Solutions for Data Structures and Algorithms Spring 2023 — Problem Sets

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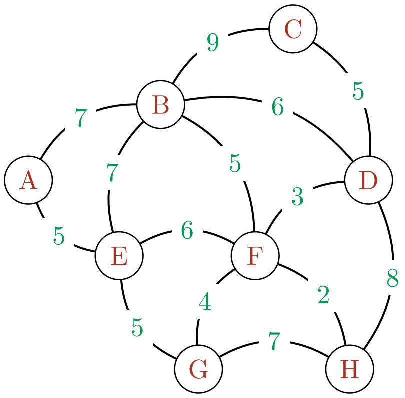
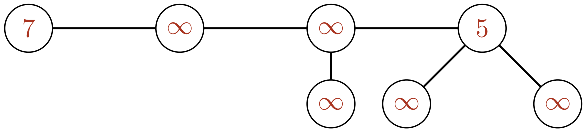
Week 12. Problem set

1. Run Prim-Jarnik algorithm [Cormen, Section 21.2] on the following graph, starting at vertex C. Assuming that the algorithm is using Fibonacci heap implementation of a priority queue, show the state of the Fibonacci heap after each iteration of the algorithm (i.e. after adding each new vertex to the MST). The graph contains 8 vertices, which means that your solution must provide 8 states of the Fibonacci heap. No justification required.

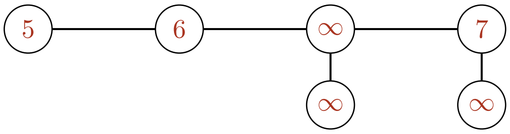


Answer.

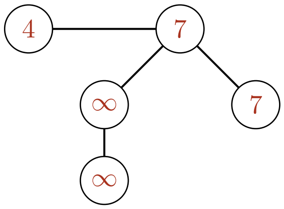
(a) State 1:



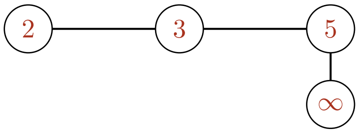
(b) State 2:



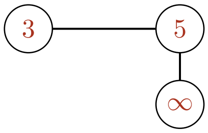
(c) State 3:



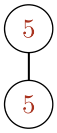
(d) State 4:



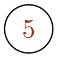
(e) State 5:



(f) State 6:



(g) State 7:



(h) State 8:

2. Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Prim-Jarnik algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W? Justify your answer in at most two paragraphs.

Answer.

If the edge weights are in some range then we can create an array with size of W and add edges to the corresponding weights. Therefore, we will get sorted edges by their weight. We can replace the $|V| \log |V|$ and $|E| \log |V|$ loops from `EXTRACT-MIN` and `DECREASE-KEY` operations in Prim's algorithm with the created weight array. This will change the original time complexity of $O(\log |V|(|V| + |E|))$ to $O(W(|V| + |E|))$.

References

[1] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition.* The MIT Press 2022.