The officer was always sitting in a trench, but the sniper knew his location and already calculated the distance to the target ($L\ meters$).

After a while cargo ship appeared, which blocked the direct vision of the trench. The officer decided to stand up to stretch his legs. The sniper assumed that it might happened and make a shot, hitting the officer. Let's check this story.

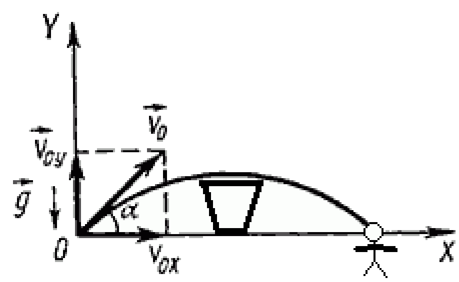
Formal description: Considering the bullet as a material point and taking into account its weight and the force of wind resistance, we need to solve the following problems:

- Find the α (initial angle of overhang) that is required to hit the target.
- At this angle, what is the maximum height the bullet will reach?

Given, that:

- $\vec{F_c} = kv \cdot \vec{v}$
- $m = 13.6\ g, L = 1500\ m, k = 1.3 \cdot 10^{-5}, v_0 = 870\ m/s.$

3.2 Solution



3.2.1 No air drag

	$initial$	$final$
t	0	?
x	0	L
$\dot{x}$	$v_0 \cdot \cos(\alpha)$	$v_0 \cdot \cos(\alpha)$
$\ddot{x}$	0	0
y	0	0
$\dot{y}$	$v_0 \cdot \sin(\alpha)$	?
$\ddot{y}$	$-g$	$-g$

Firstly, as there are no external forces applied on the body, except gravitational, its horizontal speed is constant, while vertical is dependant on g, hence the trajectory of the body will be parabola. The top point will be located halfway through the trajectory, where vertical speed will be equal to 0:

$$\begin{cases} L = v_0 \cos{(\alpha)} \cdot t \\ \sin{(\alpha)} v_0 - g\frac{t}{2} = 0 \end{cases}$$

From this, we acquire: $\alpha = \frac{1}{2} \arcsin \frac{gL}{v_0^2} \approx 0.56°$, $t = \frac{L}{v_0 \cos{(\alpha)}} \approx 1.72\ s$

Due to vertical acceleration being constant, y-coordinate of the body can be calculated like this:

$$y = -g\frac{t^2}{2} + v_{0y}t$$

And after substituting t there, we acquire maximum height $\approx$ 3.65 m

3.2.2 With air drag

	$initial$	$final$
t	0	?
x	0	L
$\dot{x}$	$v_0 \cdot \cos(\alpha)$	?
$\ddot{x}$	0	?
y	0	0
$\dot{y}$	$v_0 \cdot \sin(\alpha)$	?
$\ddot{y}$	$-g$	?

Now, as an air drag is applied to the system, everything becomes much more complicated. We need to consider 2^{nd} Law of Newton. we get:

$$\begin{cases} m\ddot{x} = -F_{cx} \\ m\ddot{y} = -F_{cy} - mg \end{cases} \quad => \begin{cases} m\ddot{x} = -k \cdot \sqrt{\dot{x}^2 + \dot{y}^2} \cdot \dot{x} \\ m\ddot{y} = -k \cdot \sqrt{\dot{x}^2 + \dot{y}^2} \cdot \dot{y} - mg \end{cases}$$

This thing can't be integrated by hands, so I used (almost) almighty Python to solve it. It gave me:

The sniper rifle has to be inclined by almost 1.86° relative to the X axis;
Maximum height of bullet's trajectory is around: 19.08 m

3.3 ANSWERS

3.3.1 Without drag force

$$h = 3.65\ m,\ \alpha = 0.56^\circ$$

3.3.2 With drag force

$$h = 19.08\ m,\ \alpha = 1.86^\circ$$