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Path integral for spin: A new approach

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Abstract

The path integral representation for the propagator of a Dirac particle in an external electromagnetic field is derived using the functional derivative formalism with the help of Weyl symbol representation for the Grassmann vector part of the variables. The proposed method simplifies the proof of the path integral representation starting from the equation for the Green function significantly and automatically leads to a precise and unambiguous set of boundary conditions for the anticommuting variables and puts strong restrictions on the choice of the gauge conditions. The same problem is reconsidered using the Polyakov and Batalin–Fradkin–Vilkovisky methods together with the Weyl symbol method and it is shown to yield the same PIR. It is shown that in the last case, the non-trivial first class constraints algebra for a Dirac particle plays an important role in the derivation, and this algebra is the limiting case of the superconformal algebra for a Ramond open string when the width goes to zero. That the approach proposed here can be applied to any point-like particle is illustrated in the propagator for the nonrelativistic Pauli spinning particle in an external electromagnetic field.

1. Introduction

The problem of constructing the path integral representation (PIR) for the relativistic point-like spinning particle propagator has a long history. One principle aim of the earlier works was to obtain a classical action for a spinning particle. The references to the early works in this direction and other works can be found in [1].

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In relativistic form the PIR for a Dirac particle propagator was first obtained in [2] employing the second order equation for the Green function (GF). The new stimulus for the investigation of the problem did appear in the 70's after the discovery of supersymmetry [3–5] and the development of strings and especially the Fermi string theories [6,7]. Historically the classical action for Fermi strings was first constructed in a form which is recognised as the action of two-dimensional supergravity. The main points of this work were the introduction of the Grassmann variables for the description of classical objects with spin and the revelation of local supersymmetry. The classical action for a Dirac particle possessing the local gauge reparametrisation and local supersymmetry invariances has almost simultaneously been identified to be the action of one-dimensional supergravity [8–12]. The quantization of this action by the PIR has been the subject of several works [13–17]. But only in the last work is the PIR for a Dirac particle propagator constructed in the presence of electromagnetic and other fields in the framework of Polyakov formalism [18] and employing the first order equation for the GF. The PIR for the propagators of Neveu–Schwarz–Ramond fermionic strings was found in [19]. One also needs to note the work [20] where the PIR for the GF of the Dirac particle is a method for treating the Grassmann variables very similar to the Weyl symbol method which is developed in [1].

Another point of interest for the PIR for the Dirac particle propagator concerns different applications of a relativistic particle in external fields. Namely, it is well known that the knowledge of the propagator of a relativistic particle in an arbitrary external field in principle enables one to compute the total GF including all the radiative corrections [21] and simplifies the solution of many problems with given configurations of the external fields.

However, in spite of an essential advance in understanding the Grassmann nature of the classical description of the Dirac particle and the development of the different methods for quantization of the system up to now, in our opinion, we still lack a general simple approach for constructing the PIR for relativistic spinning particles and a true understanding of the interconnection between different methods of quantization of such systems. The main aim of this work is to fill this gap as much as possible. In this work, which is a further development of the previous work [22], a new approach for constructing the PIR for a Dirac particle propagator is proposed which uses the functional derivative formalism [23] and the Weyl symbol (WS) representation for the grassmannian vector part of the particle variables. The interconnection between the canonical, Polyakov, and the Batalin–Fradkin–Vilkovisky (BFV) methods [24] for constructing the PIR for a Dirac particle propagator is demonstrated. In Section 2 we consider the new approach to the PIR starting from the canonical quantization method and the equation for the GF in an external electromagnetic field. Section 3 is devoted to the Polyakov method, using a slightly different approach than the one in [17]. In Section 4 we undertake an attempt to get the same PIR starting from the general BFV method. In Section 5 we consider the application of the new approach to the PIR for a Pauli particle in an external electromagnetic field. Section 6 contains concluding remarks. In the appendices some of the mathematical details are presented.

2. Dirac particle – canonical method

We start with the classical action for the Dirac particle in an external electromagnetic field, which is invariant under two local symmetries, namely the reparametrisation and the local supersymmetries:

$$S = \int_0^1 \mathrm{d}\tau \Big\{ \tfrac{1}{2}\Big[e^{-1}\dot{x}_\mu \dot{x}^\mu - em^2 - i\psi^n \dot{\psi}_n + i\big(e^{-1}\dot{x}^\mu \psi_\mu + m\psi^5\big)\chi \Big]$$
$$+ g\big[\dot{x}^\mu A_\mu(x) + \tfrac{1}{2} ieF_{\mu\nu}\psi^\mu\psi^\nu\big]\Big\}. \tag{2.1}$$

Here $x^\mu x_\mu = \eta_{\mu\nu}x^\mu x^\nu$, $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$; $e(\tau)$ is the one-dimensional metric; the Fermi fields $\psi^n(\tau)$ $(n = 0,1,2,3,5)$ and the gravitino field $\chi(\tau)$ are the Grassmann variables, and g is the electric charge of the particle. First we consider the specific peculiarities of the classical canonical description of the system. The canonical momenta are

$$\pi_x^\mu = p^\mu = \frac{\partial L}{\partial \dot{x}_\mu} = \frac{\dot{x}_\mu}{e} + \frac{i}{2e}\psi^\mu\chi + gA^\mu(x), \tag{2.2}$$

$$\pi_\psi^n = \frac{\partial L}{\partial \dot{\psi}_n} = -\tfrac{1}{2}i\psi^n; \quad \pi_\psi^n + \tfrac{1}{2}i\psi^n = 0. \tag{2.3}$$

Eq. (2.3) is a second class constraint. Two local symmetries of the action Eq. (2.1) give rise to the following two first class constraints:

$$\frac{\partial L}{\partial e(\tau)} = \frac{-1}{2}\left(\frac{\dot{x}_\mu^2}{e^2} + m^2 + \frac{i}{e^2}\dot{x}_\mu\psi^\mu\chi - igF_{\mu\nu}\psi^\mu\psi^\nu \right)$$
$$= -\tfrac{1}{2}\big(P_n^2 - igF_{\mu\nu}\psi^\mu\psi^\nu\big) = 0, \tag{2.4}$$

$$\frac{\partial L}{\partial \chi(\tau)} = \tfrac{1}{2}i\big(e^{-1}\dot{x}^\mu\psi_\mu + m\psi_5\big) = \tfrac{1}{2}i(P_n\psi_n) = 0 \tag{2.5}$$

where $P_\mu = p_\mu - gA_\mu$, $P_5 = m$. Note that we have also two additional first class constraints:

$$\pi_e = \frac{\partial L}{\partial \dot{e}} = 0, \quad \pi_\chi = \frac{\partial L}{\partial \dot{\chi}} = 0. \tag{2.6}$$

The hamiltonian of this system (2.1) is equal to zero on the surface of the constraints Eqs. (2.4) and (2.5):

$$H = -L + p\dot{x} - \tfrac{1}{2}i\psi^n\dot{\psi}_n$$
$$= \tfrac{1}{2}e\big(P_n^2 - igF_{\mu\nu}\psi^\mu\psi^\nu\big) - \tfrac{1}{2}i(P_n\psi^n)\chi = 0. \tag{2.7}$$

We have taken into account the constraints (2.3) while deriving Eq. (2.7). It is easy to show that the hamiltonian equations of motion obtained with the help of Dirac brackets,

$$\dot{x}^{\mu} = -\{x^{\mu}, H\}_{\mathrm{D}}, \quad \dot{\psi}^{\mu} = -\{\psi^{\mu}, H\}_{\mathrm{D}}, \quad \dot{p}_{\mu} = -\{p_{\mu}, H\}_{\mathrm{D}}, \tag{2.8}$$

$$\dot{e} = -\{e, H\}_{\mathrm{D}} = 0, \quad \dot{\chi} = -\{\chi, H\}_{\mathrm{D}} = 0, \tag{2.9}$$

coincide with the Lagrange equations, as can be demonstrated using Eqs. (2.1) and (2.3). Here $\{A, B\}_{\mathrm{D}} = \{A, B\}_{\mathrm{P}} + \{A, \theta^{\alpha}\}_{\mathrm{P}}\{\theta^{\alpha}, \theta^{\beta}\}_{\mathrm{P}}^{-1}\{\theta^{\beta}, B\}_{\mathrm{P}}$. For the canonical quantization of the system (2.1) it is very important to understand the interconnection between the first class constraints (2.4) and (2.5). Introducing

$$T_1 = \tfrac{1}{2}\left(P_n^2 - igF_{\mu\nu}\psi^{\mu}\psi^{\nu}\right), \tag{2.10}$$

$$T_2 = -\tfrac{1}{2}(P_n\psi^n), \tag{2.11}$$

then we observe

$$H = eT_1 + i\chi T_2 = \lambda^{\alpha}T_{\alpha}, \quad \alpha = 1, 2, \dots . \tag{2.12}$$

Here the T_{α} are hermitian. The constraints T_1 and T_2 have zero Dirac brackets:

$$\{T_1, T_2\}_{\mathrm{D}} = 0. \tag{2.13}$$

But on the other hand they form the non-trivial algebra of the first class constraints,

$$\{T_2, T_2\}_{\mathrm{D}} = -\tfrac{1}{2}iT_1, \tag{2.14}$$

due to the fact that all Dirac brackets are equal to zero on the surface $T_{\alpha} = 0$.

The analogous relations do appear in the fermionic string theories [7] due to superconformal algebra. Moreover Eq. (2.14) is a special case of the superconformal algebra for the Ramond-type open string. Since the Dirac particle can be obtained from the open Ramond string when the width goes to zero, this is quite natural.

First we consider the derivation of the PIR using the canonical quantization method. We impose the usual canonical quantization rules:

$$i\{p^{\mu}, x^{\nu}\}_{\mathrm{D}} = i\{p^{\mu}, x^{\nu}\}_{\mathrm{D}} \to [\hat{p}^{\mu}, \hat{x}^{\nu}]_{-} = i\eta^{\mu\nu}, \tag{2.15}$$

and the anticommutation relations for the fermionic fields obtained from the Dirac brackets:

$$i\{\psi^m, \psi^n\}_{\mathrm{D}} \to \left[\hat{\psi}^m, \hat{\psi}^n\right]_{+} = -\eta^{nm}, \quad \eta^{55} = 1. \tag{2.16}$$

The operators $\hat{\psi}^n$ are generators of the Clifford algebra C_5 with a unique finite matrix representation:

$$\hat{\psi}_{\mu} = \frac{i}{\sqrt{2}}\gamma_5\gamma_{\mu}, \quad \hat{\psi}_5 = \frac{i}{\sqrt{2}}\gamma_5, \quad \gamma_5^2 = 1, \quad [\gamma_{\mu}, \gamma_{\nu}]_{+} = -2\eta_{\mu\nu}. \tag{2.17}$$

One can check that all the equations (2.8), (2.9), (2.13), (2.14) are now fulfilled for the corresponding operators, if we make the following substitutions:

$$i\{A, B\} \to \left[\hat{A}, \hat{B}\right]_{\mp}, \tag{2.18}$$

for even and odd variables accordingly.

After quantization, the first class constraints Eqs. (2.4) and (2.5) become the conditions on permissible vectors of the physical states in the total Hilbert space.

It is to be stressed that Eq. (2.4) is a consequence of Eq. (2.5) due to Eq. (2.14). On the quantum level one gets

$$\left[\hat{T}_2, \hat{T}_2\right]_+ = -\tfrac{1}{2}T_1, \tag{2.19}$$

or in manifest form

$$\tfrac{1}{4}\left[\hat{P}_n\hat{\psi}^n, \hat{P}_m\hat{\psi}^m\right]_+ = -\tfrac{1}{4}\left(\hat{P}_n^2 - ig\hat{F}_{\mu\nu}\hat{\psi}^\nu\hat{\psi}^\nu\right). \tag{2.20}$$

Eq. (2.20) allows to consider only one condition, $T_2\Phi_{\mathrm{ph}} = 0$, on permissible vectors of the physical states. Thus, the equation for the Dirac propagator is

$$\left(\hat{P}_n\psi^n\right)\hat{G}(x, y) = \delta^n(x-y). \tag{2.21}$$

One notes that the constraints T_α and the corresponding operators $\hat{T}_\alpha$, once expressed in terms of phase space variables, are independent of τ and therefore they are conditions of invariance of $\hat{G}$ under reparametrisation of $e(\tau)$ and supertransformation of $\chi(\tau)$.

Starting from Eqs. (2.14), (2.20) one can write for the causal GF

$$\begin{aligned}\hat{G}(x, y) &= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k\, \exp(-i\hat{H})\delta^4(x-y)\\ &= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k\, \langle x|\exp(-i\hat{H})|y\rangle.\end{aligned} \tag{2.22}$$

Here $\hat{x}^\mu|x\rangle = x^\mu|x\rangle$, $\langle x|y\rangle = \delta^4(x-y)$ and

$$\hat{H} = \tfrac{1}{2}\lambda\left(\hat{P}^2 - i\epsilon - igF_{\mu\nu}\hat{\psi}^\mu\hat{\psi}^\nu\right) - \tfrac{1}{2}i\left(\hat{P}_n\hat{\psi}_n\right)k. \tag{2.23}$$

Operator $\hat{H}$ corresponds to the gauge choice of $e(\tau) = \lambda = \text{const.}$, $\chi(\tau) = k = \text{const.}$ in Eqs. (2.1) and (2.7).

We next introduce the auxiliary operator

$$\hat{U}(x, y; t) = \langle x|\exp(-i\hat{H}t)|y\rangle, \tag{2.24}$$

which appears in Eq. (2.22) at $t = 1$ and which can be rewritten in the interaction representation as

$$\hat{U}(x, y; t) = \langle x|\tilde{T}\exp\left(-i\int_0^t V\left(\hat{p}(\tau), \hat{x}(\tau), \hat{\psi}(\tau)\right)\mathrm{d}\tau\right)\exp\left(-i\hat{H}_0 t\right)|y\rangle, \tag{2.25}$$

where

$$\hat{H}_0 = \tfrac{1}{2}\hat{p}^2, \quad \hat{V} = \hat{H} - \hat{H}_0,$$
$$\hat{x}(\tau) = \exp\left(-i\hat{H}_0\tau\right)\hat{x}\exp\left(i\hat{H}_0\tau\right), \quad \hat{p}(\tau) = \hat{p}, \quad \hat{\psi}(\tau) = \hat{\psi}. \tag{2.26}$$

Here we have to qualify our choice of $\hat{H}_0$. It is chosen in this manner for simplicity, and for convergence of the PIR ($\hat{H}_0 \neq \hat{H}$ at $g = 0$). We introduced also the anti-T-product, $\tilde{T}$, in Eq. (2.25), since the Weyl symbol for $\hat{U}$ has a simpler definition in this case (see Eq. (2.31) below).

The operator $\hat{U}$ satisfies the Schrödinger equation

$$i\frac{d\hat{U}}{dt} = \hat{U}\hat{H}_J, \tag{2.27}$$

with the initial condition $\hat{U}(x, y; 0) = \delta^4(x - y)$.

Using the functional derivative method, one can write (we stress that $\hat{G}$, $\hat{U}$, $\hat{U}_J$ are the operators involving Grassmann variables)

$$\hat{U}(x, y; t) = \exp\left[-i\int_0^t d\tau\, V\left(i\frac{\delta}{\delta J_p}, i\frac{\delta}{\delta J_x}, i\frac{\delta}{\delta J_\psi}\right)\right]\hat{U}_J(x, y; t)\Big|_{J_\alpha = 0}, \tag{2.28}$$

where

$$\hat{U}_J(x, y; t) = \langle x \,|\, \tilde{T}\exp\left(-i\int_0^t d\tau\, J_\alpha(x)\hat{\varphi}^\alpha(\tau)\right)\exp\left(-i\hat{H}_0 t\right) |\, y\rangle. \tag{2.29}$$

Here $\alpha = p, x, \psi$; $\hat{\varphi}^p(\tau) = \hat{p}$; $\hat{\varphi}^x(\tau) = \hat{x}(\tau)$; $\varphi^\psi(\tau) = i\psi$ and J_ψ is the grassmannian source.

Thus the task of the construction of the PIR for $\hat{U}(x, y; t)$ and $\hat{G}(x, y)$ (see Eqs. (2.22), (2.24)) reduces to the construction of the PIR for the simpler transition matrix $\hat{U}_J$ with linear dependence on the operators $\hat{\varphi}^\alpha$ in the exponent in Eq. (2.29). The only issue which requires special care is connected with the Grassmann part of Eq. (2.29), namely the operator

$$\Lambda(\hat{\psi}) \equiv \tilde{T}\exp\left(\int_0^t d\tau\, J_\psi^n(\tau)\hat{\psi}_n(\tau)\right). \tag{2.30}$$

A natural recipe to use the Weyl symbol (WS) method for Grassmann variables was developed in [1]. The generalisation for the relativistic case of the corresponding formulas give for the WS of the operator $\Lambda(\hat{\psi})$

$$\Lambda^{W}(\xi) = c\int_{\psi(t)+\psi(0)=2\xi} D\psi_n(\tau)\exp\left(\int_0^t d\tau\left[\tfrac{1}{2}\psi_n\dot{\psi}^n + J_\psi^n(\tau)\psi_n(\tau)\right]\right.$$
$$\left. + \tfrac{1}{2}\psi^n(t)\psi_n(0)\right). \tag{2.31}$$

Here $\psi(t)$ and $\psi(0)$ are the boundary values of $\psi(\tau)$; ξ is the Grassmann symbol corresponding to the $\hat{\psi}$ operator. We stress that all of the dependence on ξ and $\hat{\psi}$ is introduced through the boundary conditions for $\psi(\tau)$ and we further note that it is impossible to choose two independent values for $\psi(t)$ and $\psi(0)$ due to the first order nature of the classical equation for $\psi(\tau)$. In order to compute the operator $\Lambda(\hat{\psi})$ we need to change the variables $\xi^n \to \hat{\psi}^n$ which can be formally affected as

$$\Lambda(\hat{\psi}) = \exp\left(\hat{\psi}\frac{\delta}{\delta\xi}\right)\Lambda^{W}(\xi)\big|_{\xi=0}. \tag{2.32}$$

It is easy to show (see Appendix B) that $\Lambda(\hat{\psi})$ defined in Eq. (2.32) satisfies the same equation as the $\Lambda(\hat{\psi})$, defined in Eq. (2.30):

$$\frac{\mathrm{d}\Lambda(\hat{\psi})}{\mathrm{d}t} = \Lambda(\hat{\psi})J_{\psi}^{n}(\tau)\hat{\psi}_n. \tag{2.33}$$

One notes that if we used the T-product in Eqs. (2.24), (2.29), (2.30) instead of the $\tilde{T}$-product we would have obtained for $\Lambda^{W}(\xi)$, instead of in (2.31) an opposite sign for the $\psi_n\dot{\psi}^n$ term (or another choice of the initial action (2.1), see Appendix B, (B.6)).

Using Eqs. (2.29) and (2.31) we can get the PIR for the WS of the operator $\hat{U}_J$:

$$\begin{aligned}
&\hat{U}_J^{W}(x, y; t)\\
&\quad= N\int \mathrm{d}p(\tau)\int_y^x \mathrm{D}x(\tau)\int_{\psi(t)+\psi(0)=2\xi}\mathrm{d}\psi(\tau)\\
&\qquad\times\exp\left[-i\int_0^t\mathrm{d}\tau\left(\frac{p_\mu^2}{2m} - p_\mu\dot{x}^\mu + \frac{1}{2}i\psi_n\dot{\psi}^n + J_\alpha(\tau)\varphi^\alpha(\tau)\right)\right.\\
&\qquad\left. + \tfrac{1}{2}\psi^n(t)\psi_n(0)\right],
\end{aligned}\tag{2.34}$$

$$\hat{U}_J(x, y; t) = \exp\left(\hat{\psi}\frac{\delta}{\delta\xi}\right)U_J^{W}(x, y; t)\big|_{\xi=0}. \tag{2.35}$$

From Eqs. (2.28) and (2.31) we can get the PIR for $U^{W}(x, y; t)$:

$$\begin{aligned}
&U^{W}(x, y; t)\\
&\quad= N\int \mathrm{D}p(\tau)\int_y^x \mathrm{D}x(\tau)\int_{\psi(t)+\psi(0)=2\xi}\mathrm{D}\psi(\tau)\\
&\qquad\times\exp\left(-i\int_0^t\mathrm{d}\tau\left[\tfrac{1}{2}\lambda\left(P_n^2 - igF_{\mu\nu}\psi^\mu\psi^\nu\right) - \tfrac{1}{2}i(P_n\psi^n)k - p_\mu\dot{x}^\mu + \tfrac{1}{2}i\psi_n\dot{\psi}^n\right]\right.\\
&\qquad\left. + \tfrac{1}{2}\psi^n(t)\psi_n(0)\right)\\
&\quad= N\int \mathrm{D}p(\tau)\int_y^x \mathrm{D}x(\tau)\int_{\psi(t)+\psi(0)=2\xi}\mathrm{D}\psi(\tau)\\
&\qquad\times\exp\left(-i\int_0^t\mathrm{d}\tau\left(\lambda T_1 + ikT_2 - ip_\mu\dot{x}^\mu + \tfrac{1}{2}i\psi_n\dot{\psi}^n\right) + \tfrac{1}{2}\psi^n(t)\psi_n(0)\right).
\end{aligned}\tag{2.36}$$

Taking into account Eqs. (2.22), (2.24) and introducing the WS for $\hat{G}$, we obtain

$$\begin{aligned} G^{\mathrm{W}}(x, y) &= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k\, U^{\mathrm{W}}(x, y; t=1) \\ &= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \cdot N \int \mathrm{D}p(\tau) \int_y^x \mathrm{D}x(\tau) \int_{\psi(t)+\psi(0)=2\xi} \mathrm{D}\psi(\tau) \\ &\quad \times \exp\left(-i \int_0^1 \mathrm{d}\tau \left(\lambda T_1 + ikT_2 - p_\mu \dot{x}^\mu + \tfrac{1}{2} i \psi_n \dot{\psi}^n \right) + \tfrac{1}{2} \psi^n(t) \psi_n(0) \right). \end{aligned} \tag{2.37}$$

After integrating over $p(\tau)$ in Eq. (2.37) we find

$$\begin{aligned} G^{\mathrm{W}}(x, y; t) = \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \cdot N_0 \int_{0 \leqslant \tau \leqslant 1} \mathrm{D}x(\tau) \int_{\psi(1)+\psi(0)=2\xi} \mathrm{D}\psi(\tau) \\ \times \exp\left[iS + \tfrac{1}{2} \psi^n(1) \psi_n(0) \right], \end{aligned} \tag{2.38}$$

where S is the action in (2.1) with the gauge choice $e(\tau) = \lambda = $ const., $\xi(\tau) = k =$ const..

For the GF we obtain an expression similar to that in Eq. (2.35),

$$\hat{G}(x, y) = \exp\left(\hat{\psi}^n \frac{\delta}{\delta \xi^n} \right) G^{\mathrm{W}}(x, y)\big|_{\xi=0}. \tag{2.39}$$

The determination of the normalisation constants N and N_0 requires care and is considered in Appendices A and B (see (A.9) and (B.4)). In order to demonstrate that $\hat{G}(x, y)$ is independent of the choice of gauge for $e(\tau)$ and $\xi(\tau)$ we can write another PIR for $G^{\mathrm{W}}(x, y)$:

$$\begin{aligned} G^{\mathrm{W}} = \tilde{N} \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \int_{x, 0 \leqslant \tau \leqslant 1}^{y} \mathrm{D}x(\tau) \int_{\psi(\lambda)+\psi(0)=2\xi} \mathrm{D}\psi(\tau) \\ \times \exp\left[iS_\lambda + \tfrac{1}{2} \psi(\lambda) \psi(0) \right], \end{aligned} \tag{2.40}$$

with

$$S_\lambda = \int_0^\lambda \left[\tfrac{1}{2} \left(\dot{x}_\mu^2 - m^2 - i\psi^n \dot{\psi}_n + \frac{i}{\lambda} \left(\dot{x}_\mu \psi^\mu + m\psi^5 \right) \kappa \right) + g \left(\dot{x}^\mu A_\mu + \tfrac{1}{2} i F_{\mu\nu} \psi^\mu \psi^\nu \right) \right], \tag{2.41}$$

corresponding to the gauge choice $e(\tau)$ and $\psi(\tau) = k/\lambda$ in (2.1). Thus, the PIR for the GF in different gauges has different forms, but they all yield the same final result.

3. Polyakov's method

Now we shortly consider the construction of the PIR for the Dirac particle propagator by using Polyakov's method. The first such attempt has been under-taken in [17]; here we would like to reproduce the derivation in a simpler way. Formally we can write the WS of the GF as

$$G^{\mathrm{W}}(x, y) \sim \int \frac{\mathrm{D}e\,\mathrm{D}\chi}{V_e V_\chi} \int_y^x \mathrm{D}x(\tau) \int_{\psi(1)+\psi(0)=2\xi} \mathrm{D}\psi(\tau)\, \exp\left[iS + \tfrac{1}{2}\psi^n(1)\psi_n(0)\right], \tag{3.1}$$

where S is the action given in Eq. (2.1) and V_e and V_χ are the volumes of the reparametrisation and supersymmetry groups; the term $\frac{1}{2}\psi(1)\psi(0)$ is necessarily introduced due to the definition of the WS (see Eq. (2.31)). The integration over $e(\tau)$ and $\chi(\tau)$ reduces to integrations over the groups of local transformations and also over parameters, which are analogous to the modules and supermodules in string theory (see [25,26,17]). In order to understand the appearance of the modules, one introduces two invariant lengths:

$$\lambda = \int_0^1 \mathrm{d}\tau\, e(\tau), \tag{3.2}$$

$$\chi = \int_0^1 \mathrm{d}\tau\, \chi(\tau), \tag{3.3}$$

where λ and χ are invariant under local reparametrisation and local superparametrisation correspondingly:

$$\delta_\xi e(\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}(\xi e), \tag{3.4}$$

$$\delta_\epsilon \chi(\tau) = 2\frac{\mathrm{d}}{\mathrm{d}\tau}\epsilon. \tag{3.5}$$

Here $\xi(\tau)$ and $\epsilon(\tau)$ are the infinitesimal parameters of these transformations; ϵ has the Grassmann value. Note that

$$\delta_\kappa \lambda = \int_0^1 \mathrm{d}\tau\, \delta_\epsilon \chi(\tau) = (\xi e)_1 - (\xi e)_0 \tag{3.6}$$

due to the boundary condition $\xi(1) - \xi(0) = 0$. Analogously

$$\delta_\kappa \lambda = \int_0^1 \mathrm{d}\tau\, \delta_\epsilon \xi = 2\epsilon(1) - 2\epsilon(0) = 0. \tag{3.7}$$

One can now determine global transformations of λ and k as:

$$\delta\lambda = \int_0^1 \mathrm{d}\tau\, \tilde{\delta} e = \tilde{\delta} e, \tag{3.8}$$

$$\delta k = \int_0^1 \mathrm{d}\tau\, \tilde{\delta}\chi = \tilde{\delta}\chi, \tag{3.9}$$

since $\tilde{\delta} e$ and $\tilde{\delta}\chi$ are independent of τ.

In order to prove that both of these transformations are independent of the local transformations Eqs. (3.4), (3.5), one introduces the invariant scalar product defined as

$$\langle \delta_\xi e, \delta\tilde{e} \rangle = \int_0^1 \mathrm{d}\tau\, e^{-1}(\tau)(\delta_\epsilon e\, \delta\tilde{e}), \tag{3.10}$$

$$\langle \delta_\epsilon \chi, \tilde{\delta}\chi \rangle = \int_0^1 \mathrm{d}\tau\, e^{-1}(\tau)\big(\delta_\epsilon \chi\, \tilde{\delta}\chi\big), \tag{3.11}$$

and demands that Eqs. (3.10), (3.11) are equal to zero:

$$\int_0^1 \mathrm{d}\tau\, e^{-1}(\tau)\big(\delta_\xi e\, \delta\tilde{e}\big) = -\int \mathrm{d}\tau(\xi\tau)\frac{\mathrm{d}}{\mathrm{d}\tau}\big[e^{-1}(\tau)\tilde{\delta}e\big] = 0, \tag{3.12}$$

$$\int_0^1 \mathrm{d}\tau\, e^{-1}(\tau)\big(\delta_\epsilon \chi\, \tilde{\delta}\chi\big) = -\int \mathrm{d}\tau\, 2\dot{\epsilon}\frac{\mathrm{d}}{\mathrm{d}\tau}\big(e^{-1}\tilde{\delta}\chi\big) = 0. \tag{3.13}$$

From Eqs. (3.12) and (3.13) we get

$$\dot{e}(\tau) = 0. \tag{3.14}$$

We suppose for simplicity that

$$\dot{\chi}(\tau) = 0 \tag{3.15}$$

too. We will consider in the following only the gauges of the type Eqs. (3.14), (3.15). In this case

$$e(\tau) = \lambda = \int_0^1 \mathrm{d}\tau\, e(\tau), \tag{3.16}$$

$$\chi(\tau) = k = \int_0^1 \mathrm{d}\tau\, \chi(\tau). \tag{3.17}$$

There is another way of introducing the invariant lengths:

$$\lambda = \int_0^\lambda \mathrm{d}\tau\, e(\tau), \quad k = \int_0^\lambda \mathrm{d}\tau\, \chi(\tau). \tag{3.18}$$

In the gauge given in Eq. (3.18), we get from Eqs. (3.14), (3.15)

$$\mathrm{e}(\tau) = \text{const.} = 1, \quad \chi(\tau) = \text{const.} = k/\lambda. \tag{3.19}$$

Note that this gauge option has been previously considered in Eqs. (2.40), (2.41). The parameters λ and κ are the modular parameters of the global group transformations. Thus in the general case the integral over metrics $e(\tau)$ and $\chi(\tau)$ contains the integrals over λ and k. Thus for the causal GF we have

$$\int \frac{\underline{\mathrm{D}e(\tau)}}{V_e} \int \frac{\mathrm{D}\chi(\tau)}{V_\chi} \sim \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \int \frac{\mathrm{D}e_\xi(\tau)}{V_e} \int \frac{\mathrm{D}\chi_\epsilon(\tau)}{V_\chi}. \tag{3.20}$$

It can be shown that normalisation of the volumes V_e, V_χ in Eq. (3.20) can be chosen in the following form:

$$V_e V_\chi = \sqrt{\lambda} \int \mathrm{D}\xi(\tau)\, \mathrm{D}\epsilon(\tau). \tag{3.21}$$

Below we use the hamiltonian form of the PIR instead of Eq. (3.1):

$$
\begin{aligned}
&G^{\mathrm{W}}(x, y) \\
&= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \int \frac{\mathrm{D}e(\tau)\,\mathrm{D}\chi(\tau)}{V_e V_\chi} N \int \mathrm{D}p(\tau) \int_x^y \mathrm{D}x(\tau) \int_{\psi(1)+\psi(0)=2\xi} \mathrm{D}\psi(\tau) \\
&\quad \times \exp\left(i\int_0^1 \mathrm{d}\tau\left(-eT_1 - i\chi T_2 + p_\mu \dot{x}^\mu - \tfrac{1}{2} i\psi_n \dot{\psi}^n\right) + \tfrac{1}{2}\psi(1)\psi(0)\right), \qquad (3.22)
\end{aligned}
$$

where T_1, T_2 are determined by Eqs. (2.10), (2.11). Using the Faddeev–Popov trick for introducing the gauge-fixing terms we find

$$
\begin{aligned}
G^{\mathrm{W}}(x, y) = &\int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \int N\ \mathrm{D}p(\tau) \int_y^x \mathrm{D}x(\tau) \int_{\psi(1)+\psi(0)=2\xi} \mathrm{D}\psi(\tau) \\
&\times \exp\left(i\int_0^1 \mathrm{d}\tau\left(-\lambda T_1 - ikT_2 + p_\mu \dot{x}^\mu - \tfrac{1}{2} i\psi_n \dot{\psi}^n\right) + \tfrac{1}{2}\psi(1)\psi(0)\right).
\end{aligned}
\qquad (3.23)
$$

Eq. (3.23) coincides with Eq. (2.37), i.e. we take into account also the expression

$$
\begin{aligned}
&\int \frac{\mathrm{D}e(\tau)}{V_e} \int \frac{\mathrm{D}\chi(\tau)}{V_\chi} \int \mathrm{D}\xi(\tau)\ \delta\left(e^\xi(\tau) - \lambda\right) \int \mathrm{D}\epsilon(\tau)\ \delta\left(\chi^\epsilon(\tau) - k\right) \\
&\quad \times \Delta(e(\tau), \chi(\tau)) \Phi(e(\tau), \chi(\tau)) = \Phi(\lambda, \kappa, \ldots), \qquad (3.24)
\end{aligned}
$$

where Φ is a function invariant under reparametrisation and supertransformations

$$
e^\xi(\tau) = e(\tau) + \frac{\mathrm{d}}{\mathrm{d}\tau}(\xi e), \quad \chi^\epsilon(\tau) = \chi(\tau) + 2\dot{\epsilon}(\tau), \qquad (3.25)
$$

and

$$
\Delta^{-1}(e(\tau), \chi(\tau)) = \int \mathrm{D}\xi(\tau) \int \mathrm{D}\epsilon(\tau)\ \delta\left(\frac{\mathrm{d}}{\mathrm{d}\xi}(\xi e)\right) \delta(2\dot{\epsilon}(\tau))\big|_{e=\lambda, \chi=k}. \qquad (3.26)
$$

The analogous argument can be applied to the gauges in Eqs. (3.18), (3.19) to obtain Eqs. (2.40), (2.41).

4. BFV method

Now we would like to construct a PIR for the Dirac particle propagator starting from the general quantization method of Batalin–Fradkin–Vilkovisky, employing the WS representation. We shall not discuss the details of this formalism and restrict ourselves to the issue relevant for constructing the PIR only. The method has been developed for systems with constraints in relativistic gauges. The main idea of the method is to enlarge the phase space by introducing the new ghost degrees of freedom and to construct the generating functional in a covariant manner, then to prove the unitarity of the physical S-matrix and its independence of the choice of the gauge-fixing functions using the BRST-invariant properties. In our case there are the first and the second class constraints.

The generating functional has the form

$$Z = \text{const.} \times \int \exp\left(i \int \mathrm{d}t \; \mathrm{d}\mu(P, Q) \left[P_A \dot{Q}^A - H^c(P, Q) + J_A Q^A + P_A I^A \right] \right), \tag{4.1}$$

with the measure

$$\mathrm{d}\mu(P, Q) = M(p, q) \Pi_t \Pi_A \frac{\mathrm{d}p_A(t) \; \mathrm{d}\phi_A(t)}{2\pi}, \tag{4.2}$$

$$M(p, q) = \Pi_t \left[\Pi_a \delta(\theta_a) \right] \; \mathrm{Ber} \| \{\theta, \theta\} \|^{1/2}, \tag{4.3}$$

and the total hamiltonian H^c is defined in terms of the initial hamiltonian H_{in} as

$$H^c \approx H_{\text{in}} + \{\psi, \Omega\}_{\mathrm{D}}. \tag{4.4}$$

Here the Dirac brackets are defined with respect to the second class constraints θ_a, and $\approx$ means equality up to a linear combination of θ_a. Ω is the BRST charge of the system and has the following properties:

$$\{H_{\text{in}}, \Omega\}_{\mathrm{D}} = 0, \tag{4.5}$$

$$\{\Omega, \Omega\}_{\mathrm{D}} = 0. \tag{4.6}$$

The ψ is the so-called gauge fermion. The variables P_A, Q_A are defined as

$$P_A = \left(p_i, \bar{P}_\alpha, \pi_\alpha, \bar{C}_\alpha \right), \quad Q_\alpha = \left(q_i, C^\alpha, \lambda^\alpha, P^\alpha \right). \tag{4.7}$$

The ghost parities of the auxiliary variables are: $\epsilon(\bar{C}_\alpha) = \epsilon(p^\alpha) = \epsilon(\bar{Q}^\alpha) = \epsilon(C^\alpha) = \epsilon(T^\alpha) + 1$. The solutions of Eqs. (4.5), (4.6) are represented in the form of power series in terms of the ghost variables:

$$\Omega \approx \pi_\alpha P^\alpha + T_\alpha(p, q) C^\alpha + \bar{P}_\alpha U^\alpha_{\beta\gamma} C^\beta C^\gamma + \dots, \tag{4.8}$$

$$\psi \approx \bar{C}_\alpha X^\alpha + \bar{P}_\alpha \lambda^\alpha + \dots . \tag{4.9}$$

It is very essential that Ω does not depend on the choice of gauge. The matrix $U^\alpha_{\beta\gamma}$ is determined by the algebra

$$\{T_\alpha, T_\beta\}_{\mathrm{D}} = 2 T_\gamma U^\gamma_{\alpha\beta} (-1)^{\epsilon(T_\alpha)}. \tag{4.10}$$

In our case we have

$$\theta_\alpha \equiv \theta_n = \pi^\psi_n + \tfrac{1}{2} i \psi_n, \quad \mathrm{Ber} \| \{\theta^n, \theta^m\} \| = \text{const.}, \tag{4.11}$$

$$T^\alpha = (T_1, T_2), \quad \pi_\alpha = (\pi_e, \pi_\chi), \tag{4.12}$$

$$\lambda^\alpha = (e, i\chi), \quad q^i = (x^\mu, \psi^n), \tag{4.13}$$

$$p_i = \left(p_\mu, \pi^\psi_n \right). \tag{4.14}$$

X^α are the gauge conditions for e and χ which will be elaborated later. We would like to stress two peculiarities for spin $\frac{1}{2}$ particle. Namely, $H_{\rm in}=0$ and $U^\alpha_{\beta\gamma}$ has only one non-zero component:

$$U^1_{22} = -\tfrac{1}{4}i. \tag{4.15}$$

(See also Eqs. (2.14) and (2.19), (2.20)). Using Eqs. (4.5)–(4.15) (see also Appendix C) one can check that Eq. (4.6) is fulfilled and that

$$H^{\rm c} = \lambda^\alpha(\tau)T_\alpha(\tau) + \pi_\alpha(\tau)X^\alpha(\tau) + \bar{P}_\alpha(\tau)P^\alpha(\tau) + \tfrac{1}{2}i\bar{P}_1(\tau)C^2(\tau)\lambda^2(\tau)$$
$$+\int_0^1 \mathrm{d}\tau'\, \bar{C}_\alpha(\tau)\frac{\mathrm{d}X^\alpha(\tau)}{\mathrm{d}\lambda^\beta(\tau')}P^\alpha(\tau'). \tag{4.16}$$

In order to pass from the generating functional of Eqs. (4.1)–(4.3) to the PIR for the GF, it is necessary to put $J_A I^A = 0$, and further to define a specific time interval in the action given in Eq. (4.1); for example, $0 \leqslant \tau \leqslant 1$. The next step is to carefully extract from

$$\int \mathrm{D}\lambda^1(\tau) \equiv \int \mathrm{D}e(\tau), \quad \int \mathrm{D}\lambda^2(\tau) \equiv \int \mathrm{D}\chi(\tau) \tag{4.17}$$

the integration over global modular parameters λ and k as it was done in Section 3, and then to choose a special gauge condition. X^α (see Appendix C (C.6)) together with the definition of the normalisation constant in Eq. (4.1). After this we obtain

$$G^{\rm W}(x,y)$$
$$= \int_0^\infty \mathrm{d}\lambda \int \mathrm{d}k \int \mathrm{D}e(\tau) \int \mathrm{D}\chi(\tau)\ \tilde{N}\int \mathrm{D}p(\tau)\int_y^x \mathrm{D}x(\tau)\int_{\psi(1)+\psi(0)=2\xi}\mathrm{D}\psi(\tau)$$
$$\times \int \mathrm{D}\pi_e(\tau)\int \mathrm{D}\pi_\chi(\tau)\ \Pi_\alpha \int \mathrm{D}\bar{C}_\alpha\,\mathrm{D}P_\alpha\,\mathrm{D}\bar{P}^\alpha\,\mathrm{D}C^\alpha$$
$$\times \exp\biggl(i\int_0^1 \mathrm{d}\tau\Bigl\{-eT_1 - i\chi T_2 + p_\mu \dot{x}^\mu - \tfrac{1}{2}i\psi_n\dot{\psi}^n$$
$$+\pi_e[\lambda - e(\tau)] + \pi_\chi[-\chi(\tau)+k] - \bar{C}_\alpha(\tau)P^\alpha(\tau) - \bar{P}_\alpha(\tau)P^\alpha(\tau)$$
$$-\tfrac{1}{2}i\bar{P}_1(\tau)C^2(\tau) + \bar{P}_\alpha\dot{C}^\alpha\Bigr\} + \tfrac{1}{2}\psi_n(1)\psi^n(0)\biggr). \tag{4.18}$$

After further integration over $\bar{C}_\alpha$, $\bar{P}_\alpha$, π_α we get under integration in Eq. (4.17) the factor

$$\delta(e(\tau)-\lambda)\delta(\chi(\tau)-k)\delta(P^\alpha)\delta\bigl(P^1 + \tfrac{1}{2}C^2\chi - \dot{C}^1\bigr)\delta\bigl(p^2 - \dot{C}^2\bigr). \tag{4.19}$$

It is clear that integration over $e(\tau)$, $\chi(\tau)$, P^α, C^α gives the same PIR as in Eqs. (2.38), (2.39). Thus the generalisation of the BFV method for the case of the PIR for the GF and use of the WS representation for the vector part of the Grassmann variables leads to the same PIR as in the previous approaches.

5. Nonrelativistic Pauli particle

This case has been considered in detail in [1] where one can find further references as well. Thus we focus our attention on the new derivation of the PIR for the propagator of the Pauli particle in an external electromagnetic field, together with the additional potential $U(r)$ by the functional derivative formalism. Dynamical variables, describing the nonrelativistic spin dynamics, are elements of the Grassmann algebra G_3 with three real generators ψ_k, $k = 1, 2, 3$. If we suppose that the classical action is an even real element of G_3 then in the general case we can write

$$S = \int_0^t \mathrm{d}\tau \left(\frac{\dot{x}_k^2}{2m} - U(\boldsymbol{x}) + g\dot{x}_k A_k(\boldsymbol{x}) - \tfrac{1}{2} i \psi_i \dot{\psi}_i + \mu \epsilon_{ijk} H_i(\boldsymbol{x}) \psi_j \psi_k \right), \tag{5.1}$$

where $A(\boldsymbol{x})$ is the vector potential of the EM field; $\boldsymbol{H}$ is the magnetic field, g is the electric charge and μ is the magnetic moment. The canonical variables are defined as

$$p_i = \frac{\partial L}{\partial \dot{x}_i} = \frac{\dot{x}_i}{m} + gA_i, \tag{5.2}$$

$$\pi_i^{\psi} = \frac{\partial L}{\partial \dot{\psi}_i} = -\tfrac{1}{2} i \psi_i. \tag{5.3}$$

Eq. (5.3) leads to the second class constraints

$$\theta_i = \pi_i^{\psi} + \tfrac{1}{2} i \psi_i = 0. \tag{5.4}$$

Taking into account Eq. (5.4) for the hamiltonian of the system in Eq. (5.1), we get

$$H = -L + p\dot{x} - \pi_i \dot{\psi}_i = \tfrac{1}{2} m P_i^2 + U(\boldsymbol{x}) + \mu \epsilon_{ijk} H_i \psi_j \psi_k, \tag{5.5}$$

where $P_i = p_i - gA_i$. The quantization conditions are

$$i\{ p_i, x_j \} \to \left[\hat{p}_i, \hat{x}_j \right]_- = -i\delta_{ij}, \tag{5.6}$$

$$i\{ \psi_i, \psi_j \}_{\mathrm{D}} \to \left[\hat{\psi}_i, \hat{\psi}_j \right]_+ = \delta_{ij}. \tag{5.7}$$

The operators $\hat{\psi}_i$ are generators of the Clifford algebra C_3 with the unique finite matrix representation

$$\hat{\psi}_i = \frac{1}{\sqrt{2}} \sigma_i, \tag{5.8}$$

where σ_i are Pauli matrices.

Our aim is to construct the PIR for the transition matrix:

$$\hat{U}_{\mathrm{NR}}(x,\ y;\ t) = \langle \boldsymbol{x} | \mathrm{e}^{-i\hat{H}t} | \boldsymbol{y} \rangle, \tag{5.9}$$

where NR means "nonrelativistic".

At first, one constructs the WS for the following operator (see Appendix B):

$$\begin{aligned} \Lambda(\hat{\psi}_i) &= \tilde{T} \exp \int_0^t \mathrm{d}\tau\ J_i(\tau)\hat{\psi}_i \\ &= \exp\left(\tfrac{1}{4} \int_0^t \mathrm{d}\tau\ \mathrm{d}\tau'\ J_i(\tau)\epsilon(\tau-\tau')J_i(\tau') \right) \\ &\quad \times \exp\left(\int_0^t \mathrm{d}\tau\ J_i(\tau)\hat{\psi}_i \right), \end{aligned} \tag{5.10}$$

where J_i is the external source. From Eq. (3.11), as in the relativistic case Eq. (2.31), one can obtain the PIR for the WS of $\Lambda_{\mathrm{NR}}(\hat{\psi}_i)$:

$$\Lambda^{\mathrm{W}}_{\mathrm{NR}}(\xi_i) = c \int_{\psi_i(t)+\psi_i(0)=2\xi_i} \mathrm{D}\psi_i(\tau)\ \exp\left(\int_0^t \mathrm{d}\tau \left(\tfrac{1}{2}\psi_i\dot{\psi}_i + J_i\psi_i \right) + \tfrac{1}{2}\psi_i(t)\psi_i(0) \right). \tag{5.11}$$

Now

$$\Lambda^{\mathrm{W}}_{\mathrm{NR}}(\hat{\psi}_i) = \exp\left(\hat{\psi}_i \frac{\delta}{\delta \xi_i} \right) \Lambda^{\mathrm{W}}_{\mathrm{NR}}(\xi)|_{\xi=0}. \tag{5.12}$$

In analogy with the relativistic case (see Eq. (2.25)), one can write for $\hat{U}_{\mathrm{NR}}$ in Eq. (5.8)

$$\begin{aligned} \hat{U}_{\mathrm{NR}}(x,\ y;\ t) = \langle \boldsymbol{x} | \tilde{T} &\exp\left(-i \int_0^t \mathrm{d}\tau\ V\left(\hat{p}_i(\tau),\ \hat{x}_i(\tau),\ \hat{\psi}_i(\tau) \right) \right) \\ &\times \exp\left(-i\hat{H}_0 t \right) | \boldsymbol{y} \rangle, \end{aligned} \tag{5.13}$$

where

$$\begin{aligned} &\hat{H}_0 = \frac{\hat{p}^2}{2m},\quad \hat{V} = \hat{H} - \hat{H}_0,\quad \hat{x}_i(\tau) = \exp\left(-i\hat{H}_0\tau \right)\hat{x}_i \exp\left(i\hat{H}_0\tau \right), \\ &\hat{p}_i(\tau) = \hat{p}_i,\quad \hat{\psi}_i(\tau) = \hat{\psi}_i, \end{aligned} \tag{5.14}$$

and then

$$\hat{U}_{\mathrm{NR}}(x,\ y;\ t) = \exp\left[-i \int_0^t \mathrm{d}\tau\ V\left(i\frac{\delta}{\delta J_p},\ i\frac{\delta}{\delta J_x},\ i\frac{\delta}{\delta J_\psi} \right) \right] \hat{U}_J^{\mathrm{NR}}(x,\ y;\ t)|_{J_\alpha=0}, \tag{5.15}$$

where

$$\hat{U}_J^{\mathrm{NR}}(x, y; t) = \langle x | \tilde{T} \exp\left(-i\int_0^t \mathrm{d}\tau\left[J_p(\tau)\hat{p} + J_x(\tau)\hat{x} + iJ_i(\tau)\hat{\psi}_i\right]\right)$$
$$\times \exp\left(-i\hat{H}_0 t\right) | y\rangle. \tag{5.16}$$

Repeating the same steps as in the relativistic case, we get for the WS of $\hat{U}_{\mathrm{NR}}$

$$U_{\mathrm{NR}}^{\mathrm{W}}(\xi) = \tilde{N}\int \mathrm{D}p_i(\tau)\int_y^x \mathrm{D}x_i(\tau)\int_{\psi_i(t)+\psi_0=2\xi_i} \mathrm{D}\psi_i(\tau)$$
$$\times \exp\left[-i\int_0^t \mathrm{d}\tau\left(\frac{P_i^2}{2m} + U(x) + \mu\epsilon_{ijk}H_i\psi_j\psi_k - p\dot{x} + \tfrac{1}{2}i\psi_i\dot{\psi}^i\right)\right.$$
$$\left. + \tfrac{1}{2}\psi_i(t)\psi_i(0)\right]. \tag{5.17}$$

After integrating over $p_i(\tau)$ one finds

$$U_{\mathrm{NR}}^{\mathrm{W}}(x, y; \tau) = \tilde{N}_0\int_y^x \mathrm{D}x_i(\tau)\int_{\psi_i(t)+\psi_0=2\xi_i} \mathrm{D}\psi_i(\tau)\, \exp\left[i\int_0^t \mathrm{d}\tau\left(\frac{\dot{x}_i^2}{2m} - U(x)\right.\right.$$
$$\left.\left. + g\dot{x}_k A_k(x) + \tfrac{1}{2}i\psi_i\dot{\psi}_i + \mu\epsilon_{ijk}H_i(x)\psi_i\psi_k\right) + \tfrac{1}{2}\psi_i(t)\psi_i(0)\right], \tag{5.18}$$

and finally for $\hat{U}_{\mathrm{NR}}$ one obtains

$$\hat{U}_{\mathrm{NR}}(x, y; t) = \exp\left(\hat{\psi}_i\frac{\delta}{\delta\xi_i}\right)U_{\mathrm{NR}}^{\mathrm{W}}(x, y; t)\big|_{\xi_i=0}. \tag{5.19}$$

Now we can construct the retarded and advanced GFs of the Schrodinger equation,

$$G_{\mathrm{R,A}}(x, y; t)\cdot\left(i\frac{\overleftarrow{\partial}}{\partial t} - \hat{H}_J\right) = \delta(x-y)\delta(t) \tag{5.20}$$

as

$$G_{\mathrm{R,A}}(x, y; t) = \mp i\theta(\pm t)\hat{U}_{\mathrm{NR}}(x, y; t). \tag{5.21}$$

It is easy to write down all the analogous formulas for the case of nonrelativistic spinless point-like particles or for a system of interacting nonrelativistic particles. For this it is necessary to neglect in the previous expressions the contributions of the Grassmann variables and in the case of the many-particle system to introduce a summation over all particles.

6. Concluding remarks

We discussed three different methods for constructing the PIR for a Dirac particle propagator and have shown that all of them lead to the same result. In the canonical method, we fixed the gauge from the beginning and then applied the functional derivative formalism together with the Weyl symbol representation. In the Polyakov and BFV approaches, we started with not so well defined mathematical objects, Eqs. (3.1) and (4.1), which involved the integrals over one-dimensional metrics. Then, to overcome these shortcomings, we introduced the invariant lengths (modular parameters), the WS representation for Grassmann variables and went into details about the structure of the group measures. Furthermore, starting from the PIR for the GF it is easy to prove, by the functional derivative formalism, that they satisfy the same equation for the GF as in the canonical case.

Analogous results for the relativistic scalar particle propagator can be obtained if we neglect the contributions of the variables involving spin.

The main advantage of the proposed approach is that the construction of the PIR for the GF of the particles with any local interaction reduces to the construction of the PIR for the simplest transition matrix $\hat{U}_J(x, y; t)$ with the linear dependence on the particle operators in the exponent.

The use of the WS representation for the Grassmann vector part of the variables significantly simplifies the construction of the PIR for the propagator of any spinning point-like particle and automatically leads to a precise and unambiguous form of the boundary conditions for the anticommuting variables and puts strong restrictions on the choice of the gauge conditions.

The important conclusion is the understanding of the gauge-invariant nature of the Green function in terms of the canonical variables, under $e(\tau)$ and $\chi(\tau)$ transformations.

The analysis using Polyakov and BFV methods leads to the conclusion that there exist only two possibilities for gauge choices, depending on the definition of the invariant lengths λ and k.

We found that for the Dirac particle problem the first class constraints form a non-trivial algebra which is a special case of the superconformal algebra for Ramond open fermi string theory when the width of the string goes to zero (see [18]). This algebra leads to the appearance in the BFV methods of the third degree terms involving ghost fields in the BRST charge Ω.

The use of the hamiltonian (phase space) representation for the PIR for a propagator of any spinning point-like particle enables us to avoid the appearance of the ill-defined variables of the type $\dot{x}_\mu$ in the PIR.

Finally, the proposed approach can be applied to any point-like nonrelativistic particle as well, as is demonstrated in Section 5.

Appendix A

In this appendix we will prove that $\hat{U}_j(x, y; t)$ in Eq. (2.29) has the PIR given in Eqs. (2.34) and (2.35), without discretisation of the time interval. Since the $\hat{\psi}$ commutes with $\hat{p}_\mu$ and $\hat{x}_\mu$ one can write [2]

$$\hat{U}_j(x, y; t) = U_{J_{pq}}(x, y; t)\Lambda(\hat{\psi}), \tag{A.1}$$

where

$$U_{J_{pq}}(x, y; t) = \langle x | \tilde{T} \exp\left(-i\int_0^t \mathrm{d}\tau\left[J_p(\tau)\hat{p} + J_x(\tau)\hat{x}(\tau)\right]\right) \times \exp\left(-i\hat{H}_0 t\right) | y\rangle. \tag{A.2}$$

Here we will show that

$$U_{J_{pq}}(x, y; t) = \tilde{N}\int \mathrm{D}p(\tau)\int_y^x \mathrm{D}x(\tau) \times \exp\left(-i\int_0^t \mathrm{d}\tau\left(\tfrac{1}{2}p^2 - p\dot{x} + J_p p + J_x x\right)\right), \tag{A.3}$$

where the constant $\tilde{N}$ is defined in Eq. (A.9), leaving the proof concerning $\Lambda(\hat{\psi})$ to Appendix B.

We introduce the new variables, via $\tilde{x}(\tau) = x(\tau) - \varphi(\tau)$, $\varphi(\tau) = \int_0^\tau \mathrm{d}\tau' J_p(\tau')$. Then Eq. (A.3) reads (we have dropped the $\sim$ on x for notational simplicity)

$$\begin{aligned} U_{J_{pq}}(x, y; t) &= \tilde{N}\int \mathrm{D}p(\tau)\int_y^{x+\varphi(t)} \mathrm{D}x(\tau) \\ &\quad\times \exp\left(-i\int_0^t \mathrm{d}\tau\left[\tfrac{1}{2}p^2 - p\dot{x} + J_x(x+\varphi)\right]\right) \\ &= \exp\left(-i\int_0^t \mathrm{d}\tau\, J(\tau)\varphi(\tau)\right)\tilde{N}\int \mathrm{D}p(\tau)\int_y^{x+\varphi(t)} \mathrm{D}x(\tau) \\ &\quad\times \exp\left(-i\int_0^t \mathrm{d}\tau\left(\tfrac{1}{2}p^2 - p\dot{x} + J_x x\right)\right) \\ &= \exp\left(-i\int_0^t \mathrm{d}\tau\, J(\tau)\varphi(\tau)\right)\langle x + \varphi(t) | \tilde{T} \\ &\quad\times \exp\left(-i\int_0^t \mathrm{d}\tau\, J(\tau)\hat{x}(\tau)\right)\exp\left(-i\hat{H}_0 t\right)| y\rangle. \end{aligned} \tag{A.4}$$

[2] From the definition of $U_{J_{pq}}$ one can show that one has two different representations:

$$\begin{aligned} U_{J_{pq}}(x, y; t) &= \langle x | \exp(-i\hat{H}_0 t) T \exp\left(-i\int_0^t \mathrm{d}\tau\left[J_p\hat{p} + J_x\underline{\hat{x}}(\tau)\right]\right)| y\rangle \\ &= \langle x | \tilde{T}\exp\left(-i\int_0^t \mathrm{d}\tau\left[J_p\hat{p} + J_x\hat{x}(\tau)\right]\right)\exp\left(-i\hat{H}_0 t\right)| y\rangle, \end{aligned}$$

where $\hat{x}(\tau) = \exp(i\hat{H}_0 t)\hat{x}\exp(-i\hat{H}_0 t)$, $\underline{\hat{x}}(\tau) = \exp(-i\hat{H}_0 t)\hat{x}\exp(i\hat{H}_0 t)$.

Thus the PIR for $U_{J_{pq}}$ reduces to a simpler form. We are still left with the task of proving the following equation:

$$U_{J_x}(x,\,y;\,t)=\langle x\,|\,\hat{T}\,\exp\Big(-i\int_0^t \mathrm{d}\tau\,J_x(\tau)\,\hat{x}(\tau)\Big)\,\exp\big(-i\hat{H}_0 t\big)\,|\,y\rangle$$
$$=\tilde{N}\int \mathrm{D}p(\tau)\int_y^x \mathrm{D}x(\tau)\,\exp\Big(-i\int_0^t \mathrm{d}\tau\big(\tfrac{1}{2}p^2-p\dot{x}+J_x x\big)\Big). \qquad \text{(A.5)}$$

To prove this, we carry out the integration over $p(\tau)$ in Eq. (A.5) and get

$$U_{J_x}=\tilde{N}\int_y^x \mathrm{D}x(\tau)\,\exp\Big(i\int_0^t \mathrm{d}\tau\big[\tfrac{1}{2}\dot{x}^2-J(\tau)\,x(\tau)\big]\Big). \qquad \text{(A.6)}$$

The solution to Eq. (A.6) is

$$U_{J_x}(x,\,y;\,t)=\tilde{N}\int \mathrm{D}z(\tau)\,\exp\Big(i\int_0^t \mathrm{d}\tau\big(\tfrac{1}{2}\dot{z}^2\big)\Big)\,\exp\big[iS_{\mathrm{cl}}(J,\,x,\,y)\big], \qquad \text{(A.7)}$$

where $z(\tau)=z(0)=0$, and

$$S_{\mathrm{cl}}=\int_0^t \mathrm{d}\tau\big(\tfrac{1}{2}\dot{x}_{\mathrm{cl}}^2-J_x x_{\mathrm{cl}}\big).$$

We can get the following equation from S_{cl}:

$$\dot{x}_{\mathrm{cl}}(\tau)=-J_x(\tau),\quad x_{\mathrm{cl}}(0)=y,\quad x_{\mathrm{cl}}(t)=x. \qquad \text{(A.8)}$$

The solution to Eq. (A.8) is

$$x_{\mathrm{cl}}(\tau)=-\phi(\tau)+\alpha\tau+y,$$
$$\alpha=\big[(x-y)-\phi(t)\big]t^{-1},\quad \phi(\tau)=\int_0^\tau \mathrm{d}\tau'\int_0^{\tau'}\mathrm{d}\tau''\,J(\tau'')$$

The constant $\tilde{N}$ is determined in the following way:

$$\tilde{N}\int \mathrm{D}z(\tau)\,\exp\Big(i\int_0^t \mathrm{d}\tau\;\tfrac{1}{2}\dot{z}^2\Big)=\tilde{N}\big[\det\big(-\partial_\tau^2\big)\big]^{-2}=2\pi t^{-2}, \qquad \text{(A.9)}$$

after using, for example, the ξ-regularization method. Now it can be shown that Eq. (A.7) satisfies the following equations:

$$\Big(i\frac{\partial}{\partial t}+\frac{1}{2}\frac{\partial^2}{\partial x^2}+J_x(t)\,x\Big)U_{J_x}(x,\,y;\,t)=0$$

or

$$\Big(i\frac{\partial}{\partial t}+\frac{1}{2}\frac{\partial^2}{\partial y^2}+J_x(t)\,y\Big)U_{J_x}(x,\,y;\,t)=0,$$

with the initial condition

$$U_{J_x}(x,\,y;\,t=0)=\delta^4(x-y),$$

which coincides with Eq. (A.5).

Appendix B

In this appendix we would like to demonstrate that Eqs. (2.31), (2.32) coincide with Eq. (2.30). We first note that

$$\frac{\partial \Lambda(\hat{\psi})}{\partial t} \equiv \dot{\Lambda} = \Lambda J^n_{\psi}(t)\hat{\psi}_n = \Lambda \rho^n \hat{\psi}_n \tag{B.1}$$

and look for a solution in the following form:

$$\Lambda = f(t)\,\exp\left(\int_0^t \mathrm{d}\tau\; \rho^n(\tau)\hat{\psi}_n\right). \tag{B.2}$$

From Eqs. (B.1), (B.2) we find

$$\dot{f} = -\tfrac{1}{2}\int_0^t \mathrm{d}\tau\; \rho^n(\tau)\rho_n(\tau)$$

and

$$f = \exp\left(\int_0^t \mathrm{d}\tau \int_0^t \mathrm{d}\tau'\; \rho^n(\tau)\epsilon(\tau-\tau')\rho_n(\tau')\right).$$

Thus

$$\Lambda = \exp\left(\tfrac{1}{4}\int_0^t \mathrm{d}\tau \int_0^t \mathrm{d}\tau'\; \rho^n(\tau)\epsilon(\tau-\tau')\rho_n(\tau')\right)\exp\left(\int_0^t \mathrm{d}\tau\; \rho^n(\tau)\hat{\psi}_n\right). \tag{B.3}$$

It is easy to show that

$$\begin{aligned}\Lambda &= \exp\left(\tfrac{1}{4}\int_0^t \mathrm{d}\tau \int_0^t \mathrm{d}\tau'\; \rho^n(\tau)\epsilon(\tau-\tau')\rho_n(\tau')\right)\\ &= C\int_{\varphi(t)+\varphi(0)=0} \mathrm{D}\varphi(\tau)\,\exp\left(\int_0^t \mathrm{d}\tau\left[\tfrac{1}{2}\varphi_n(\tau)\dot{\varphi}^n(\tau) + \rho_n(\tau)\varphi^n(\tau)\right]\right),\end{aligned} \tag{B.4}$$

where

$$C^{-1} = \int \mathrm{D}\varphi(\tau)\,\exp\left(\int_0^t \mathrm{d}\tau\,\tfrac{1}{2}\varphi\dot{\varphi}\right)$$

and is independent of t. Consequently from Eqs. (B.3) and (B.4) we have

$$\Lambda^{\mathrm{W}}(\xi) = C\int_{\varphi(t)+\varphi(0)=0} \mathrm{D}\varphi\,\exp\left(\int_0^t \mathrm{d}\tau\left[\tfrac{1}{2}\varphi\dot{\varphi} + \rho(\varphi+\xi)\right]\right).$$

After a change of variables $\varphi + \xi = \psi$ we get

$$\begin{aligned}\Lambda^{\mathrm{W}}(\xi) &= C\int_{\psi(t)+\psi(0)=2\xi} \mathrm{D}\psi\,\exp\left(\int_0^1 \mathrm{d}\tau\left[\tfrac{1}{2}(\psi-\xi)\dot{\psi} + \rho\psi\right]\right)\\ &= C\int_{\psi(t)+\psi(0)=2\xi} \mathrm{D}\psi\,\exp\left(\int_0^1 \mathrm{d}\tau\left(\tfrac{1}{2}\psi\dot{\psi} + \rho\psi\right) + \tfrac{1}{2}\psi(1)\psi(0)\right),\end{aligned} \tag{B.5}$$

which coincides with Eq. (2.31) since $\rho^n \equiv = J^n_\psi$. If we have started with the T-product, instead of the $\tilde{T}$-product, for $\lambda(\hat{\psi})$ in Eq. (2.30) we would have gotten

$$\Lambda^{\mathrm{W}} = C\int_{\psi(t)+\psi(0)=2\xi} \mathrm{D}\psi\ \exp\left(\int_0^t \mathrm{d}\tau\left(-\tfrac{1}{2}\psi\dot{\psi} + \rho\psi\right) - \tfrac{1}{2}\psi(1)\psi(0)\right). \tag{B.6}$$

Appendix C

In this appendix we will provide the details of Eqs. (4.4)–(4.6). First note that since $H_{\mathrm{in}} = 0$, we have

$$H^{\mathrm{c}} = \{\psi, \Omega\}_{\mathrm{D}}, \tag{C.1}$$

where

$$\psi = \overline{C}_\alpha X^\alpha + \overline{P}_\alpha \lambda^\alpha \tag{C.2}$$

and

$$\Omega = \int_0^1 \mathrm{d}\tau\left(\pi_\alpha P^\alpha + \tau_\alpha C^\alpha + \tfrac{1}{2}\overline{P}_\alpha U^\alpha_{\beta\gamma} C^\beta C^\gamma\right). \tag{C.3}$$

Using

$$\left\{T_\alpha, T_\beta\right\}_{\mathrm{D}} = T_\gamma U^\gamma_{\alpha\beta}(-1)^{\epsilon(T_\alpha)} \tag{C.4}$$

and Eq. (4.15), we can compute

$$\begin{aligned}\{\Omega, \Omega\}_{\mathrm{D}} &= \{T_2, T_2\}_{\mathrm{D}}(C^2)^2 + \tfrac{1}{4}i\left\{T_1 C^1, \overline{P}_1(C^2)^2\right\}_{\mathrm{D}} + \tfrac{1}{4}i\left\{\overline{P}_1(C^2)^2, T_1 C^1\right\}_{\mathrm{D}} \\ &= -\tfrac{1}{2}iT_1(C^2)^2 + \tfrac{1}{2}iT_1(C^2)^2 = 0.\end{aligned}$$

Here C^2 is even and C^1 and $\overline{P}_1$ are odd variables. Next from Eqs. (C.1)–(C.3), we find

$$\begin{aligned}H^{\mathrm{c}}(\tau) &= \int_0^1 \mathrm{d}\tau'\left\{\overline{C}_\alpha(\tau)X^\alpha(\tau) + \overline{P}_\alpha(\tau)\lambda^\alpha(\tau), \pi_\alpha(\tau')P^\alpha(\tau')\right. \\ &\quad \left. + T_\alpha(\tau')C^\alpha(\tau') + \tfrac{1}{2}\overline{P}_\alpha(\tau')U^\alpha_{\beta\gamma}C^\beta C^\gamma\right\}_{\mathrm{D}} \\ &= \int_0^1 \mathrm{d}\tau'\left(\{\overline{C}_\alpha X^\alpha, \pi_\alpha P^\alpha\}_{\mathrm{D}} + \overline{P}_\alpha\{\lambda^\alpha, \pi_\alpha\}_{\mathrm{D}}P^\alpha + \overline{C}_\alpha\{X^\alpha, T_\beta\}_{\mathrm{D}}C^\beta\right. \\ &\quad \left. + \lambda^\alpha T_\alpha\{\overline{P}_\alpha, C^\alpha\}_{\mathrm{D}} - \tfrac{1}{2}\overline{P}_1\left\{\overline{P}_2, (C^2)^2\right\}_{\mathrm{D}} U^1_{22}\lambda^2\right) \\ &= \int_0^1 \mathrm{d}\tau'\left(\overline{C}_\alpha(\tau)\frac{\partial X^\alpha(\tau)}{\partial\lambda^\beta(\tau')}P^\alpha(\tau') + \overline{P}_\alpha(\tau)P^\alpha(\tau)\right. \\ &\quad \left. + \lambda^\alpha(\tau)T_\alpha(\tau)\pi_\alpha(\tau)X^\alpha(\tau) + \overline{P}_1(\tau)C^2(\tau)\chi(\tau)\right).\end{aligned} \tag{C.5}$$

Above we have chosen X^α in such a way that $\{X^\alpha, T_\beta\}_{\mathrm{D}} = 0$ is satisfied.

For simplicity we consider only the contribution to the PIR for the GF in Eqs. (4.1) and (4.3) from all ghost fields and $e(\tau)$, $\chi(\tau)$. Taking into account Eq. (C.5), we get

$$\prod_{\alpha=1}^{2}\int \mathrm{D}\lambda^\alpha\,\mathrm{D}\pi_\alpha\,\mathrm{D}\bar{P}_\alpha\,\mathrm{D}C^\alpha\,\mathrm{D}\bar{C}_\alpha\,\mathrm{D}P_\alpha\,\exp\Bigg[i\int_0^1\mathrm{d}\tau\Bigg(-e(\tau)T_1-i\chi(\tau)T_2$$
$$+\pi_\alpha\dot{\lambda}^\alpha+\bar{C}_\alpha\dot{P}^\alpha+\bar{P}_\alpha\dot{C}^\alpha+\bar{P}_\alpha P^\alpha-\pi_\alpha X^\alpha-\bar{P}_1C^2\chi(\tau)$$
$$-\int_0^1\mathrm{d}\tau'\,\bar{C}_\alpha(\tau)\frac{\partial X^\alpha(\tau)}{\partial\lambda^\beta(\tau')}P_\beta(\tau')\Bigg)\Bigg]. \tag{C.6}$$

As we have shown in Section 3 in order to get the PIR for the GF from the general expression in Eqs. (4.1)–(4.3) it is necessary to choose the special gauge conditions of the type in Eqs. (3.16) and (3.17). From Eq. (C.6) it follows that we must set

$$X^1(\tau)=e(\tau)-\lambda+\dot{e}(\tau),$$
$$X^2(\tau)=\chi(\tau)-\kappa+\dot{\chi}(\tau). \tag{C.7}$$

Then, in Eq. (C.6) we have

$$\bar{C}_\alpha\dot{P}^\alpha-\int_0^1\mathrm{d}\tau'\,\bar{C}_\alpha(\tau)\frac{\partial X^\alpha(\tau)}{\partial\lambda^\beta(\tau')}P_\beta(\tau')=-\bar{C}_\alpha(\tau)P^\alpha(\tau). \tag{C.8}$$

Thus integration in Eq. (C.6) over $\pi_\alpha(\tau)$, $\bar{P}^\alpha(\tau)$, $\bar{C}^\alpha(\tau)$ gives

$$\prod_{\alpha=1}^{2}\int \mathrm{D}e(\tau)\,\mathrm{D}\chi(\tau)\,\mathrm{D}P^\alpha(\tau)\,\mathrm{D}C^\alpha(\tau)\,\delta(P^\alpha)\delta(e(\tau)-\lambda)\delta(\chi(\tau)-\kappa)$$
$$\times\delta(\dot{C}^1-C^2\chi)\delta(\dot{c}^2)\,\exp\left(-i\int_0^1\mathrm{d}\tau[e(\tau)T_1+i\chi(\tau)T_2]\right)$$
$$=\text{const.}\times\exp\left(i\int_0^1\mathrm{d}\tau(\lambda T_1+i\kappa T_2)\right). \tag{C.9}$$

Taking into account the total expression for $G^{\mathrm{W}}(x, y)$ we get the same PIR as in Eqs. (2.38), (2.39). Another choice of the time interval in Eq. (4.1), $\lambda\leqslant t\leqslant 0$, and the special gauge condition given in Eq. (3.19) leads to the same PIR for $G^{\mathrm{W}}(x, y)$ in Eq. (2.40).

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