

# Accurate Fast Heartrate Detection Based on Fourier Bessel Series Expansion During Radar-based Sleep Monitoring

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**Abstract**—Continuous fast heart rate (HR) detection can provide heart rate variability (HRV), which indicates the autonomous activity system and has been found to change during different sleep stages. However, the accurate fast HR measurement is still challenging due to the frequency resolution limitation and high algorithm complexity of existing methods. In this paper, a novel respiration-reduced Fourier Bessel series expansion (FBSE) technique is proposed to realize fast detection of HR without filtering process using short-time (less than 5s) window length. It is theoretically illustrated that the proposed method has better spectrum resolution. The simulation results also show the proposed method has accurate spectrum representation of heart motion signal. With a custom-designed 24GHz Doppler radar, the overnight sleep experiment was carried out under the clinical standard. The results show the HRV obtained by proposed technique has the good correspondence with the polysomnography signal and has the potential on the future automated HRV-based sleep stage classification.

**Keywords**—Doppler radar sensor, Fourier-Bessel series expansion (FBSE), heartrate, heartrate variability (HRV), polysomnography (PSG)

## I. INTRODUCTION

Non-contact vital signs monitoring based on radar system have been widely discussed in the recent year. With the advantages of penetrability and non-contact properties, more and more millimeter-wave radar sensors have been used in daily life monitoring of vital signs in recent years. Especially for sleep scenes, radar does not invade privacy, and can directly penetrate the quilt to detect human vital signs, making radar an ideal solution for daily sleep monitoring.

The accurate vital signs detection based on radar relies on the chest-wall motion recovery from the back-scattered signal. The displacement of the thorax is the sum of the displacement of the chest caused by breathing and heartbeat. And heart rate (HR) can be estimated in the frequency domain of the recovered chest-wall motion signal. For healthcare applications, the fast detection of HR is the basic of accurate frequency-based heartrate variability (HRV) detection, which is highly desired for the sleep staging and emotion/stress detection [1].

Fast heart rate acquisition requires a time window of less than 5 seconds, so that changes in heart rate can be detected in time. Due to the nonlinearity of the hardware and the characteristics of the respiratory movement itself, there may be high-order harmonics in the respiratory signal, and some

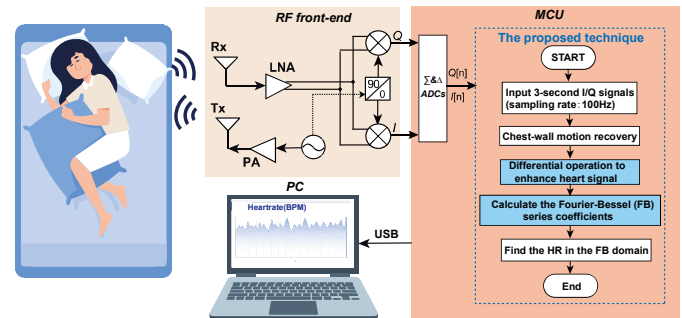


Fig. 1. Sleep monitor system using proposed technique with Doppler radar. The blue zones of flow chart indicate the core part of the proposed technique.

harmonics' frequency will occur near the heart rate frequency, which interferes with the estimation of the heart rate, especially in the condition with a poor frequency resolution. Many current methods using the Fourier transform (FT)-based algorithm require a long time-window to achieve a suitable spectrum resolution. In this case, the heart rate value obtained is an average value within the long-windowed signal, which cannot meet the requirements of accurate HRV detection.

Recently, the wavelet transform (WT)-based technique was reported to extract HR with 2-5s data length [2], which needs the selection of basis function and effective support in advance. The discrete cosine transform (DCT)-based technique is also reported to obtain the 2 times better than FT-based methods in frequency resolution and achieve to measure the HR with the less than 3-s window, which chose the cosine function as the base function [3].

In this paper, we proposed an effective technique to enhance the heart signal and improve the frequency resolution based on the Fourier Bessel series expansion (FBSE) during radar-based HR monitoring. With no need to set parameters based on prior knowledge, the FBSE technique uses Bessel functions as bases, also can achieve the compact frequency representation as compared to DCT-based representation. Moreover, due to the asymmetric and time-varying characteristics of the Bessel function itself, the simulation results show the proposed technique can obtain more accurate heart rate estimation in a short time window. With a custom-designed 24GHz Doppler radar, the overnight sleep experimental results show the HRV analysis results obtained by proposed technique has the good

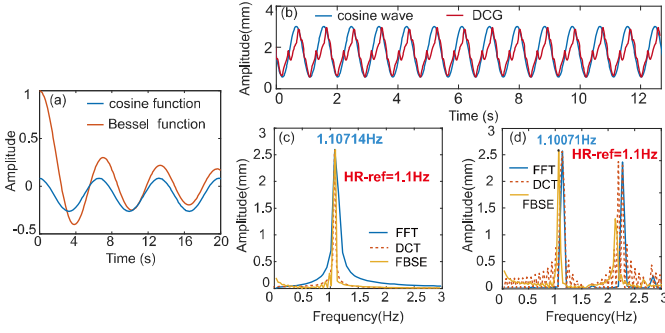


Fig. 2. (a) The basis functions in DCT, FFT and FBSE. (b) The heart motion signals used in simulation, the red line is mimicked according to the Doppler cardiogram (DCG) in panel III of Fig. 1(b) in [4], and the blue line is commonly used sine waveform. (c) The spectrums handled by FFT, DCT and FBSE of sine wave in (b). (d) The spectrums handled by FFT, DCT and FBSE of DCG in (b).

correspondence with the reference signal of PSG, and has the relation with professional sleep staging result.

## II. THEORY

### A. Principle

In the detection of vital signs by radar, the chest-wall motion actually caused by the heartbeat is due to the heart chambers' time-variant volume change which is complicated and periodic. However, almost most simulations treat the heartbeat signal as a sinusoidal signal. Based on the accurate detection of the heart motion in [4], the Doppler cardiogram (DCG) obtained from the MRI data is chosen to be used as the heartbeat signal in the simulation, which is plotted in red line in Fig. 2 (b).

The FBSE uses zero-order Bessel function as basis function, which is plotted as red line in Fig. 2 (a). It is seen that Bessel function has an asymmetric shape and time-varying amplitude in nature. Mathematically, it can be expressed as follow [5]:

$$x(n) = \sum_{i=1}^M C_i J_0\left(\frac{\beta_i n}{M}\right), n = 0, 1, \dots, M-1 \quad (1)$$

Where  $C_i$  is the Fourier Bessel series coefficients of digital signal  $x(n)$ , which can be expressed mathematically as follow [5]:

$$C_i = \frac{2}{M^2 (J_1(\beta_i))^2} \sum_{n=1}^{M-1} n x(n) J_0\left(\frac{\beta_i n}{M}\right) \quad (2)$$

Where,  $J_0(\cdot)$  and  $J_1(\cdot)$  denote zero and first-order Bessel functions, respectively. The ascending order positive roots corresponding to zero-order Bessel function ( $J_0(\beta) = 0$ ) are denoted by  $\beta_i$  with  $i=1, 2, \dots, M$ . It should be noted that order  $i$  of the Fourier Bessel series coefficients is related to continuous-time frequency  $f_i$  and can be expressed by the following equation:

$$\beta_i \approx \frac{2\pi f_i M}{f_s} \quad (3)$$

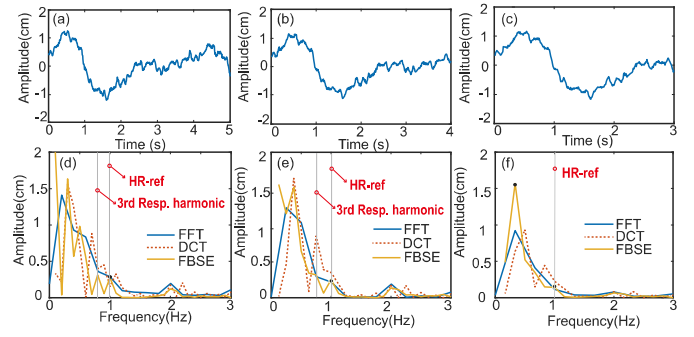


Fig. 3. (a), (b), (c) are the simulated chest-wall motion signals in time domain with different window length. (d) (e) (f) are the spectrums handled by different techniques of signals in (a), (b) and (c), respectively.

where  $\beta_i \approx \beta_{i-1} + \pi \approx i\pi$ ,  $f_s$  denotes the sampling rate.

Then, the equation (3) can be written as:

$$i \approx \frac{2f_i M}{f_s} \quad (4)$$

In equation (4),  $i$  should be varied from 1 to  $M$  for covering the whole bandwidth of  $x(n)$ . Thus the frequency resolution  $\Delta f$  of FBSE is:

$$\Delta f = \frac{f_s}{2 \cdot M} = \frac{f_s}{2 f_s \cdot T} = \frac{1}{2T} \quad (5)$$

where  $T$  denotes the time length of  $x(n)$ , which is also can be seen as the time window length. So the frequency resolution of FBSE is the twice better than that of FFT, which is  $\frac{1}{T}$ .

Fig. 2 (c), (d) show the spectrums obtained by FFT, DCT and FBSE techniques of different heartbeat waveform in Fig. (b). Fig. 2 (c) shows the results of sinusoidal signal as shown in blue line in Fig. 2 (b). It is seen that the spectrum obtained by all the technique has the specific centre frequency, and the spectrums of DCT and FBSE have 2 times shorter main-lobe width than that of FFT. However, when the heartbeat signal changes to DCG, as shown in Fig. 2 (c), the HR estimations of FFT and DCT methods have a slight deviation from the true value. This might due to the DCG is asymmetrical while the basis function of FFT and DCT has the absolutely symmetrical waveform.

### B. Algorithm and simulation

The whole signal processing flowchart is shown in Fig 1. After the motion recovery of the target motion  $x(t)$  by arcsine algorithm [6], the differential operation is performed on the  $x(t)$  to eliminate the effects of the respiration harmonic interference according to [7]. And with the FBSE representation of respiration-reduced signal, the heart rate is estimated in the compact FBSE spectrum. The proposed technique can run in MCU to realize the real-time processing. Then, we numerically simulated 5, 4 and 3-second chest-wall motion which consists of the heartbeat signal with 1.1-Hz frequency and 5-mm amplitude and respiration signal with 0.25-Hz frequency and 3-cm amplitude. It should be noted that the simulated respiratory signal has high order harmonics to match the actual scene [4].

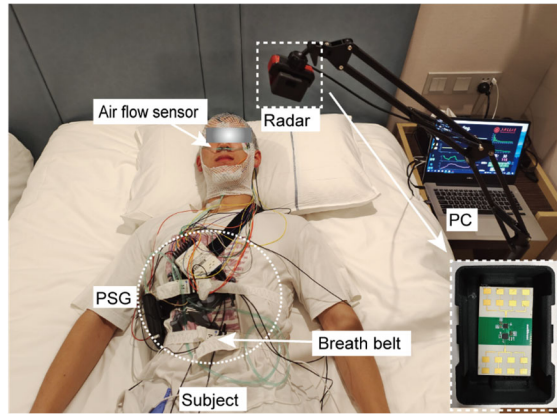


Fig. 4. Sleep experiment setup in home environment. (b) Inset is the implementation of 24-GHz radar sensor.

The simulated chest-wall motion can then be generated by adding the respiration signal and the heartbeat signal. Gaussian noises were simultaneously added to the simulation with 40-dB SNR.

It can be seen that the chest-wall signal of 5-second window, as shown in Fig. 3 (a), has a large time-varying trend due to the respiratory signal does not have a complete period. It can be seen in Fig. 3 (d) that the FFT spectrum completely mixes the third harmonic of respiration and HR, so that the HR cannot be distinguished. DCT spectrum wrongly presents the heart rate peak, while the FBSE spectrum can clearly distinguish the third harmonic of respiration and HR, and the estimated HR value is consistent with the ground truth. As the time window gradually shortens, the spectrum precision of FBSE gradually decreases until only a small HR peak can be seen at 3-s window. However, the other two methods still failed to estimate the exact HR.

### III. EXPERIMENTS AND DISCUSSIONS

In sleep medicine, sleep stage is commonly classified as wake (W), four non-rapid eye movement (NREM) stages (S1–S4), and rapid eye movement (REM). NREM can be roughly divided into light sleep (S1, S2) deep sleep (S3, S4). In clinical practice, the professional sleep technician manually denotes the signals from the golden standard polysomnography (PSG) to achieve the overnight sleep stages classification. Recently, some home-based sleep monitor using HRV to provide low-cost sleep stage classification. The metrics of HRV shows good relation with different stages, like the LF/HF ratio [1].

To verify the feasibility of the proposed method for sleep monitor, the overnight sleep experiment was carried out with the assistance of professional sleep technician. Fig. 4 shows the experimental scene setup. Overnight sleep experiment was performed on a 22-year-old subject using Embletta MPR (Comprehensive Portable PSG) data acquisition and analysis system. Physiological signals monitored included EEG; EOG; submental EMG; heartrate, ECG, breathe effort and etc. The insert of Fig. 4 shows the used 24GHz radar sensor [4]. Taking advantage of the digital signal processor (DSP) in MCU, the real-time heartrate data can be obtained by proposed technique.

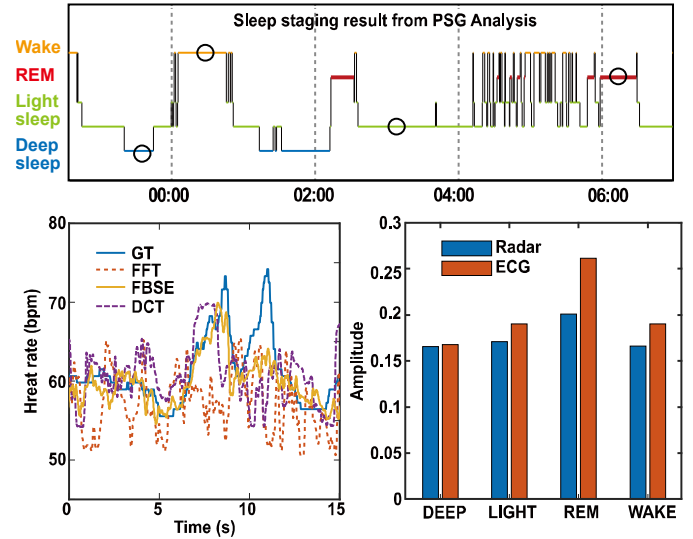


Fig. 5. (a) The sleep phase diagram from sleep technician. (b) Fast HR measurement of different techniques in 5 minutes. (c) LF/HR ratio values of different signals of sleep stages corresponding to the part circled in black in (a) from radar signal and ECG signal.

The PSG and radar sensor recorded data simultaneously. After the experiment, the professional sleep technologist makes the sleep staging results according to the recorded PSG data using American academy of sleep medicine guidelines. The subject provided written informed consent before inclusion.

Fig. 5 shows the experimental results from PSG and radar data. Fig. 5 (a) shows the sleep staging result from PSG signals. Fig. 5 (b) shows the fast HR measurement of different techniques in 5 minutes. It can be seen that the results obtained by FBSE technique shows the best accuracy than the other techniques with the respect to the ground truth (GT) from PSG. Fig. 5 (c) shows the LF/HF ratio values calculated from radar signal and ECG signal of PSG. It is seen that the values from radar has the same trend with that from ECG, and there is a clear difference in different sleep stages.

### IV. CONCLUSION

In this work, a novel respiration-reduced Fourier Bessel series expansion (FBSE) technique to realize fast detection of HR without filtering process using short-time window length is proposed. The simulation and experimental results shows the proposed technique has the better accuracy than the existing methods. To verify the feasibility of radar-based sleep monitor, the overnight sleep experiment was performed under clinical standards. Experimental results show the HRV obtained by proposed technique has the good correspondence with the reference signal, which has potential on the future automated HRV-based sleep stage classification.

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