Solutions for Data Structures and Algorithms Spring 2023 — Problem Sets

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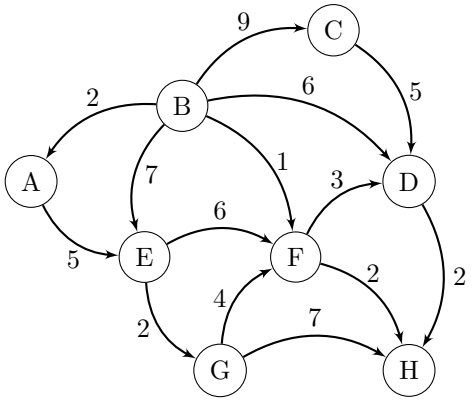
April 26, 2023

Week 14. Problem set

Run Edmonds-Karp algorithm [Cormen, Section 24.2] on the following network:

1. Identify the source and the sink of the network.
2. Construct the residual network.
3. For every iteration of the algorithm
   (a) show the augmenting path,
   (b) show the flow after the iteration,
   (c) show the residual network after the iteration
4. Write down the maximum flow value after the last iteration.
5. Show that the flow is maximum by demonstrating a minimum cut of the network.



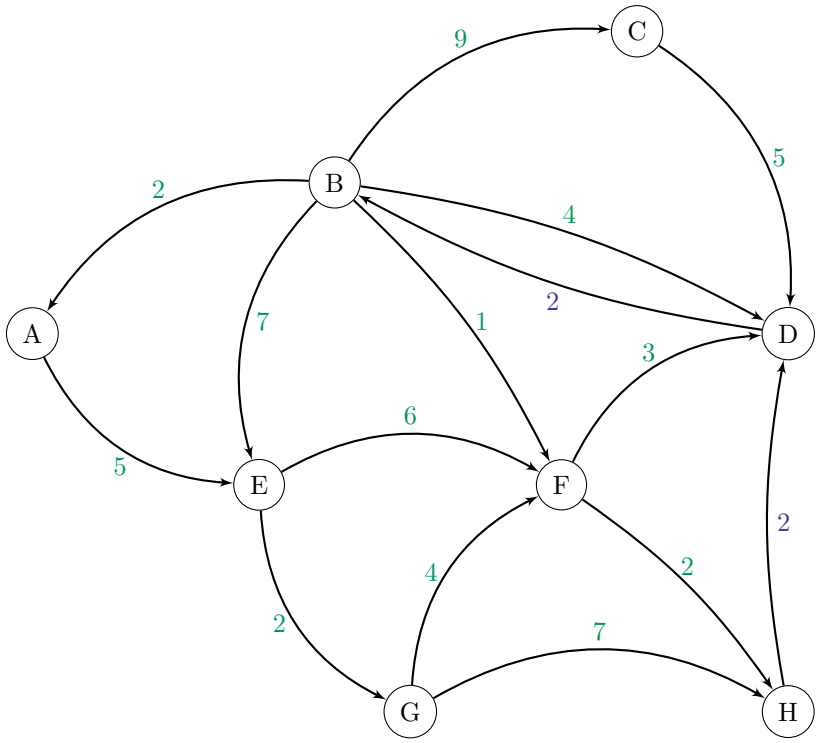
Solution

Source: B
Sink: H
Algorithm iterations:

1. Iteration 1:
   (a) BDH

(b) 2

(c) Residual network:

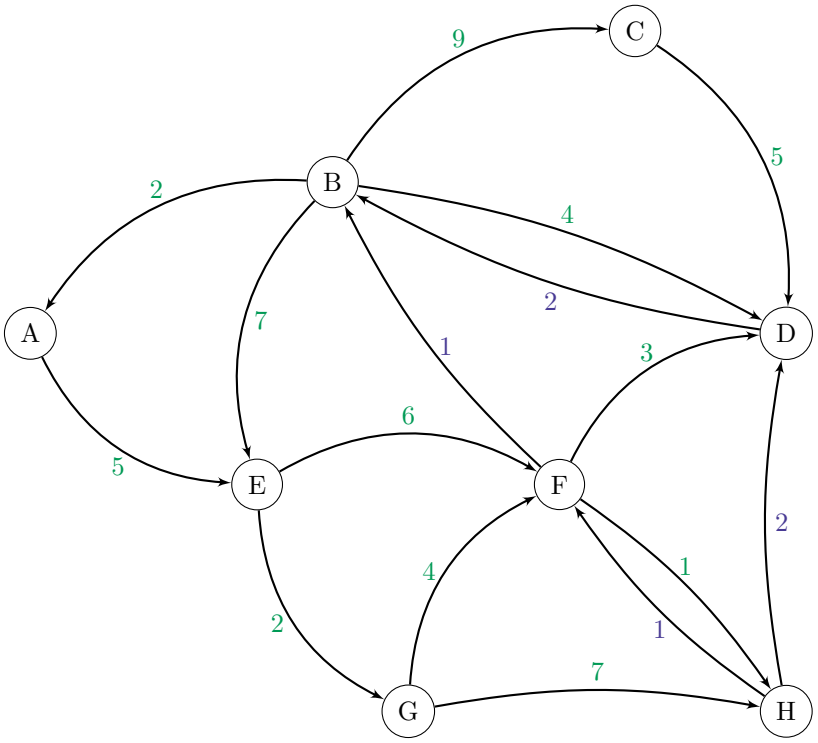


2. Iteration 2:

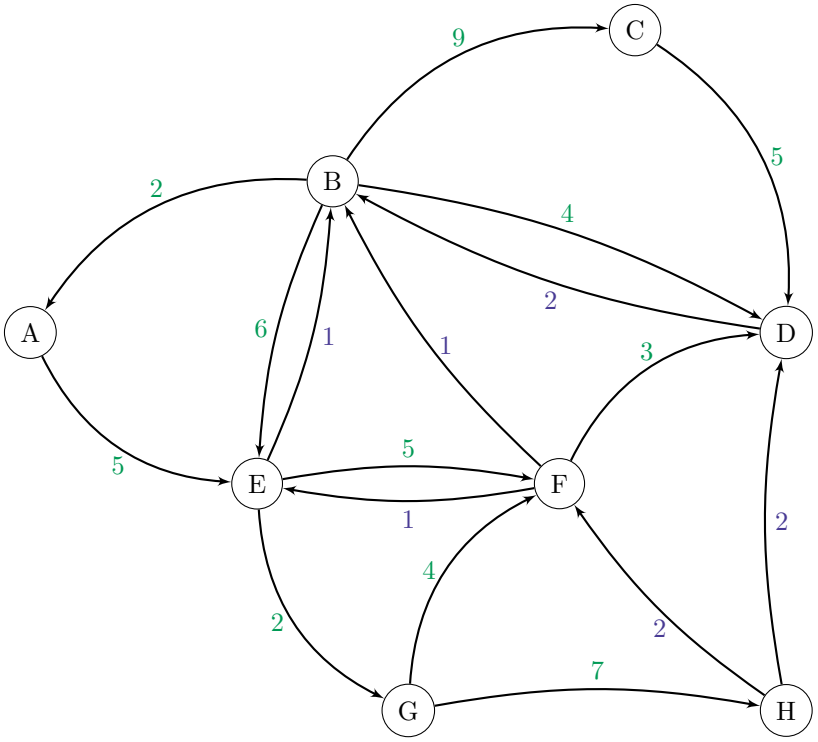
(a) BFH

(b) 3

(c) Residual network:



3. Iteration 3:
   (a) $BEFH$
   (b) 4
   (c) Residual network:

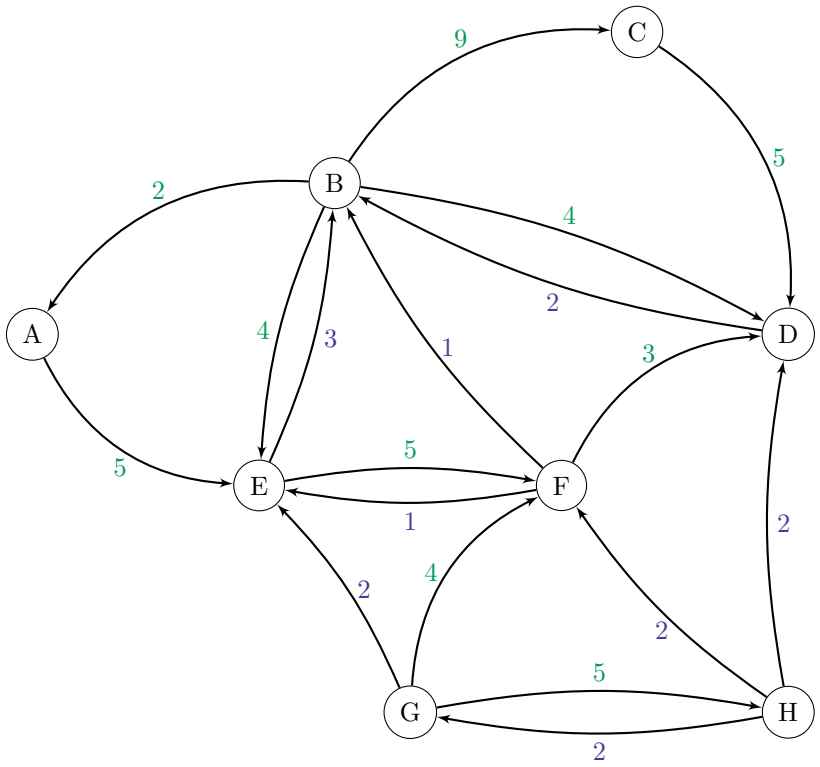


4. Iteration 4:

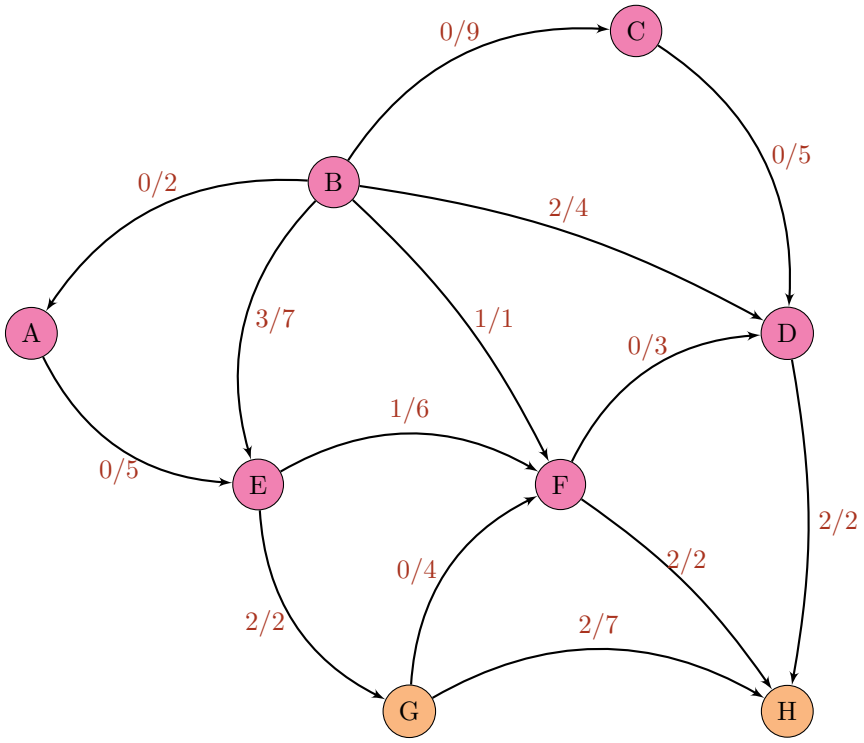
   (a) $BEGH$

   (b) 6

   (c) Residual network:



Maximum flow value: 6
Minimum cut of the network:



The flow across the cut is equal to $2 + 2 + 2 - 0 = 6$, therefore, the found value is correct.

References

[1] T. H. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein. *Introduction to Algorithms, Fourth Edition.* The MIT Press 2022.