P170B115 Numerical Methods and Algorithms
Task of Project II

General requirements

- Report must be uploaded to the Moodle till the defense day. In the report the student should provide task, results, segments of the program code which implements the algorithm.
- In all cases at the defense the student can use their own written programs as support material.
- During defense the student must be able to explain any line of the program source code. Otherwise, the work is evaluated as 0.
- The program code should be implemented in any language (C#, JAVA, …), but **python** is preferred.

1 Linear equation systems

Write program which solves system of linear equations (Gaussian algorithm for even task numbers / Reflection for odd). The program must work without crashes (errors or exceptions) with any equation provided in table 1. If it has infinite number of solutions, at least one of them should be provided. If there are no solutions, program must identify this case and output the message about situation. In the report provide algorithm description and at least 5 random systems of linear equations from table 1 solved including obtained results and its validation. **(2 points)**

2 Non-linear equation system

Graphically solve system of non-linear equations provided in table 2. In different graphics represent the surfaces $Z_1(x_1, x_2)$ and $Z_2(x_1, x_2)$. Solve the system of non-linear equations graphically and using Newtons (for odd task numbers) or Quasi-Newton (for even task numbers). Validate obtained results using external sources. (**2 points**)

3 Optimization

According to the task provided in Table 3, create the objective function which represents the goodness of the solution and solve it using gradient descend (even task numbers) or steepest gradient descend method (odd task numbers). Graphically visualize (animate point movements) optimization process of given and added points represented by different colors. In the report provide initial and final point configurations, analytical expression of objective function and graph how objective function depends on iteration. (**~6 points**)

NOTE: Part 3 (Optimization) can be completed by a group of 1-4 students. If this part is planned to be completed by a group, please notify the teacher at least 2 weeks before the defense date. Send one email with a list of all students in the group, and CC the email to all group members. Additionally, ensure that all students in the group are present during the defense.

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1 table. Systems of linear equations

Nr.	Eq. system	Nr.	Eq. system
1	$\begin{cases} x_1 - 2x_2 + 3\,x_3 + 4\,x_4 = 11 \\ x_1 - x_3 + x_4 = -4 \\ 2x_1 - 2x_2 + 2\,x_3 + 5x_4 = 7 \\ -7x_2 + 3\,x_3 + x_4 = 2 \end{cases}$	2	$\begin{cases} 3x_1 + 7x_2 + x_3 + 3\,x_4 = 37 \\ x_1 - 6x_2 + 6x_3 + 9x_4 = 11 \\ 4x_1 + 4x_2 - 7\,x_3 + x_4 = 38 \\ -x_1 + 3x_2 + 8x_3 + 2x_4 = -1 \end{cases}$
3	$\begin{cases} x_2 + 2x_3 + x_4 = 2 \\ 6x_1 - 2x_2 + 3x_3 + 4x_4 = -15 \\ 3x_2 + 4\,x_3 - 3\,x_4 = 10 \\ -4x_2 + 3\,x_3 + x_4 = -2 \end{cases}$	4	$\begin{cases} 3x_1 + 7x_2 + x_3 + 3\,x_4 = 40 \\ x_1 - 6x_2 + 6x_3 + 8\,x_4 = 19 \\ 4x_1 + 4x_2 - 7\,x_3 + x_4 = 36 \\ 4x_1 + 16x_2 + 2x_3 = 48 \end{cases}$
5	$\begin{cases} x_1 + 2x_2 + x_3 = -4 \\ 2x_1 + 5x_2 + 4\,x_4 = 3 \\ 14x_1 - 8x_2 + 4x_3 + x_4 = 7 \\ 4x_1 + 10x_2 + 8\,x_4 = 2 \end{cases}$	6	$\begin{cases} 3x_1 + x_2 - x_3 + 5x_4 = 20 \\ -3x_1 + 4x_2 - 8\,x_3 - x_4 = -36 \\ x_1 - 3x_2 + 7\,x_3 + 6x_4 = 41 \\ 5x_2 - 9\,x_3 + 4x_4 = -16 \end{cases}$
7	$\begin{cases} 2x_1 + 5x_2 + x_3 + 2x_4 = 14 \\ -2x_1 + 3\,x_3 + 5\,x_4 = 10 \\ x_1 - x_3 + x_4 = 4 \\ -3x_1 - 4x_2 + x_3 + x_4 = -6 \end{cases}$	8	$\begin{cases} 4x_1 + 12x_2 + x_3 + 7x_4 = 171 \\ 2x_1 + 6x_2 + 17\,x_3 + 2\,x_4 = 75 \\ 2x_1 + x_2 + 5\,x_3 + x_4 = 30 \\ 5x_1 + 11x_2 + 7x_3 = 50 \end{cases}$
9	$\begin{cases} 3x_1 + 7x_2 + x_3 + 3x_4 = 11 \\ x_1 - 6x_2 + 6\,x_3 + 8x_4 = 3 \\ 4x_1 + 4x_2 - 7\,x_3 + x_4 = 1 \\ -x_1 + 3x_2 + 8x_3 + 2\,x_4 = 1 \end{cases}$	10	$\begin{cases} 4x_1 + x_2 + x_3 + 7x_4 = 148 \\ x_1 + 2\,x_3 - 2\,x_4 = -37 \\ 2x_1 + 2x_2 - 7\,x_3 + x_4 = 21 \\ 4x_1 + 14x_2 + 7\,x_3 = 53 \end{cases}$

Table 2. Systems of non-linear equations

Task number	Eq. system
1	$\begin{cases} (x_1-1)^2 + x_2 - 10 = 0 \\ x_1^2 + x_2^2 + 4x_1\cos(x_1+x_2) - 40 = 0 \end{cases}$
2	$\begin{cases} x_1^2 + 2(x_2 - \cos(x_1))^2 - 20 = 0 \\ x_1^2 x_2 - 2 = 0 \end{cases}$
3	$\begin{cases} \left(\frac{x_1}{4}\right)^4 + \left(\frac{x_2}{4}\right)^4 - \left(\left(\frac{x_1}{2}\right)^2 + \left(\frac{x_2}{2}\right)^2\right) + 5 = 0 \\ x_1^2 + x_2^2 + x_1x_2 - 8(x_1+x_2) - 4 = 0 \end{cases}$
4	$\begin{cases} x_2\sin\left(\frac{x_1}{2}\right) - 0.1 = 0 \\ x_1^2 + \left(\frac{x_2}{4}\right)^4 - 12 = 0 \end{cases}$
5	$\begin{cases} -\frac{5x_2}{x_1^2+1} + x_2^2 - x_1^2 = 0 \\ x_1^2 + x_2^2 - 12 = 0 \end{cases}$
6	$\begin{cases} 2\sin(x_1) + x_1 + x_2 = 0 \\ 4\cos(2x_2) - x_2 + 0.5x_1 = 0 \end{cases}$
7	$\begin{cases} (x_1-3)^2 + x_2 - 8 = 0 \\ \frac{x_1^2+x_2^2}{2} - 6(\cos(x_1) + \cos(x_2)) - 10 = 0 \end{cases}$
8	$\begin{cases} x_1^2 + \frac{x_2^2}{2} + 4x_1\cos(x_2-1) - 4 = 0 \\ 5\sin(x_1) + x_2 + x_1 = 0 \end{cases}$
9	$\begin{cases} 10\sin(x_1)\cos\left(\frac{x_2}{2}\right) = 0 \\ \frac{x_1^2}{4} + \frac{x_2^2}{2} - 4 = 0 \end{cases}$
10	$\begin{cases} x_2^2 - x_1^2 - 5x_1\cos(x_2+1) - 10 = 0 \\ x_1^2 + x_2^2 + x_1x_2 - 20 = 0 \end{cases}$
11	$\begin{cases} x_2^2 + x_1^2 + 100e^{-x_1^2} - 20 = 0 \\ 2\cos(x_2) - x_2 + x_1 + 1 = 0 \end{cases}$
12	$\begin{cases} x_1^2 + 10(\sin(x_1) + \cos(x_2))^2 - 10 = 0 \\ (x_2-3)^2 + x_1 - 8 = 0 \end{cases}$
13	$\begin{cases} \sin^3\left(\frac{x_1}{2}\right) + \cos^2\left(\frac{x_2}{2}\right) - 0.5 = 0 \\ (x_2-3)^2 + x_1^2 + x_1x_2 - 4 = 0 \end{cases}$
14	$\begin{cases} x_1(x_2 + 2\cos(x_1)) - 1 = 0 \\ x_1^4 + x_2^4 - 64 = 0 \end{cases}$
15	$\begin{cases} \frac{10x_1}{x_2^2+1} + x_1^2 - x_2^2 = 0 \\ x_1^2 + 2x_2^2 - 32 = 0 \end{cases}$
16	$\begin{cases} \frac{x_1^2+x_2^2}{5} - 2\cos\left(\frac{x_1}{2}\right) - 6\cos(x_2) - 8 = 0 \\ \left(\frac{x_1}{2}\right)^5 + \left(\frac{x_2}{2}\right)^4 - 4 = 0 \end{cases}$
17	$\begin{cases} \frac{x_1^2+x_2^2}{5} - 4\sin(2x_1) - 4 = 0 \\ \frac{100}{x_1^2+x_2^2+5} - x_1 - x_2 = 0 \end{cases}$

Table 3. Optimization problem

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Optimization problem
Placement of the Recycling Containers. The proper placement of the recycling containers leads to the increased quantity of the recycled materials (paper, glass, plastic) and therefore contributes to circular economy. However, it is rather difficult to find the proper placement for the recycling spots, considering the network of the recycling spots, the population density, the waste collection issues (especially in the old town district with narrow streets and high urbanization level). Help city policy makers to decide on where **10** new recycling containers should be installed. Create the target function and find the best coordinates for the new containers using gradient optimization. Provide comments on whether the obtained places are good suggestions. If possible, visit at least one suggested place and evaluate the possibilities to install recycling containers there (it is suggested to take a photo of the environment to prove later discussion). What difficulties could the policy makers approach following Your suggestion? In the file **Annex.docx** You will find the coordinates X and Y of currently existing recycling containers (paper, glass, plastic). You **must** also use additional demographic and economic parameters, for example, population density, average income in the area, and others. You can find grid with the population and housing censuses here: https://open-data-ls-osp-sdg.hub.arcgis.com/datasets/a5df434c024e4bde8472c5ffcb781fa8_0/explore?location=54.958145%2C23.932964%2C11.04 The finest grid (100 m) of data is recommended although coarse grids (500 m, 1 km) are also available. You can find example on how to read the downloaded files in Data.zip.