Measuring the Duration of an Internationally Diversified Bond Portfolio

The conventional measure is meaningless.

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How does one calculate the duration of a portfolio including bonds denominated in different currencies? Indeed, what exactly do we mean by the term "duration" when it is applied to a mixed bond portfolio?

These questions are of obvious relevance to global bond portfolio managers. They are also important for the much larger audience of fixed-income managers who hold portfolios substantially composed of U.S. dollar bonds, but who substitute foreign bonds opportunistically for a small portion of their U.S. holdings, depending on market conditions. This latter practice — treating foreign bonds as a tactical asset class into which U.S. holdings can sometimes be shifted — has become common in the U.S. fixed-income management community. These investors need to know how to adjust the duration of their largely U.S. portfolios to reflect their foreign bond holdings.

We explain how the duration of a mixed portfolio of bonds is commonly calculated, and critique the normal practice. We discuss various ways of defining duration of an international portfolio, depending on what the measure is to be used for, and we describe and calculate an adjusted measure of duration useful for foreign bonds held in a substantially U.S. portfolio containing foreign bonds. We present estimates of the adjustment factor for various foreign bond markets and discuss the practical application of our measure of the duration of mixed U.S./foreign portfolios. We also discuss the limitations of our proposed measure of duration, and sketch what to do if the assumptions we make are violated.

THE CONVENTIONAL MEASUREMENT OF DURATION

The duration of a U.S.-only bond portfolio gives the percentage change in the portfolio's market value, given a 1% parallel shift in the U.S. yield curve. Any portfolio's duration is easily calculated by first computing the duration of each component bond, and then weighting those individual durations by the relevant portfolio shares. In symbols:

$$D_p = \sum_i w_i D_i \tag{1}$$

where

D_p = portfolio duration;
w_i = market value of holdings in the i-th bond, divided by the market value of the portfolio; and
D_i = duration of the i-th bond.

Now suppose some of the bonds are not denominated in dollars. How then should the duration be calculated? Conventional practice is to use the same procedure and formula. In this case, the duration of the i-th bond, D_i, is taken to be the bond's duration measured in its local currency.

This procedure is commonly applied even to well-diversified global portfolios in which foreign bonds constitute a majority of the position. To illustrate, the published durations of some commonly used global bond indexes are presented in Exhibit 1. For example, the duration of the Salomon World Government bond index (4.88) indicates that the market value of the constituent bonds will change by about 4.88% for a given 1% parallel shift in all the underlying yield curves.

Knowing how much the portfolio's value would change if all the world's yield curves shift simultaneously in the same direction and by the same amount is not of much practical us, for two reasons. First, all the world's yield curves seldom move the same way. Second, some markets are more volatile than others, so all the world's markets seldom move by the same amount.

We are interested in duration because duration shows a portfolio's sensitivity to changes in interest rates. When only one interest rate is involved, there is no ambiguity about what this means. A diversified portfolio's value is sensitive, however, not only to changes in U.S. interest rates, but also to changes in foreign interest rates. So the portfolio may have a duration as measured with respect to a change in the U.S. rates, a duration with respect to a change in German rates, a duration with respect to a change in Japanese rates, and so on. We will term these sensitivities the portfolio's "U.S. duration," "German duration," and "Japanese duration." In practice these are the durations of most use to managers responsible for mixed domestic/foreign portfolios.[1]

EXHIBIT 1
DURATIONS OF COMMON INTERNATIONAL BOND INDEXES

Lehman Global Index	4.97
J.P. Morgan Global	4.91
Merrill Lynch Global	4.93
Salomon Brothers WGBI	4.88
Salomon Brothers WBGI (Non-$)	4.80

Source: Bloomberg as of August 1, 1996.

Suppose we wish to calculate the change in the portfolio's market value for a 1% change in U.S. yields, i.e., what we have termed its U.S. duration. Then the conventional calculation of duration, which was used to produce Exhibit 1, is accurate for calculating a mixed portfolio's U.S. duration only when foreign interest rate changes are perfectly correlated with changes in U.S rates, and foreign interest rates have the same volatility as U.S. rates. Obviously, these are highly restrictive conditions that are unlikely to be met in practice. How can we measure a diversified portfolio's U.S. (or other) duration when foreign interest rates do not move in lockstep with U.S. interest rates?

COUNTRY BETAS

Suppose U.S. and foreign interest rates are not perfectly correlated, but we wish to calculate the U.S. duration of a mixed U.S./foreign bond portfolio. In this case, we first must adjust each foreign bond's duration. We term the required adjustment factors "country betas." The formula for calculating the portfolio's U.S. duration becomes:

$$D_p = \sum_i w_i \beta_i D_i \tag{2}$$

where β_i, called the bond's country beta, is formally derived in the appendix.

For all U.S. bonds, β_i is one. For foreign bonds, β_i will generally not be one. There is a different adjustment factor for each country, but assuming all yield curve changes are parallel, all German bonds share the same country beta, as do all Japanese bonds, and so on.[2] That is, the adjustment factor depends only on the currency in which the bond is denominated, and not on any other characteristic of the bond, such as its maturity or coupon. Accordingly, in order to perform the required duration adjustment you need know only one beta for each country. Estimated country betas for many major bond markets are given in Exhibit 2.

We are aware that the term "beta" has a specific meaning within the equity portfolio management community, and we have chosen deliberately to use the term here. Our beta is, in fact, analogous to a particular stock's capital asset pricing model beta. Like a stock's beta, our country betas are the product of a volatility term and a correlation term. In our case, the relevant volatility is the relative volatility of changes in foreign and U.S. interest rates, and the relevant correlation is the correlation of changes in foreign and U.S. rates. As we will illustrate, country bond betas can be estimated from a linear regression equation, as equity betas usually are.

From the appendix, beta is:

$$\beta_i = \frac{\sigma_i}{\sigma_{us}} \rho_i \qquad (3)$$

where σ_i is the volatility of interest rate changes in the i-th country, σ_{us} is the volatility of interest rate changes in the U.S., and ρ_i is the correlation between interest rate changes in the U.S. and the i-th country.[3]

EXHIBIT 2
SOME INTERNATIONAL BOND CORRELATIONS, VOLATILITIES, AND BETAS

	Correlation	Relative Volatility	Beta
Australia	0.75	1.60	1.20
Canada	0.70	1.50	1.05
France	0.65	1.05	0.65
Germany	0.55	0.80	0.45
Japan	0.30	1.00	0.30
The Netherlands	0.65	0.85	0.55
U.K.	0.55	1.30	0.70

To see why global duration might be misleading, let us consider some examples. Suppose two portfolios have identical global durations, but one is well-diversified while the other consists entirely of one country's bonds. Which is riskier? If all the world's interest rates were perfectly correlated, they would be equally risky — as is implied by their identical global durations. Since different bond markets do not always move in tandem, or even in the same direction, however, the diversified portfolio's risk is likely to be less. In other words, global duration does not account for differences in risk that result from some risk being diversifiable.

Now suppose we compare two portfolios with the same global duration but composed of different bonds. The first is of bonds from highly volatile markets such as Australia and Canada. The second uses bonds from Germany and the Netherlands — relatively low-volatility markets. Obviously, we expect the first portfolio to be more volatile than the second, even though they have the same global duration. This is because global duration assesses the portfolio's sensitivity to a 1% change in yields, but it does not factor in how likely a change of that magnitude is. That is, it does not account for different bond market volatilities.

It is instructive to consider the components of beta and what they represent. The term σ_i/σ_{us} represents the relative volatility of the foreign interest rate compared to the U.S. interest rate. If this ratio is close to one, then the magnitude of interest rate changes in country i is similar to that in the U.S. To the extent foreign rates are more (less) volatile than U.S. interest rates, the duration of the portfolio of foreign bonds will be greater (less), holding everything else constant.

The term ρ_i represents the direction and strength of the relationship between foreign and U.S. interest rates. The closer this term is to positive one, the more likely it is that U.S. and foreign interest rates move in tandem.

Beta is the product of two terms representing the magnitude and U.S.-relatedness of foreign rate changes. Both are important. Even a weakly correlated foreign market can contribute significant U.S. duration if that market is unusually volatile.

Notice that having the product $(\sigma_i/\sigma_{us})\rho_i = 1$ is not equivalent to having both terms equal to one. For instance, suppose the correlation is 0.25, and the relative volatility is 4.0. Then β_i, the product, is one. A foreign bond with these component terms will generally behave quite differently from a U.S. dollar bond, or from a foreign bond with correlation equal to one and relative volatility equal to one ($\beta_i = 1$). When the correlation is only 25%, often yields on this bond will actually move in a different direction from yields on the U.S. bond. So, this bond may track a U.S. bond poorly. To secure low tracking error it is critical that the correlation be close to one, regardless of the value of β_i.

MEASURING COUNTRY BETAS

Beta can be rewritten as:[4]

$$\text{Beta} = \frac{\text{cov}\left(\Delta y_i, \Delta y_{us}\right)}{\sigma_{us}^2} \tag{4}$$

Readers will recognize this as the form of a CAPM beta. As with the CAPM beta, this beta term equals the expectation of the $\hat{\beta}_i$ term solved for in the regression:

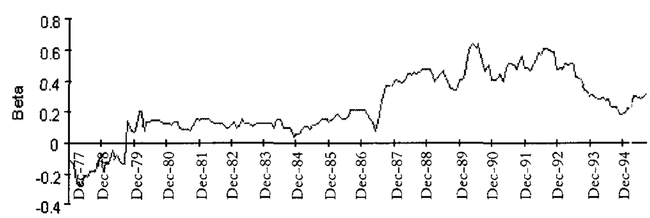
$$\Delta y_i = \hat{\beta}_i \Delta y_{us}$$

This is a convenient result since we can use any standard regression routine to calculate $\hat{\beta}_i$, the best estimate for the actual beta β_i. In practice we note that yield curve changes are not all parallel so we must choose a point along the yield curve on which to regress changes in yield.[5] We like to use the point with duration corresponding to that of our benchmark.

Exhibit 2 presents estimated bond betas for ten-year government bonds. A common procedure is to estimate regressions such as this over an arbitrary window of data. The implicit assumption is that beta is constant throughout this window. Our two-factor regression approach lets the data determine the best weights to apply to the recent and the more distant observations.

We find that the best model includes a 5% weighting on the long-run average beta and a 95% weighting on the most recent three-year period. The fact that the best estimate is obtained by giving greater weight to the most recent data indicates that betas are not stationary. This is illustrated in Exhibits 3-6, which show the time series of beta estimates for Japanese, U.K., German, and Canadian bonds. Consequently,

EXHIBIT 3
JAPAN



beta estimates should be updated periodically.

Exhibit 2 also shows the components of country bond beta: relative volatility and correlation. Notice that in practice the value of beta is not determined only by correlation. For example, the correlation of the U.K. market with the U.S. (0.55) is less than that of France (0.65) or the Netherlands (0.65), but the beta for the U.K. (0.70) exceeds that for either France (0.65) or the Netherlands (0.55), because the U.K. market is more volatile. Exhibit 2 shows clearly that beta depends both on correlation with U.S. rates and relative volatility of the foreign market.

It may be surprising to some that Australia has a slightly higher correlation with the U.S. than does Canada, but Australia is also more volatile than Canada. What Exhibit 2 does not show directly is the Australian bond's performance variability that is *not* associated with U.S. interest rate changes. This residual risk turns out to be lower for Canadian bonds than for Australian bonds. (We discuss residual risk and its importance later.)

APPLYING BOND COUNTRY BETAS TO PORTFOLIO MANAGEMENT

A consequence of Equation (2) and the property of covariance is that, like CAPM beta, a bond portfolio's

EXHIBIT 4
UNITED KINGDOM

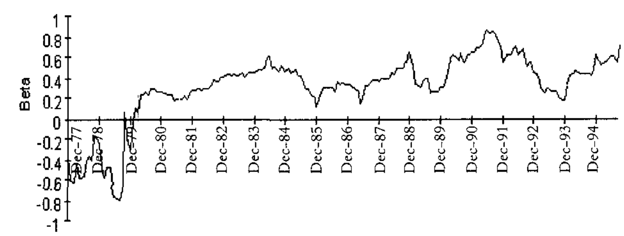
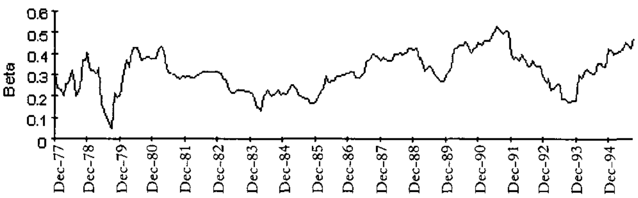


EXHIBIT 5
GERMANY



beta is a simple value-weighted average of its individual country bond betas.[6] We can easily calculate the portfolio beta and the adjusted U.S. durations of the indexes given in Exhibit 1. These are presented in Exhibit 7.

Exhibit 7 indicates two sources of variation between the published and the U.S. durations. First, for each index the published global duration and the U.S. duration differ significantly. This is because the U.S. duration measures the interest rate sensitivity of a foreign bond portfolio to U.S. interest rate changes.

Second, there is much more variation among the indexes' U.S. durations. Instead of all durations lying within 0.27 years of each other, they now vary by as much as 1.17 years.

All foreign bond indexes are not created equal, at least as measured by their sensitivity to changes in the U.S. interest rates, even though their published global durations are similar.

LIMITATIONS

U.S. duration is useful for describing the risks of a portfolio including foreign bonds, but it is not by itself sufficient to describe those risks completely. U.S. duration captures the component of a foreign bond's interest rate risk that is related to U.S. interest rate changes. The remaining risks are ordinarily significant. Even though this country-specific interest rate risk is not related to

EXHIBIT 6
CANADA

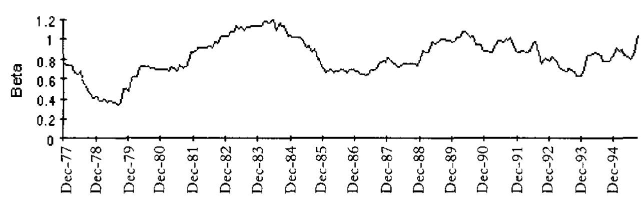


EXHIBIT 7
STANDARD AND ADJUSTED DURATIONS OF COMMON INTERNATIONAL BOND INDEXES

	Average Beta	Published Duration	Adjusted U.S. Duration
Lehman Global Index	*	4.97	3.64
J.P. Morgan Global	*	4.91	3.49
Merrill Lynch Global	*	4.93	3.42
Salomon Brothers WGBI	*	4.88	3.33
Salomon Brothers WBGI (Non-$)	*	4.80	2.47

U.S. interest rate changes — which is the major bond risk factor for most U.S. bond managers — it is nevertheless a real source of portfolio performance variation.

An estimate of the proportion of the total risk that is attributable to unique country risk, relative to risk derived from U.S. interest rate changes alone, is given by the expression:[7]

$$\frac{2\hat{a}}{\text{std. dev.}\left(\Delta y\right)} \tag{5}$$

Here the $\hat{a}$ comes from Equation (3). It represents the average change in the foreign interest rate during a period in which U.S. rates and constant foreign interest rate changes of this kind might be driven by factors such as local need for capital, foreign monetary policy, non-U.S. trade flows, and the like.

Exhibit 8 presents percent country risk estimation for the countries of Exhibit 2. Notice that the data in Exhibit 8 are not substantially explained by the correlations with the U.S. market, reported in Exhibit 2. In summary, this statistic measures the importance of interest rate risk factors not shared with the U.S. market, and thus likely to be of influence out of concert with the factor affecting the U.S.

While Exhibit 8 is quite useful, it also does not tell the whole story. It aggregates all the factors not related to U.S. interest rates into one summary statistic. In fact, this non-U.S. interest rate factor really represents many factors not related to U.S. interest rates but potentially interrelated with other countries in the portfolio. A full-blown factor analysis and an estimation of an international variance covariance matrix would be required to better assess the residual risk.[8] To keep

EXHIBIT 8
ESTIMATED PERCENT OF UNRELATED FOREIGN INTEREST RATE MOVEMENT VERSUS U.S.

	Risk Percentage
Australia	17
Canada	7
France	21
Germany	20
Japan	35
The Netherlands	24
U.K.	22

things tractable, we have proposed the simple country risk estimate given in Exhibit 8.

The residual interest rate risk not represented in the measure of U.S. duration we propose is likely to be small whenever:

1. The correlation of the foreign holding with U.S. rates is high.
2. The country risk percentage is low.
3. The portfolio includes a small foreign allocation.

When these conditions obtain, the U.S. duration measure will fairly accurately describe duration performance of a U.S. bond portfolio with a foreign holding. The higher the correlation, the lower the country risk percentage. Also, of course, the smaller the foreign holding, the closer the duration-based variability to that of a U.S. bond portfolio with duration equal to beta-adjusted duration. Information such as that in Exhibits 7 and 8 can provide valuable assistance in reducing such risk.

SUMMARY

Conventionally, the duration of a mixed portfolio of foreign and U.S. bonds is calculated in much the same way that duration of a U.S.-only portfolio is calculated. You weight the duration of each component of the portfolio by its portfolio share, and then sum across all the components of the portfolio.

This approach has two attractions. First, it is familiar. Second, the calculation is simple to perform. One defect is crucial: The resulting duration measure is meaningless. It is related to the usual understanding of the term duration only when extreme (and unrealistic) conditions obtain. Practically speaking, this measure of duration is useless for risk management.

To obtain a meaningful measure of portfolio duration, you should adjust the durations of foreign bonds before performing the conventional calculation of duration. We demonstrate how to adjust the durations of foreign bonds so they can be meaningfully aggregated with U.S. bond durations. It is shown that the duration of a portfolio including foreign bonds depends on 1) the duration of the foreign bonds, as it is calculated conventionally in their home currencies; 2) the volatilities of foreign interest rates, compared to the volatility of U.S. rates; and 3) the correlation between foreign and U.S. interest rate changes.

We call the adjustment factors we propose "country betas," and we provide recent estimates of these bond betas in Exhibit 2. They range in value from very small — 0.30 in Japan, for example — to greater than 1.00 in Australia and Canada. In other words, for some countries, local currency measured duration should be adjusted upward, and in some countries downward, before aggregating with the durations of the U.S. bonds in the portfolio.

Users of foreign bond betas need to be sensitive to their limitations. When foreign bonds become a substantial fraction of the portfolio, the usefulness of foreign bond betas as a single statistic summarizing a portfolio's riskiness declines. For a well-diversified global portfolio, even if the portfolio's duration is calculated with the adjustment we propose, it may not be an adequate measure of the portfolio's risk.

Why? Very simply, no single duration measure can accurately summarize risk when the portfolio is exposed to multiple sources of interest rate risk.[9] For a well-diversified bond portfolio you must monitor the portfolio's sensitivity to changes in each of the world's major interest rates.

APPENDIX
DERIVATION OF THE FORMULA FOR BETA

We write the price of a portfolio of bonds P, dependent on U.S. interest rates through its yield y_{us}, as $P(y_{us})$. P may be comprised of U.S. and foreign bonds. We denote the price of a bond in the i-th country as P_i. Such a bond is directly dependent on the interest rate of the i-th country through y_i, the yield of the bond.

Then:

$$P(y_{us}) = \Sigma w_i P_i(y_i) \tag{A-1}$$

where w_i is the market value weight in each country.

To find the U.S. duration we must differentiate (A-1) with respect to y:

$$\frac{P(y_{us})}{dy_{us}} = \Sigma w_i \frac{\partial P_i(y_i)}{\partial y_i} \frac{dy_i}{dy_{us}}$$

The term of interest is dy_i/dy_{us} since it is not otherwise defined. Assume for simplicity that interest rates are governed by a simple Wiener process. Then we assert:[10]

$dy_{us} = \sigma dz$ and $dy_i = \sigma_i dz_i$ with $E[dz x dz_i] = \rho_i dt$

Now applying a Taylor expansion to y_i we get:

$$dy_i = \frac{dy_i}{dy_{us}} dy_{us} + \frac{1}{2} \frac{d^2 y_i}{dy_{us}^2} \sigma^2$$

Substituting for dy_i and dy, we get:

$$\sigma_i dz_i = \frac{dy_i}{dy_{us}} \sigma dz + \frac{1}{2} \frac{d^2 y_i}{dy_{us}^2} \sigma^2 dt$$

Multiplying both sides by dz, we get:

$$\sigma_i dz_i dz = \frac{dy_i}{dy_{us}} \sigma dt + \frac{1}{2} \frac{d^2 y_i}{dy_{us}^2} \sigma^2 z_1 dt^{3/2}$$

Itô's lemma assures us that the term at the far right vanishes so that we get:

$$\sigma_i dz_i dz = \frac{dy_i}{dy_{us}} \sigma dt$$

or

$$\frac{dy_i}{dy_{us}} dt = \frac{\sigma_i}{\sigma} dz_i dz$$

Taking expectations yields:

$$\frac{dy_i}{dy_{us}} = \frac{\sigma_i}{\sigma} \rho_i$$

We define $(\sigma_i/\sigma)\rho_i$ as beta.

ENDNOTES

[1]Logically these should be called the portfolio's "dollar duration," its "deutschemark duration," its "yen duration," and so on. The term "dollar duration," however, is commonly used to mean something quite different.

[2]Of course, the yield curve may change in non-parallel ways, but then we are only highlighting the insufficiency of duration itself as a measure of interest rate sensitivity. See Willner [1996] and Thomas and Willner [1997].

[3]To be specific, σ_i means the standard deviation in the changes in yield over a given period, say, monthly, for a specific country.

[4]$\beta_i = (\sigma_i/\sigma_{us})\rho_i = \sigma_{us}\sigma_i\rho_i/\sigma_{us} = [cov(\Delta y_i \Delta y_{us})]/\sigma_{us}^2$.

[5]For an extension of this analysis to a more comprehensive framework that considers non-parallel shifts in the yield curve, see Willner [1998].

[6]In performing this calculation, the country beta for the U.S. components of the portfolio is, of course, one. Also, if you wish, you could calculate a weighted average of the adjusted bond durations to find the portfolio duration.

[7]The intuition behind this estimate is that the additional proportion (to the U.S.) is given by the unexplained portion of local interest rate movements, both positive and negative, relative to typical U.S. interest rate movements.

[8]Presenting its own problems in estimation and stability.

[9]In a global context, a useful single duration number would be one defined against a global interest rate factor, when the global factor represents one (the primary) factor within a multifactor model.

[10]Actually, we should formally relate interest rate movements to yield movements. We could avoid this technical problem by assuming that our bonds are pure discount bonds and that we are considering spot rates and not yields.

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