A simple example of Kalman filter for independent tracker movements in 2D.

The Kalman filter, under its original form, recursively filters a linear dynamic system under Gaussian noise and is a nonrobust, minimum mean-square error filter. You will apply two identical one-dimensional Kalman filters. One for the horizontal movements and one for the vertical movements. For example, in the horizontal movements the y_k and $\dot{y}_k$ together give $\mathbf{x}_k$, the state variable vector along the horizontal. The measurements are the scalar z_k.

The state equation of the filter is

$$\mathbf{x}_k = \begin{pmatrix} y_k \\ \dot{y}_k \end{pmatrix} = \mathbf{F} \begin{pmatrix} y_{k-1} \\ \dot{y}_{k-1} \end{pmatrix} + \mathbf{w}_k \ .$$

A constant-velocity state transition model is applied to the previous state $\mathbf{x}_{k-1}$

$$\mathbf{F} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix} \qquad system\ matrix.$$

The Gaussian noise will be constant all the frames. We take $\sigma = 5$ and thus $\mathbf{w}_k \sim \mathbf{N}(\mathbf{0},\ 5^2\mathbf{I}_2) = \mathbf{N}(\mathbf{0},\ \mathbf{Q})$.

The scalar measurement equation applied to the current state vector

$$z_k = \mathbf{H} \begin{pmatrix} y_k \\ \dot{y}_k \end{pmatrix} + v_k$$

The transpose vector is

$$\mathbf{H} = [\,1\ 0\,] \qquad measurement\ matrix$$

measured under Gaussian noise $v_k \sim \mathbf{N}(0,\ \sigma_k^2)$. The value of σ_k is inferred from the k-frame after convergence of the mean shift. A Gaussian fit to the local peak and the two points at half the target dimension give the standard deviation. See Section 6.2 in the paper. The value $R_k = \sigma_k$ is a scalar.

The Kalman filter first does *prediction* followed by the *update* of all parameters.

Predict

The state prediction. $\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1}$

The covariance estimate prediction. $\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1}\mathbf{F}^\top + \mathbf{Q}$

Update

The optimal Kalman gain is the minimum of $\mathrm{E}[(\mathbf{x}_k - \hat{\mathbf{x}}_k)^2]$ which is equivalent minimizing the trace of $\mathbf{P}_k = cov(\mathbf{x}_k - \hat{\mathbf{x}}_k)$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}^\top(\mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^\top + R_k)^{-1} \,.$$

The state estimate update.

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(z_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1})$$

The covariance update.

$$\mathbf{P}_k = (\mathbf{I}_2 - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k|k-1}$$

The Fig.10 and Fig. 12 in the paper, the green crosses show the uncertainty, the two standard deviations, for the horizontal and vertical movements.