Advanced course of Measurement and Signal Processing
計測信号処理特論
4. Statistical signal processing (Noise Filtering and Correlation function)

- You can comment and ask questions any time in "Chat“.
- Please mute your mic to avoid noise and use “rise hand” button and wait when you want to speak.
- Answer a quiz in clicker to register your attendance.
- Your video image will not be recorded. Your voice will be recorded.

2021-05-11

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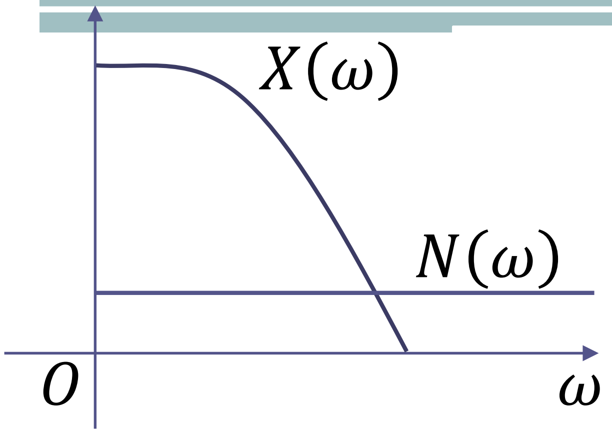
1Q, Tuesday 14:20～16:00

Statistical noise removal

- Modeling of signal and noise
  - Yes
    - Weiner filter
    - Adaptive filter
      - movie, picture, music, human voice,…
    - Estimation of original signal considering physical phenomena. → Later
  - No
    - Linear filter
    - Nonlinear filter
    - Averaging

Wiener inverse filter

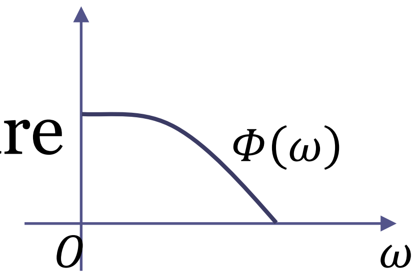


Modeling of measured signal

$Y(\omega) = X(\omega) + N(\omega), \quad y_i = x_i + n_i$

Y, y: measured signal, X, x: signal, N, n: noise

Find φ_i that make $\hat{x}_i = y_i \varphi_i$ using least square method.



$\mathrm{E}\langle(\hat{x}_i - x_i)^2\rangle = \mathrm{E}\langle\sum_i [(x_i + n_i)\varphi_i - x_i]^2\rangle$

$= \sum_i \left\{\mathrm{E}\left\langle x_i^2\right\rangle(1-\varphi_i)^2 + \mathrm{E}\left\langle n_i^2\right\rangle\varphi_i^2\right\} - 2\sum_i \varphi_i \boxed{\mathrm{E}\langle n_i x_i\rangle}$

$= 0$

No correlation between noise and signal

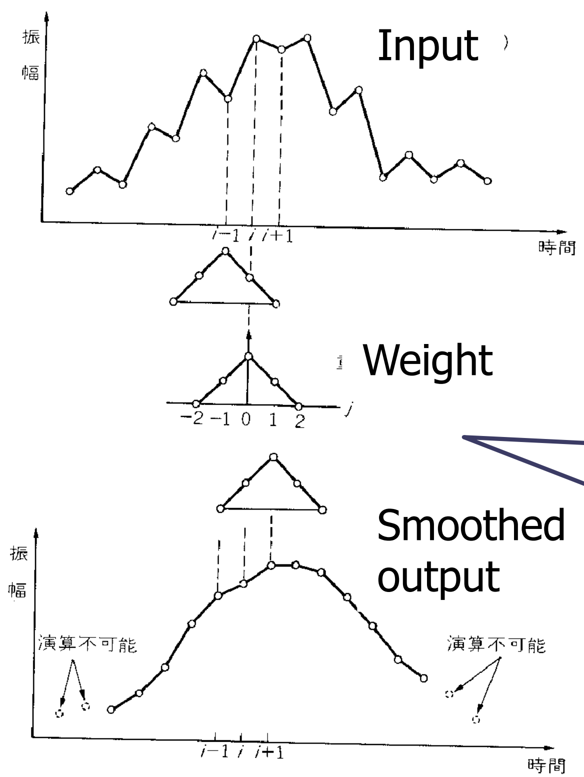
Take partial differential to minimize error

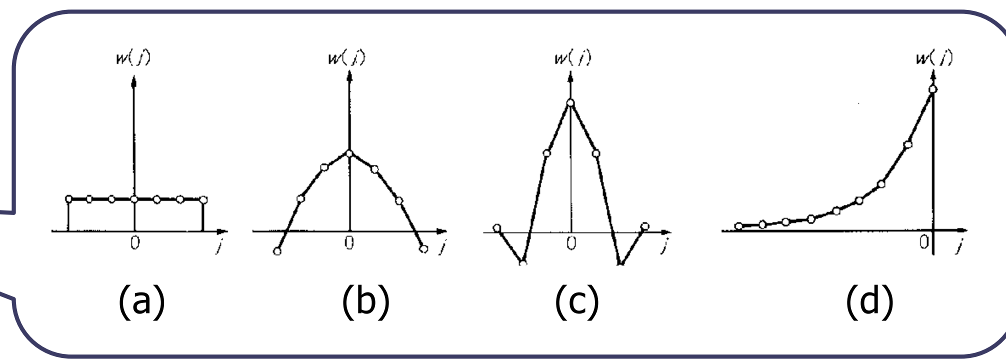
$\varphi_i = \frac{\langle x_i^2\rangle}{\langle x_i^2\rangle + \langle n_i^2\rangle}, \quad \Phi(\omega) = \frac{P_X(\omega)}{P_X(\omega) + P_N(\omega)}$

$P_N(\omega)$ *is asuumed constant*

Moving average（移動平均）

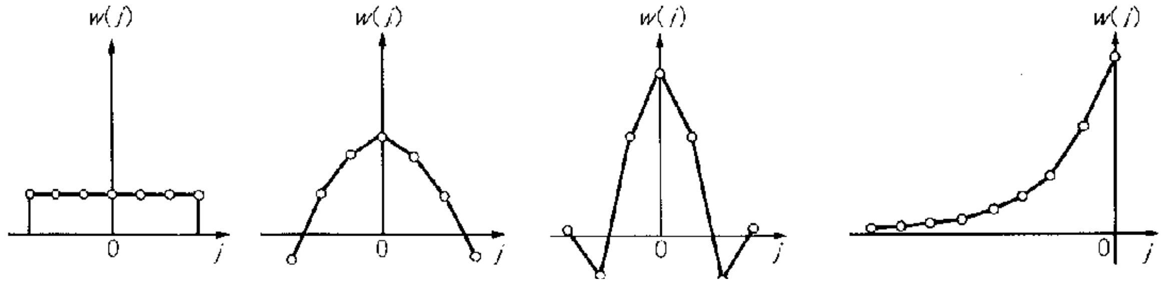
- Moving average = Filtering operation





(a) Moving average
(b,c,d) Weighted moving average
(b) 2nd, 3rd order polynomial fitting
(c) 4th, 5th order polynomial fitting
(d) Pseudo analogue filter

Moving average



(a) (b) (c) (d)

Moving average

Weighted moving average

(a) $Z_x = \frac{1}{2m+1} \sum_{i=-m}^{m} z_{x+i}$

(b, c, d) $Z_x = \sum_{i=-m}^{m} \left(w_i \ z_{x+i} \right) \Big/ \sum_{i=-m}^{m} w_i$

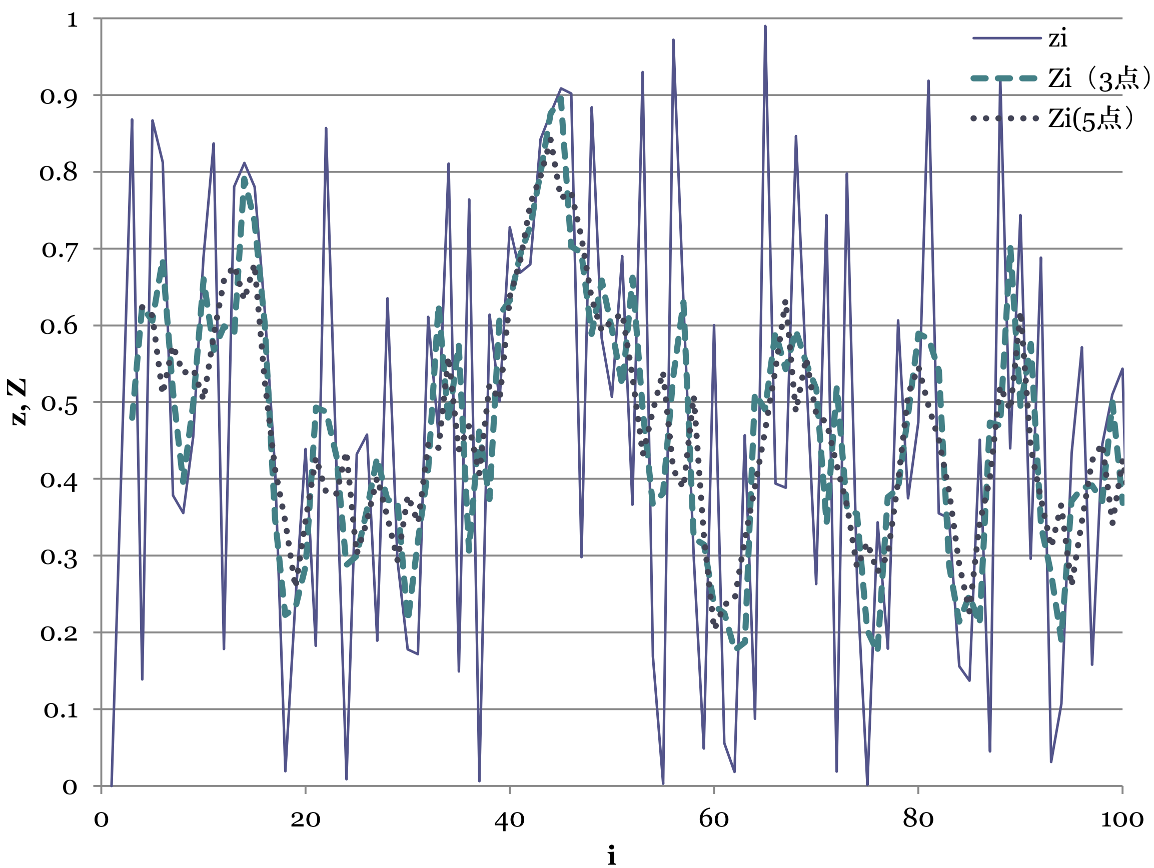
Moving average

- Noise suppression of simple moving average

$$\sigma_y^{\,2} = \sigma_x^{\,2} / N \qquad \sigma_y / \sigma_x = 1 / \sqrt{N}$$

  - σ_x, σ_y: standard deviation of the noise before / after the operation
  - N: points of averaging
  - Supposes noise have no correlation to each other
- The choice of the range of averaging and weight distribution depends on user's experience.
- Waveform may be distorted.

Moving average



Original

$\sigma_z = 0.31$

3 points

$\sigma_Z = 0.18$

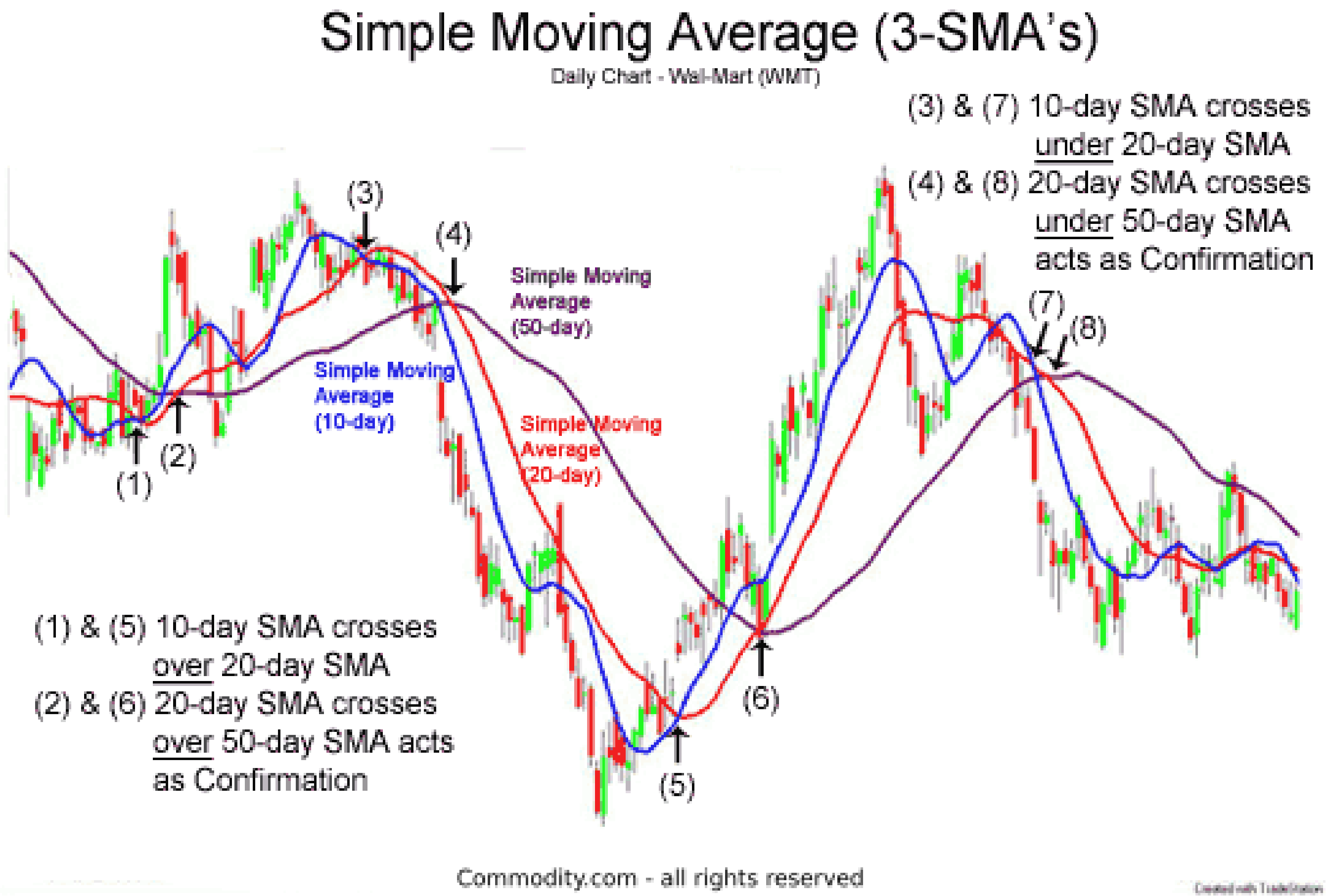
$\approx \sigma_z/\sqrt{3}$

5 points

$\sigma_Z = 0.14$

$\approx \sigma_z/\sqrt{5}$

Example of moving average (Hidden slide)



Median filter

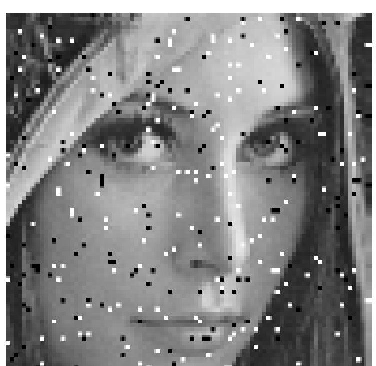
- Definition

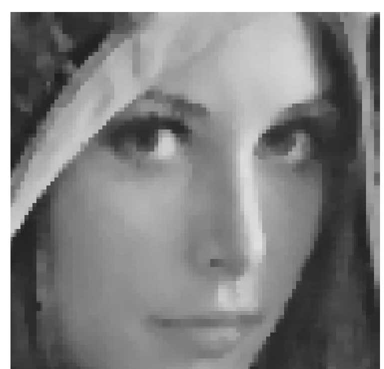
$$Z_x = \text{median}\left[z_{x-m}, \cdots, z_{x-1}, z_x, z_{x+1}, \cdots, z_{x+m}\right]$$

$$Z_{x,y} = \text{median}\begin{bmatrix} z_{x-1,y-1}, z_{x,y-1}, z_{x+1,y-1}, \\ z_{x-1,y}, z_{x,y}, z_{x+1,y}, \\ z_{x-1,y+1}, z_{x-1,y+1}, z_{x-1,y+1} \end{bmatrix}$$

(case of 2D, m=1

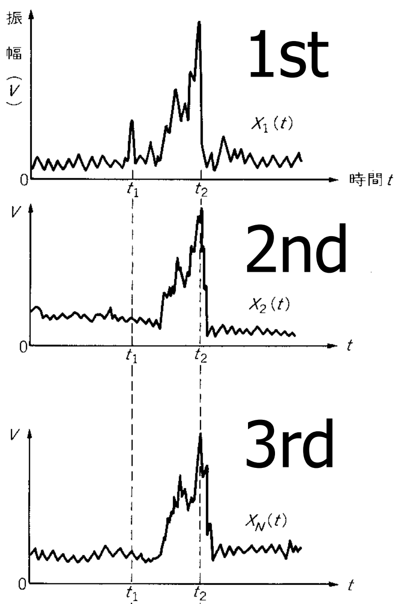
- Works good for independent noise, while keeping fine details.
- Non-linear operation

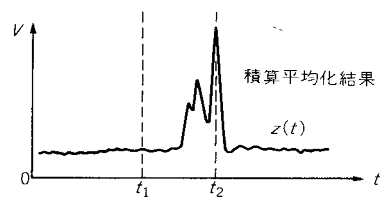




Cumulative averaging
(Ensemble Averagingアンサンブル平均)

- Averaging the multiple results





Result of cumulative averaging

- Example: Estimation of current distribution in brain

Cumulative averaging

- Averaging the multiple results
- Total of the signal $\sum_{i=1}^{N} s_i = Ns$
- Total of Noise $\sqrt{\sum_{i=1}^{N} n_i} = \sqrt{N}\,n$
- S/N ratio $Ns \Big/ \sqrt{N}\,n = \sqrt{N}\; s \big/ n$
- N times measurement
  - → S/N ratio improvement $\sqrt{N}$

Note

- Filtering causes loss of information.
  Therefore, try reducing noise in the experimental setup first.

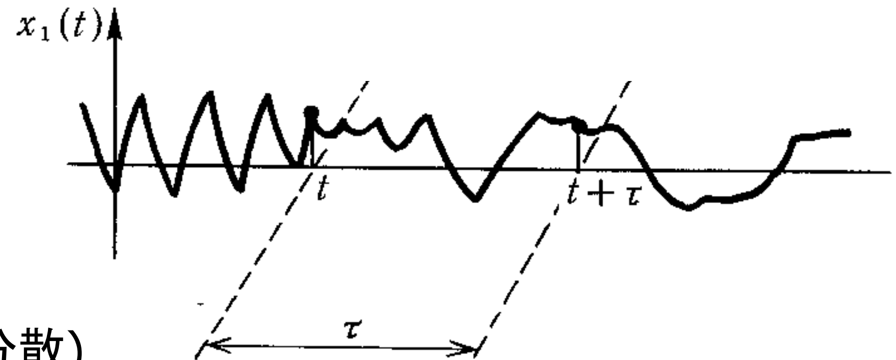
Auto Covariance (自己共分散)

expected value (期待值)

Definition

$$R_x(\tau) = E[x(t)x(t+\tau)]$$

$$\approx \frac{\int_0^{\ell} x(t)x(t+\tau)dt}{\ell}$$



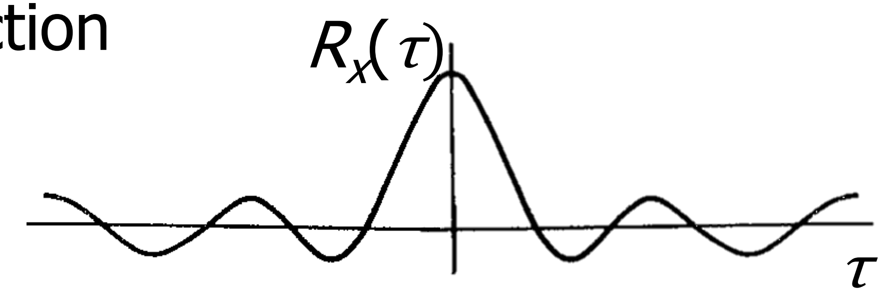
note:

variance (分散)

$$R_x(0) = E[x(t)^2] = V[x(t)]$$

$$R_x(-\tau) = R_x(\tau)$$ Even function

$$R_x(0) \geq |R_x(\tau)| \quad \forall : \tau$$



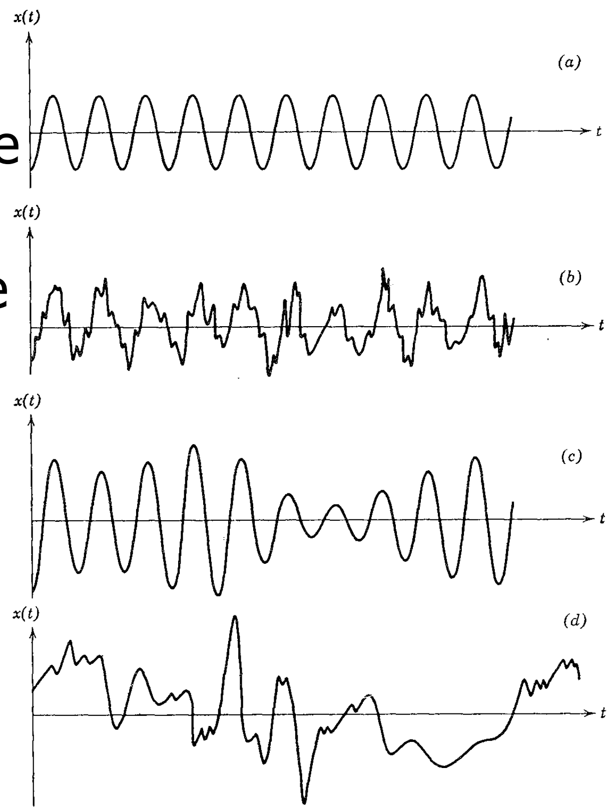
Examples of Autocovariance

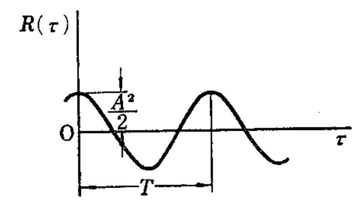
Sine wave

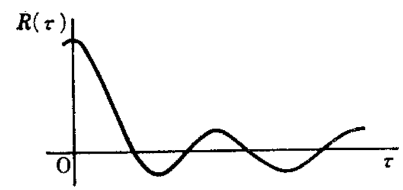
sine wave + noise

Bandlimited noise

White noise







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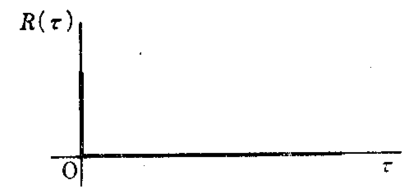


Practical Calculation of Autocovariance

- Calculation
  - Continuous

$$R_x(\tau) = \frac{\int_a^b x(t)x(t+\tau)dx}{b-a}$$

  - Discrete

$$R_{x\tau} = \frac{\sum_{t=a}^{b} x_t \, x_{t+\tau}}{b-a+1}$$

computational complexity: $O[N^2]$
O is Landau's symbol

(a) Small $\tau \rightarrow$ Large *b-a* available

(b) Large $\tau \rightarrow$ Limited to small *b-a*

Data length limitation

Auto Correlation Function (ACF)（自己相関関数）

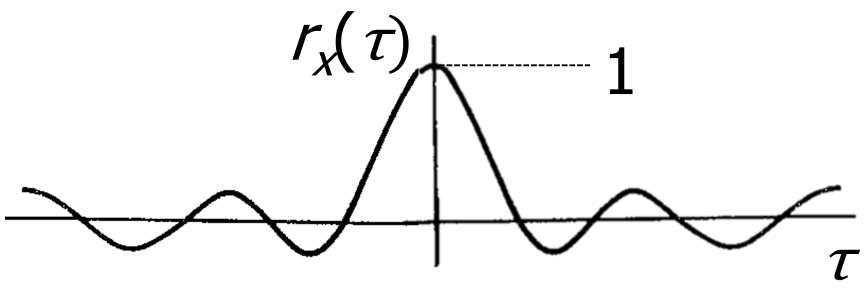
- Normalized Autocovariance

$$r_x(\tau) = R_x(\tau) / R_x(0) = R_x(\tau) / V[x]$$

  - Note:

$$r_x(0) = 1$$

$$-1 \le r_x(\tau) \le 1$$



Correlation length（相関長）

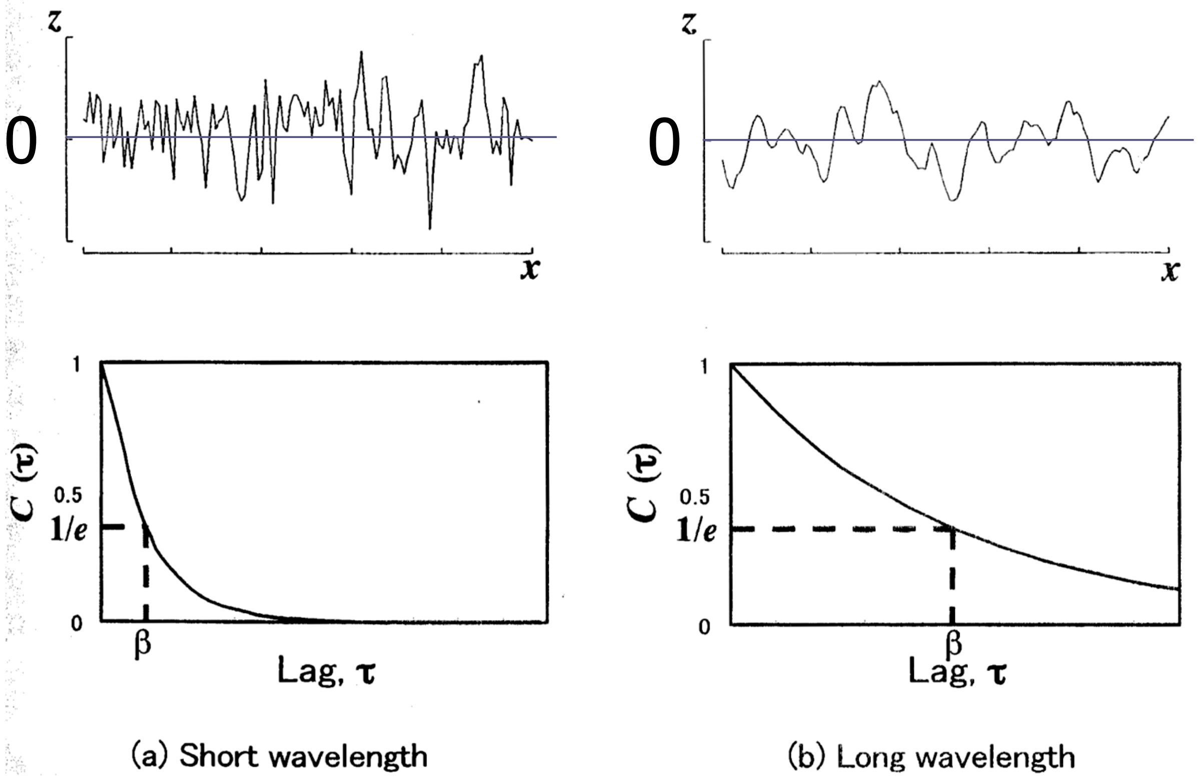
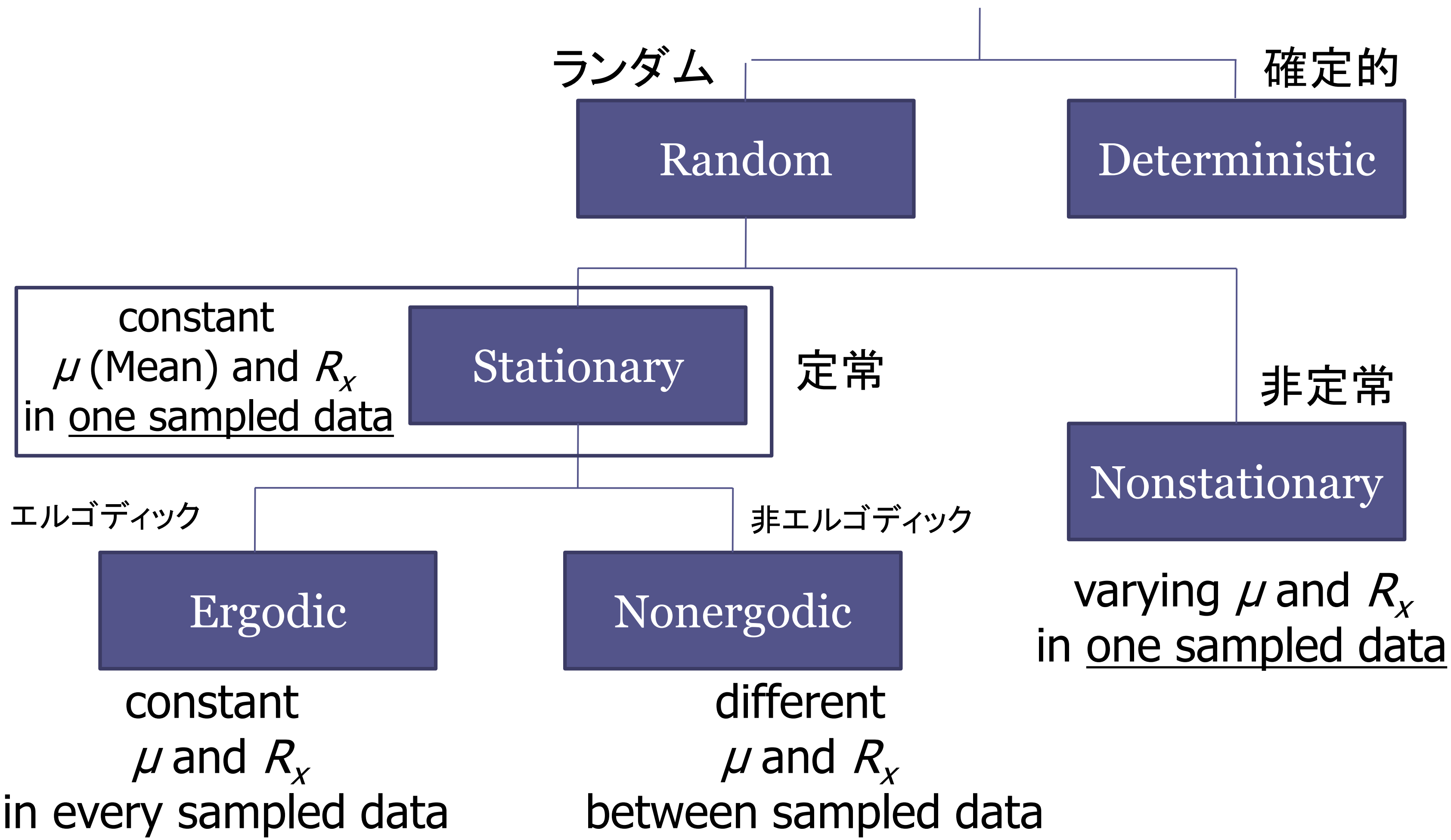


Fig. 3–5 Difference of ACC between two profiles with different wavelength

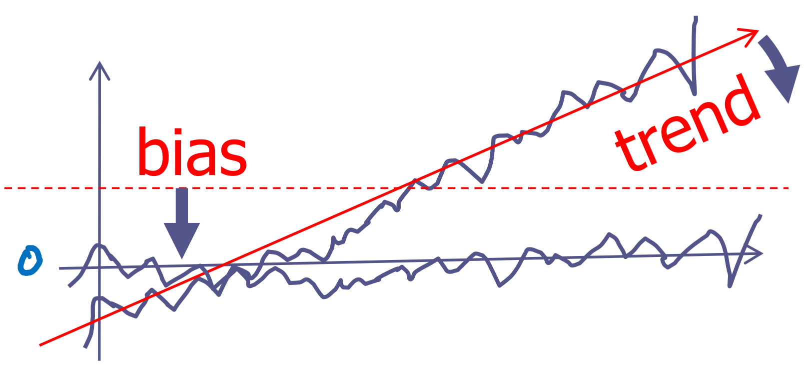
Correlation value = 1/e, 0.5, 0.8.
Different values are used depending on application field

Classification of Random data



Calculation of ACF

- Data Requirement
  - Long data is required (b-a >> τ)
    - otherwise you must “note” that
  - “Bias” and “Trend” should be removed
    - E[x(t)]=0 (mean shoud be zero)
    - Bias → Subtract mean value
    - Trend → Linear regression
      - Most signals can be regarded as stationary signal (多くの信号が定常ランダム信号と見なせるようになる)



Why the line is a cur

Application of ACF

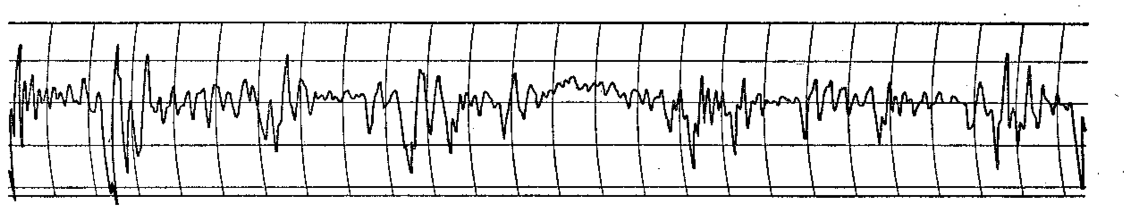
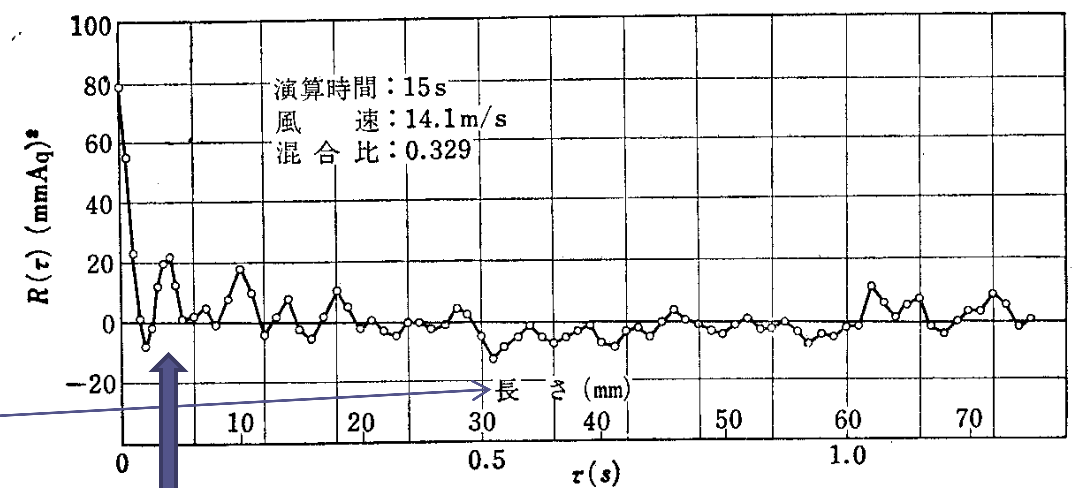


Fig. Fluctuation of the pressure in a air-duct recorded on roll paper



Length on recording paper

$\approx 16\mathrm{Hz}$

Fig. Auto correlation of the above fluctuation

Analog instrument calculating ACF

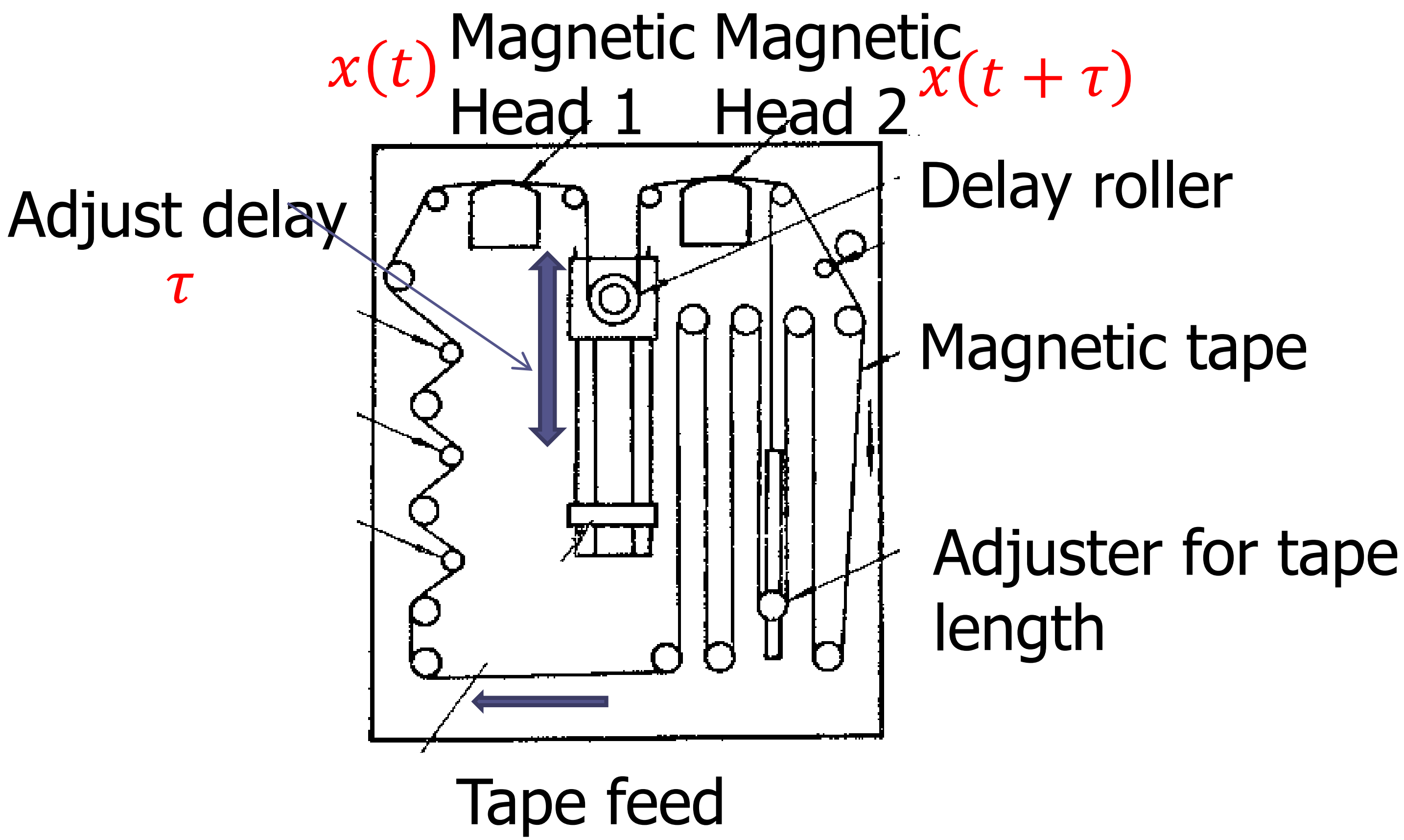
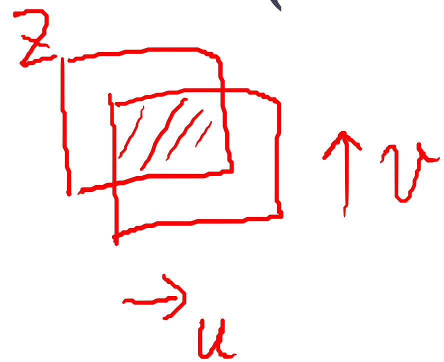


Fig. Analog instrument for ACF

Areal Auto Correlation Function (AACF)



- Definition

$$R_z(u,v) = E[z(x,y)z(x+u,y+v)]$$

note:

$$R_z(u,v) = R_z(-u,-v), \quad R_z(u,v) \neq R_z(u,-v), R_z(u,v) \neq R_z(-u,v)$$

Point symmetry, 点対称

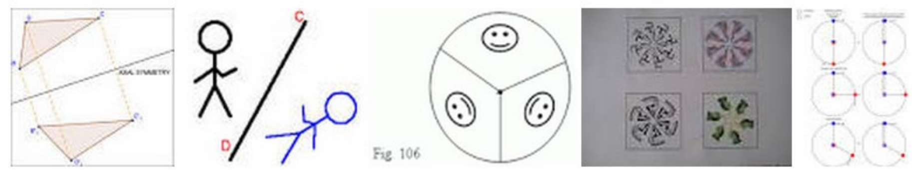
Not line symmetry, 線対称ではない

- Application
  - 3-D surface topography, 2-D Image analysis
  - Find ray direction and period for each direction

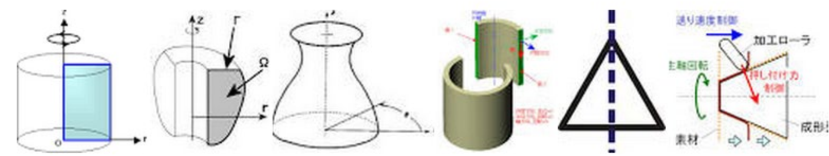
対称の例 Example of symmetry

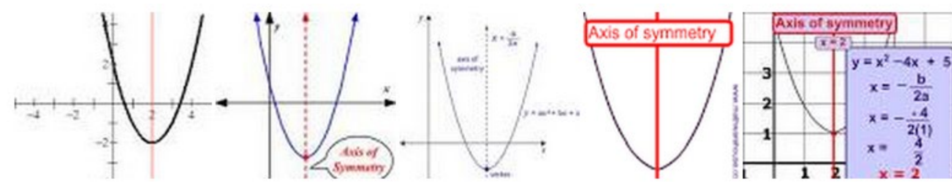
- 点対称Point symmetry

- 線対称Line symmetry

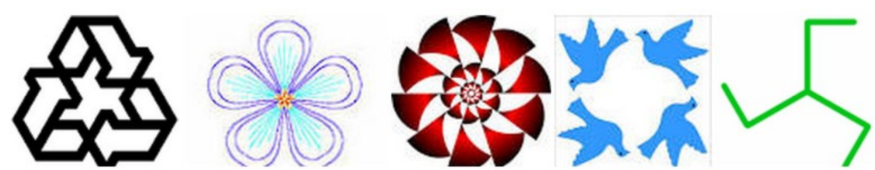


- 軸対称Axis Symmetry



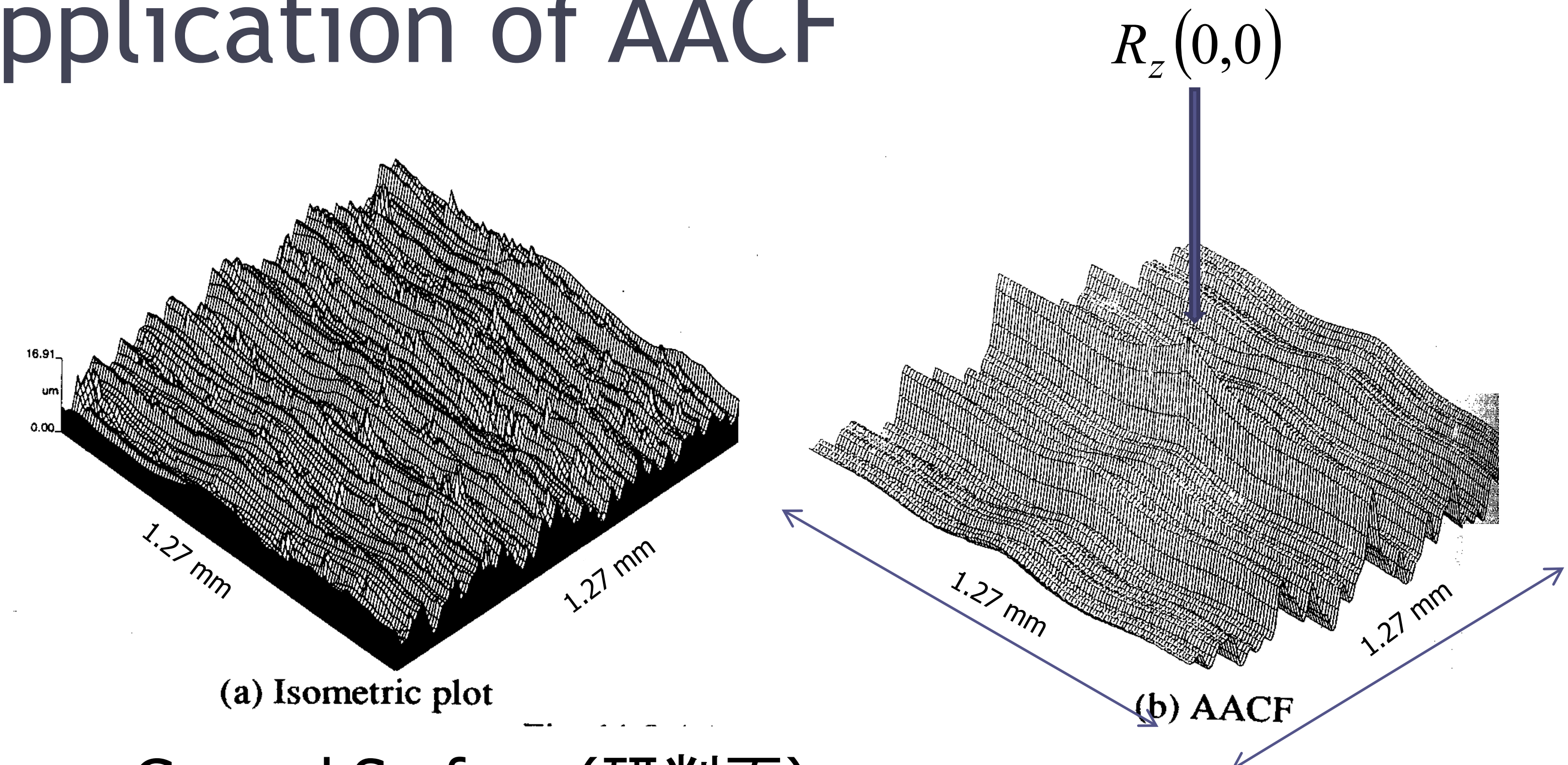


- 回転対称Rotational symmetry



- 回転体solid of revolution

Application of AACF



Ground Surface (研削面)

Fig. Machined surface
and its areal autocorrelation function

Application of AACF in 3D surface topography parameters

auto-correlation length

Sal

horizontal distance of the $ACF(tx, ty)$ which has the fastest decay to a specified value s, with $0 \leq s < 1$

$$Sal = \min_{tx,\, ty \in R} \sqrt{tx^2 + ty^2} \quad \text{where } R = \left\{ (tx, ty) : ACF(tx, ty) \leq s \right\}$$

texture aspect ratio

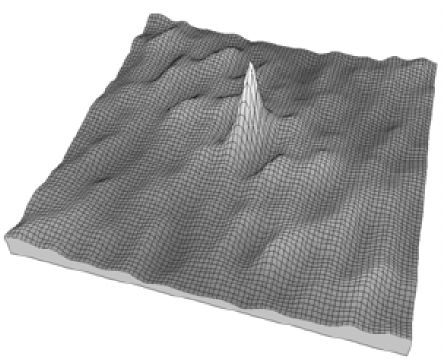
Str

ratio of the horizontal distance of the $ACF(tx, ty)$ which has the fastest decay to a specified value s to the horizontal distance of the $ACF(tx, ty)$ which has the slowest decay to s, with $0 \leq s < 1$

$$Str = \frac{\displaystyle\min_{tx,\, ty \in R} \sqrt{tx^2 + ty^2}}{\displaystyle\max_{tx,\, ty \in Q} \sqrt{tx^2 + ty^2}} \quad \text{where} \quad \begin{aligned} R &= \left\{ (tx, ty) : ACF(tx, ty) \leq s \right\} \\ Q &= \left\{ (tx, ty) : ACF(tx, ty) \geq s \; \& \; ** \right\} \end{aligned}$$

Str ≈ 1: Isotopical
(等方性)

Str >> 1: Anisotopical, Directional
(非等方性)



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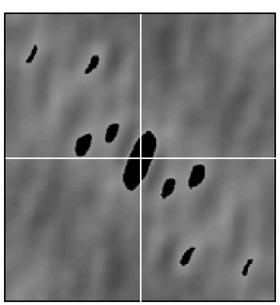

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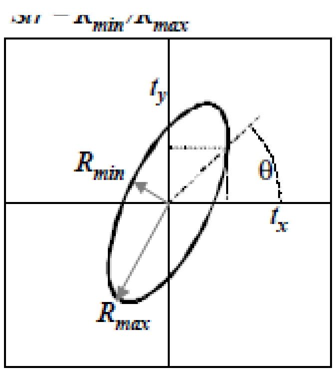


ISO 25178-2

Covariance(共分散)

- Definition

$$R_{xy}(\tau) = E[x(t)\, y(t+\tau)]$$

- Calculation

$$R_{xy}(\tau) = \frac{\int_a^b x(t) y(t+\tau) dx}{b-a}$$

$$R_{xy\,\tau} = \frac{\sum_{t=a}^{b} x_t\; y_{t+\tau}}{b-a+1}$$

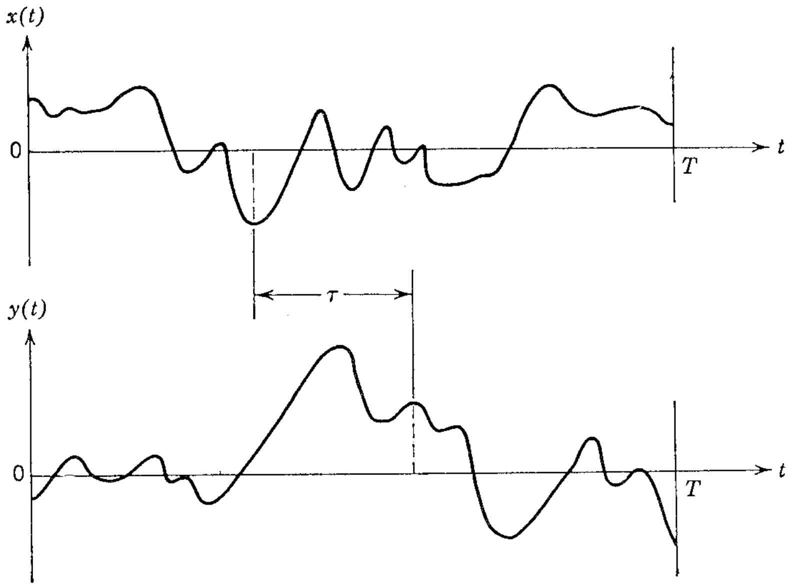


Fig. Calculation of Cross Correlation

Cross Correlation Function(相互相関)

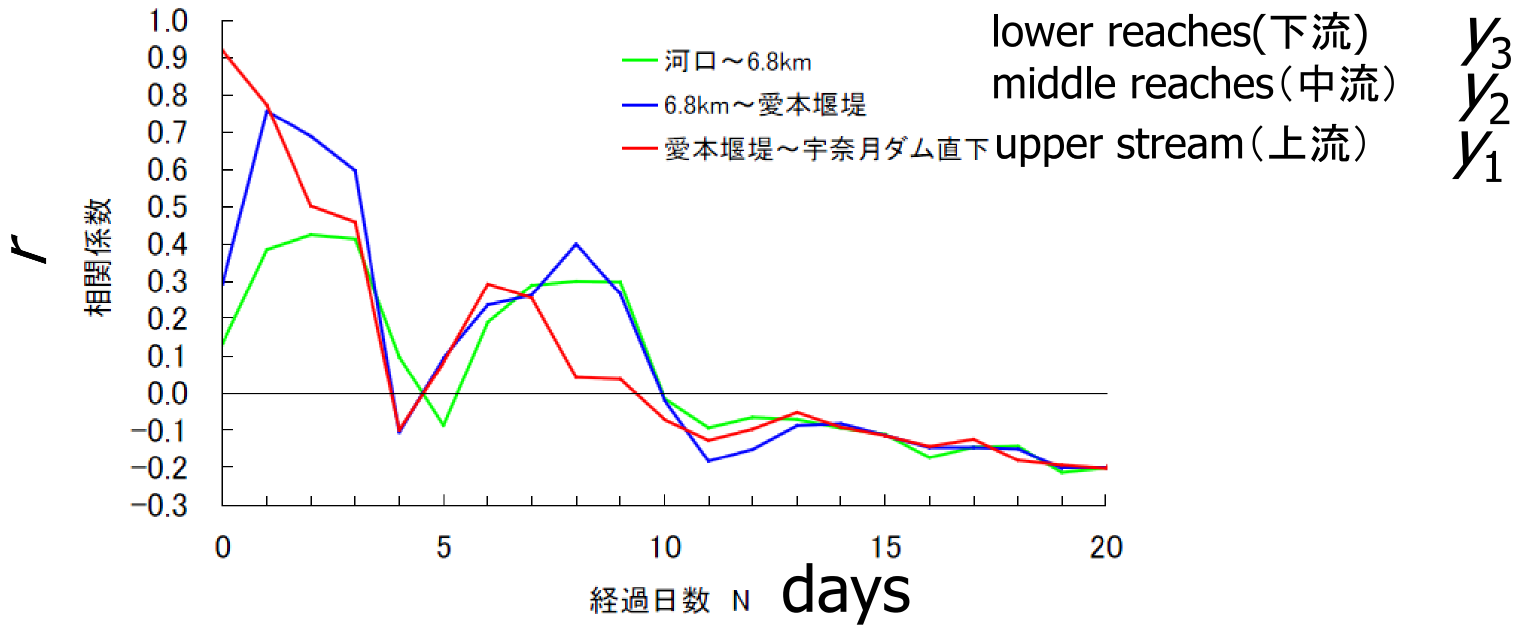
- Normalization of covariance

$$r_{xy\,\tau} = R_{xy\,\tau} \Big/ \sqrt{R_{x0}\ R_{y\,0}}$$

If 2 signals are same $r_{xy\,0} = 1$

otherwise $r_{xy\,0} \neq 1$

Application of cross correlation



図−6　ダムからの年間最大日流出土砂量とその日から N 日後の区間平均日河床変動量絶対値の相関係数

Correlation between the soil discharge of the dam and variation of the riverbed

x　　y

貯水池からの排出土砂の下流への伝播特性
京都大学大学院 ○南修平（㈱ニュージェック）
京都大学防災研究所 藤田正治 (2008)

Choose the range to plot carefully. For example, it is too long to set the horizontal axis of the plot to one month to observe a spectrum with a period of about half a day or to observe the peak deviation of the tide level for a few hours.

Report

1. Calculate and display autocorrelation function, amplitude spectrum and power spectrum of hourly tide observational data for 1 month. Explain the phenomenon revealed by the analysis. In addition, explain the cause of the phenomenon.

   Notes:
   - You may use tidal observational data from Japan Meteorological Agency http://www.data.jma.go.jp/kaiyou/db/tide/genbo/index.php
   - You can choose any location and time (Year, Month). Specify city and time of the data.
   - Don't forget to remove trend and bias before calculation.

2. Calculate and display cross correlation function of the hourly tide observational data between two locations for 1 month. Explain the phenomenon revealed by the analysis. In addition, explain the cause of the phenomenon.

   Notes:
   - Choose a location where the peak tide time is at least 2 hours different.
   - It will be interesting if you choose one location within the Seto inland sea and one location outside the Seto inland sea.
   - Don't forget to remove trend and bias before calculation.

- In English or Japanese. You can attach Mathematica file, MATLAB file, program code, etc.
- Submission limit: Wednesday, May 26
- Submission to: OCW-i
- If you have questions, refer to the lecturer at least 5 days prior to the submission limit.