Image Reconstruction Group

Digital PET Imaging Lab

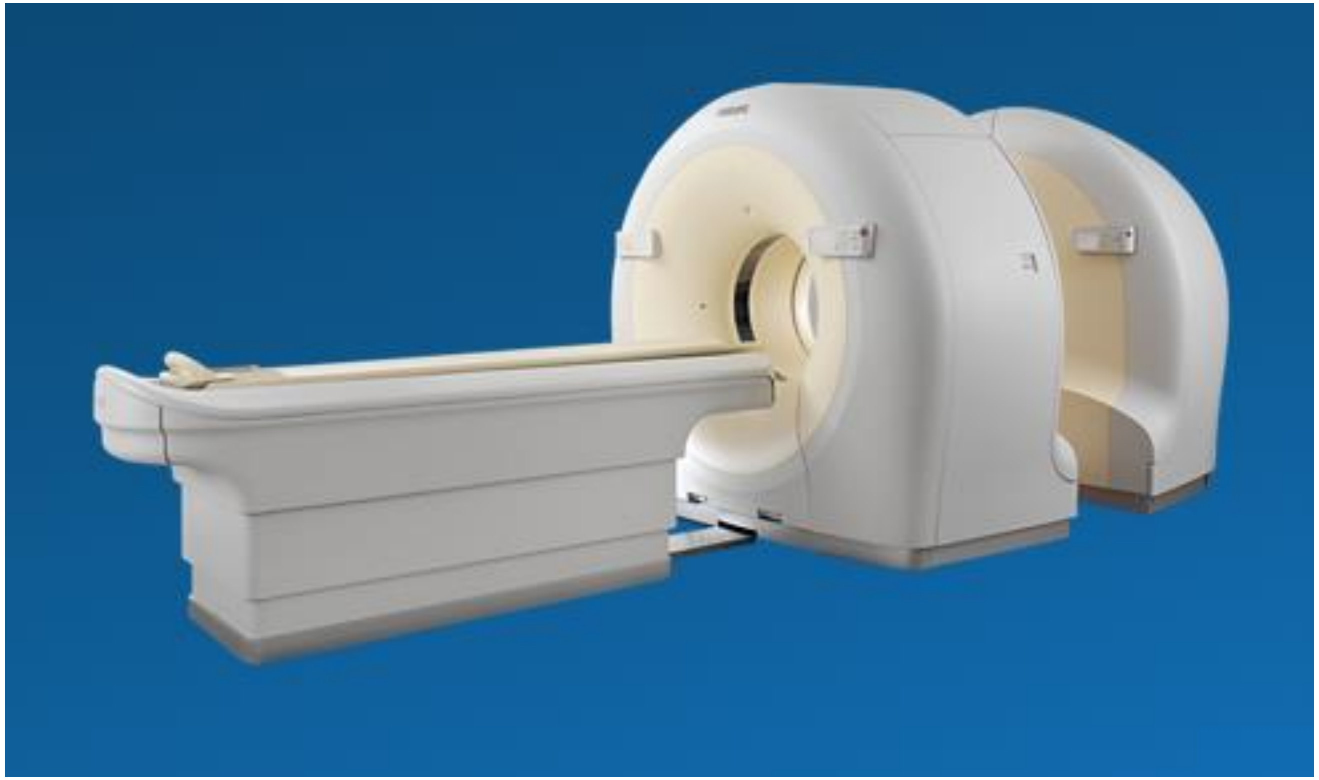
The first sight of PET

mechine & image

Positron Emission Tomography

SIMENS

PHILIPS

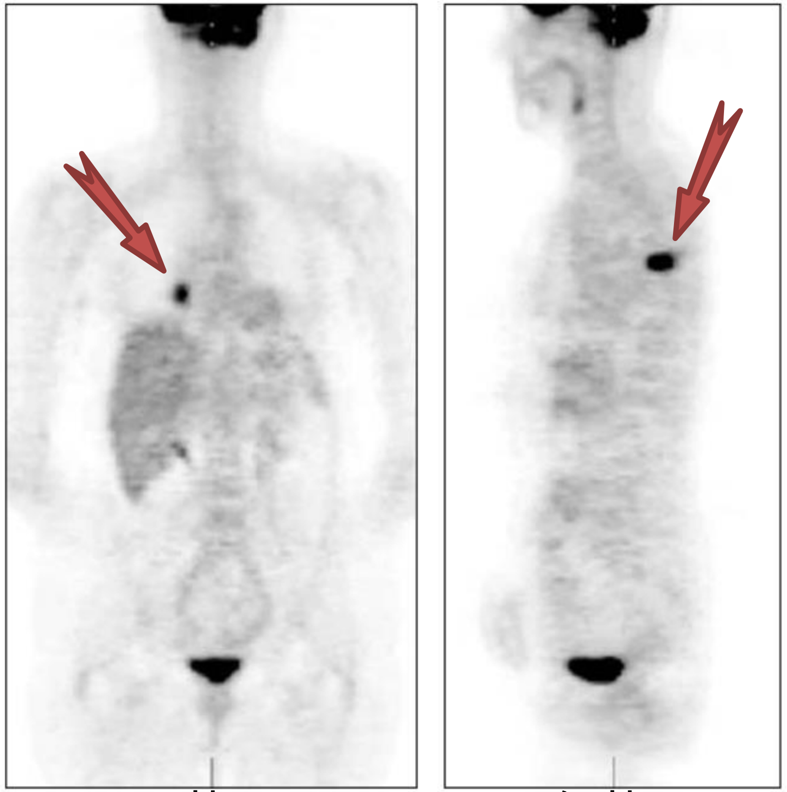


Positron Emission Tomography

RAYCAN

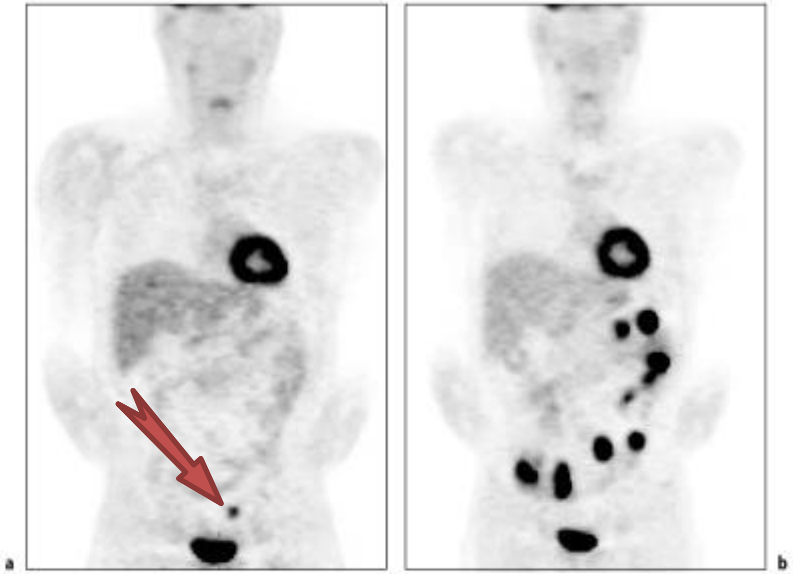
Rays

Clinic PET image



冠状面　　矢状面

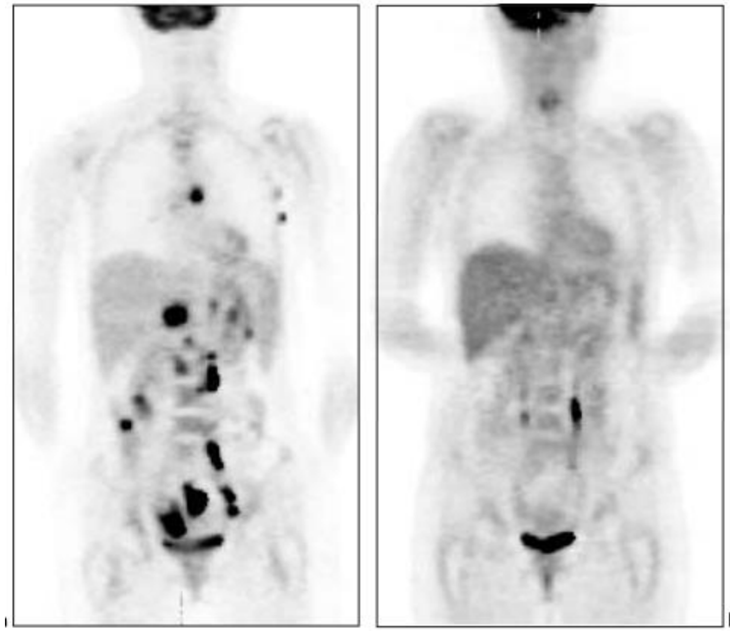
直肠癌切除患者复发的图像
CT未检测出异常，活检证实直肠癌复发
PET检查用于术前分期



术后一个月　　术后一年

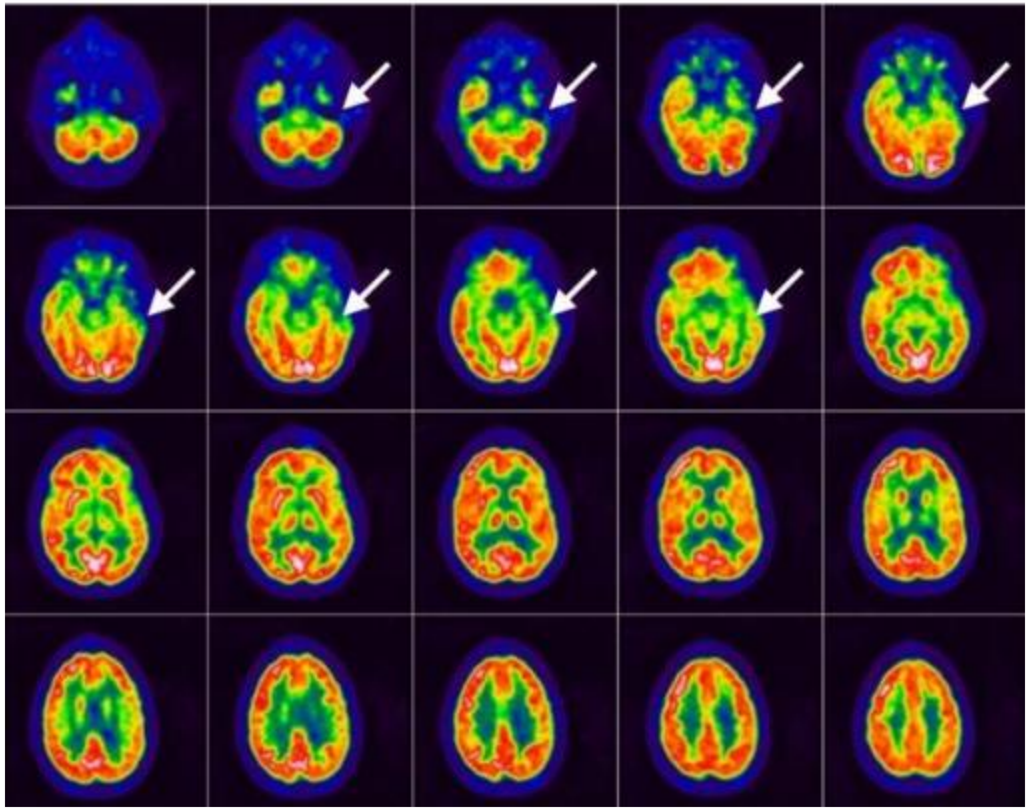
患者从右大腿切除克拉克III级黑色素瘤一个月后左侧骨盆（左）摄取增加。右骨盆也有类似的病灶。患者无症状，骨盆CT扫描阴性。
一年后患者出现胃肠道出血，发现胃粘膜有肿块，活检证实为复发性黑色素瘤。再做PET扫描发现（右）...

对非霍奇金病患者的治疗反应。治疗前扫描(a)显示膈上和膈下遍布肿瘤，而治疗后扫描(b)显示无异常示踪剂定位，表明治疗完全有效

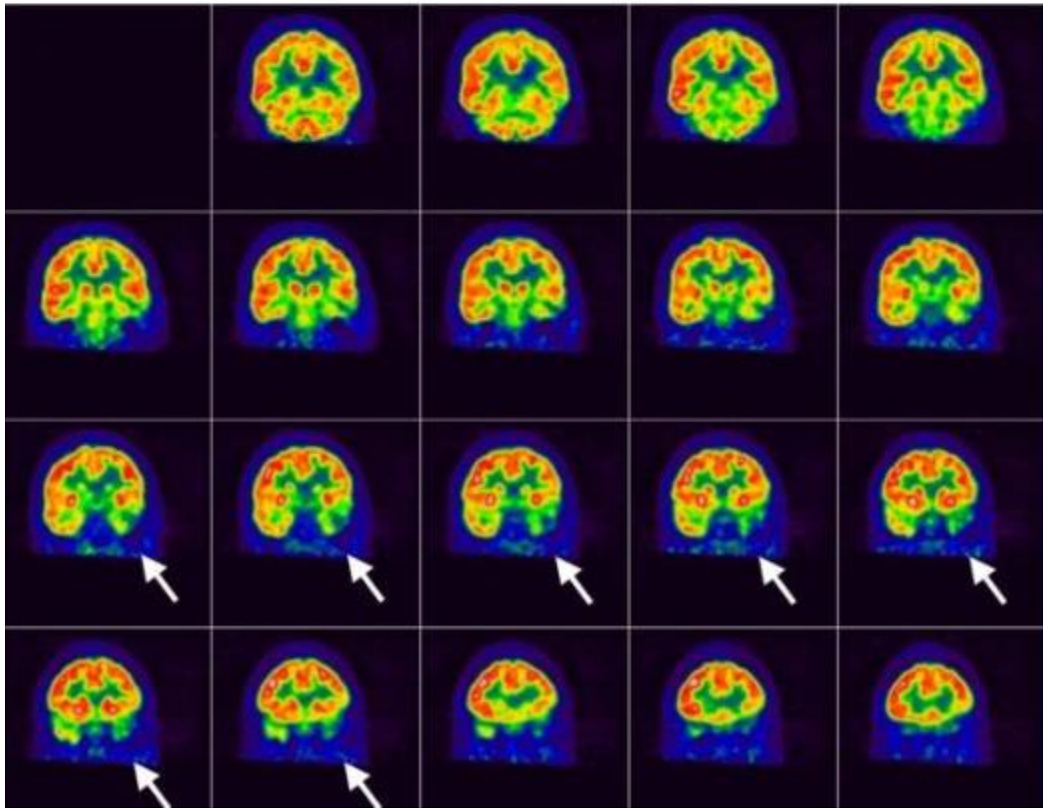


Brain PET image

- 癫痫 Epilepsy

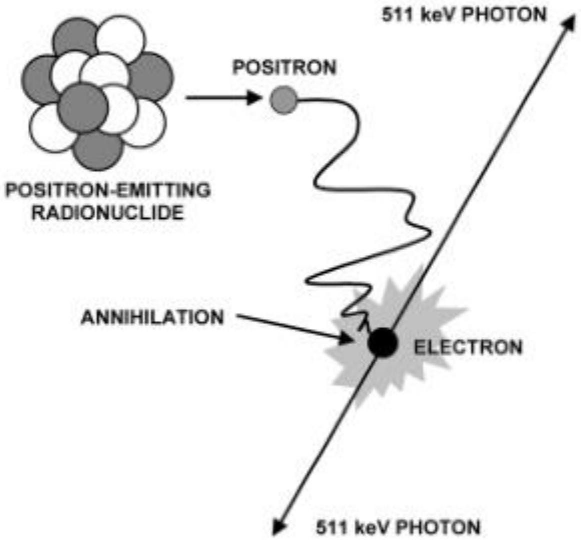


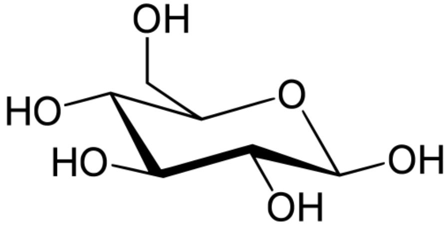
- 癫痫 Epilepsy

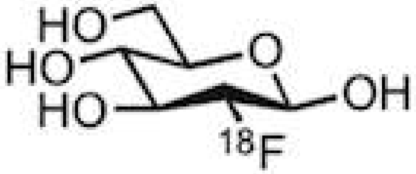


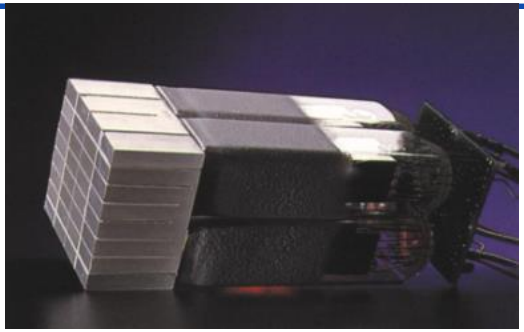
PET imaging

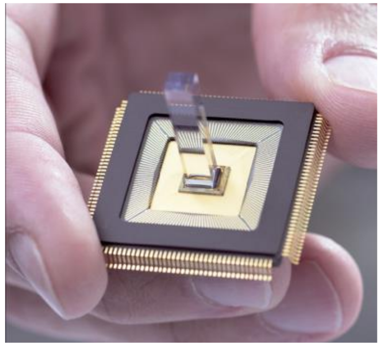
射源与探测器

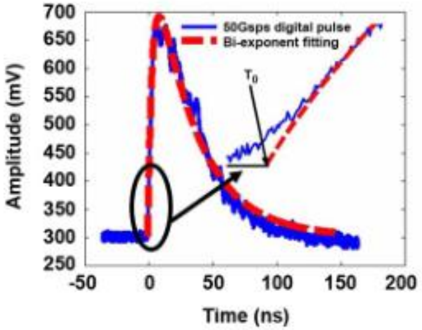


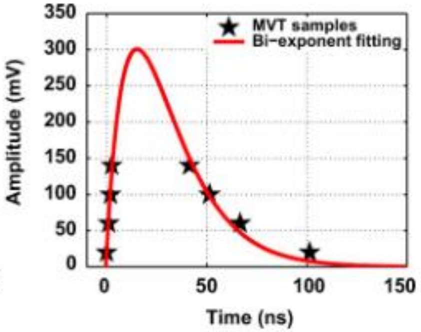










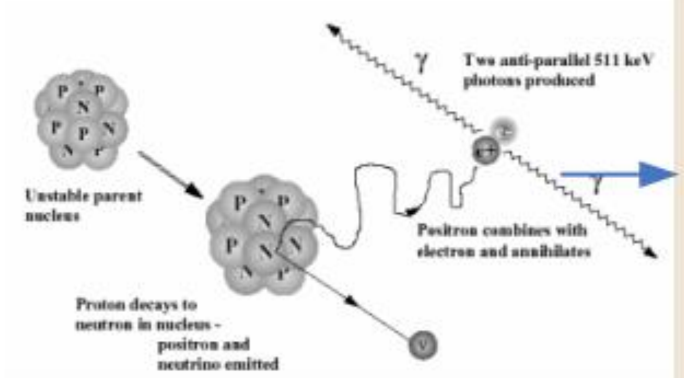


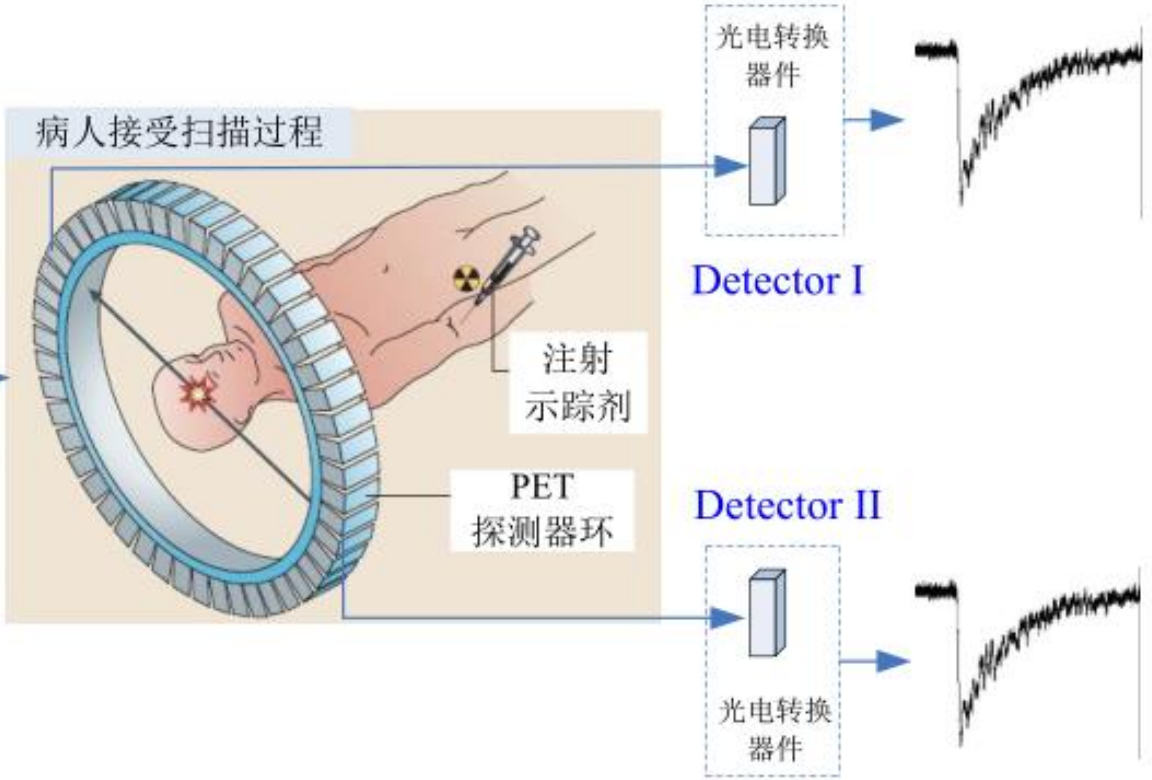
射源：能发射运动方向相反且能量已知的伽马光对子的物质。伽马光子通常由正电子湮灭得到。
例：F-18（FDG，氟代脱氧葡萄糖），O-15，C-14。

$$p \rightarrow n + e^{+} + v$$

探测器：探测射源发射的伽马光子的装置。
通常分为：闪烁晶体+弱光探测器。
闪烁晶体：LSO，LYSO，YSO。
弱光探测器：PMT，SiPM。

PET成像原理





给病人注射示踪剂FDG

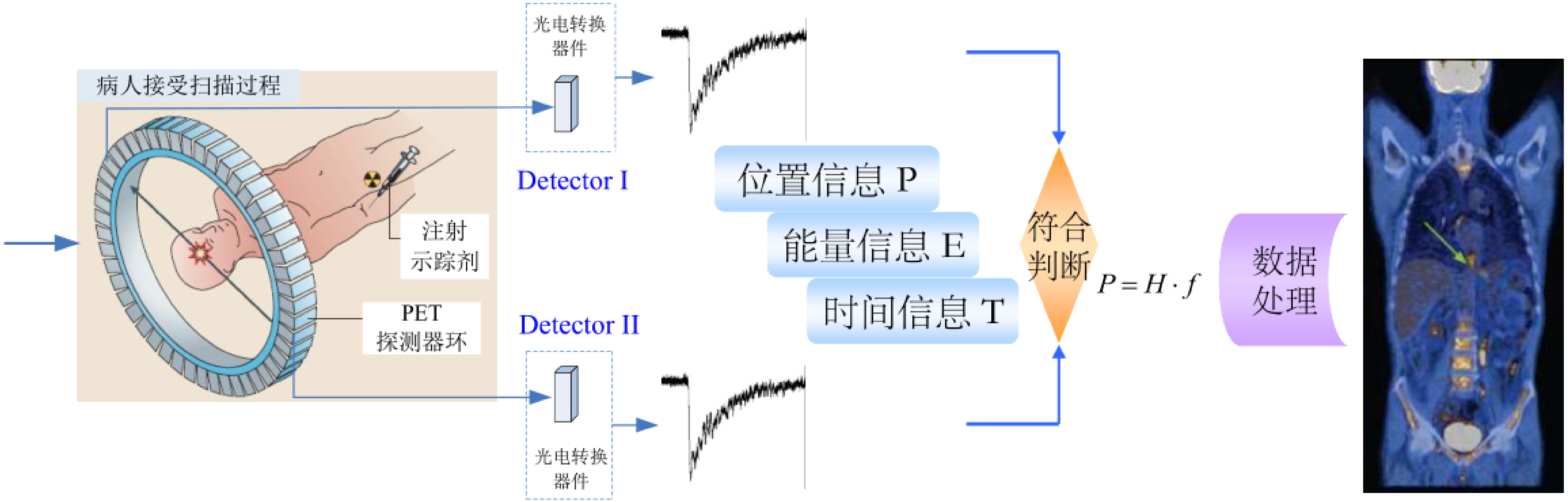
示踪剂β+衰变得到正电子
正电子湮灭产生一对511keV的γ光子

$$p \rightarrow n + e^{+} + \nu$$

$$e^{+} + e^{-} = \gamma + \gamma$$

探测器扫描产生的γ光子，产生闪烁脉冲信号

PET成像原理

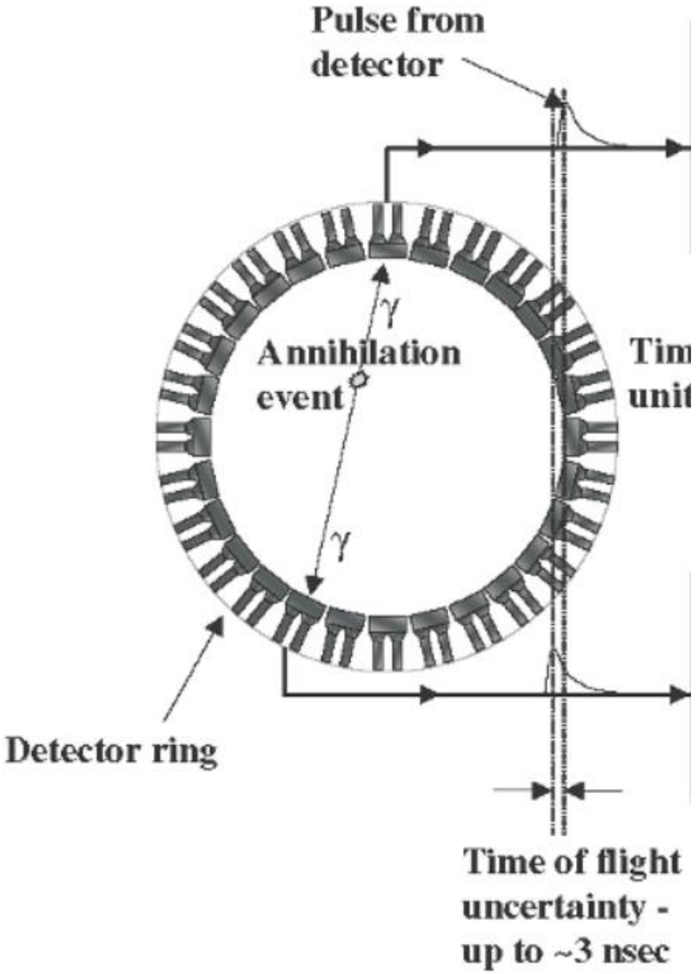


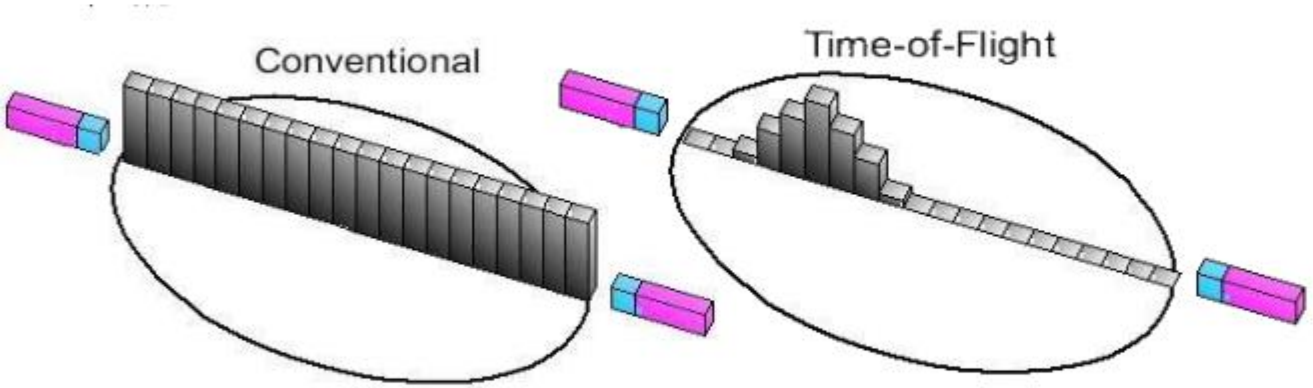
探测器扫描产生的γ光子，产生闪烁脉冲信号

对闪烁脉冲信号进行符合配对

数据处理重建出图像

时间差定位？

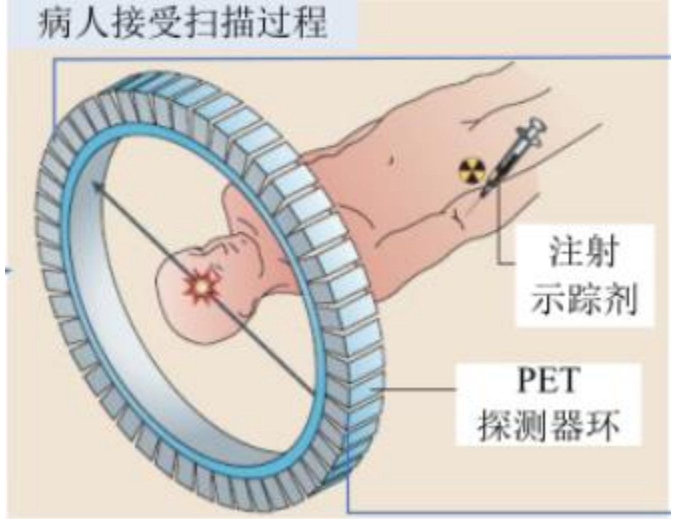


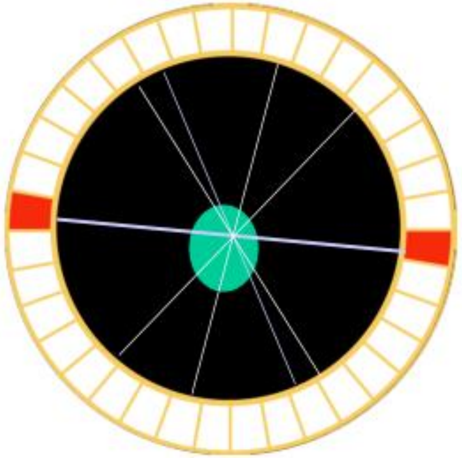


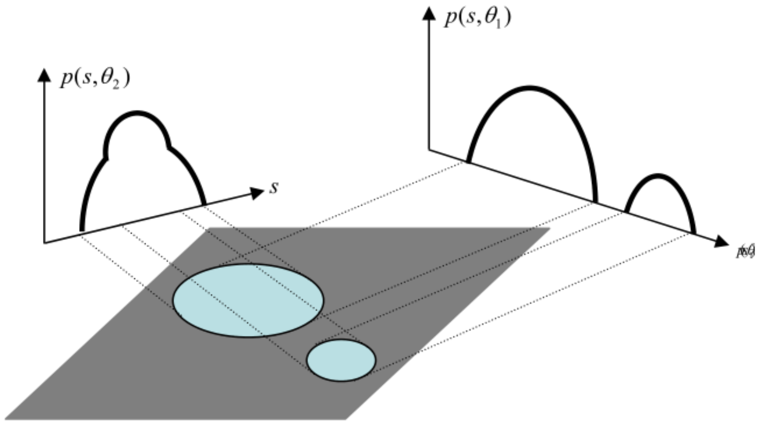
硬件时间精度不够！

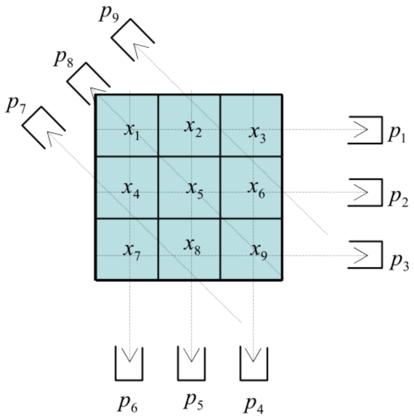
一段时间扫描后的光子对数的规律

假设射源朝各个方向发射的gamma光子一样多。
那么：对于某条直线上探测到的gamma光子对数，就是这条直线上射源活度的求和。





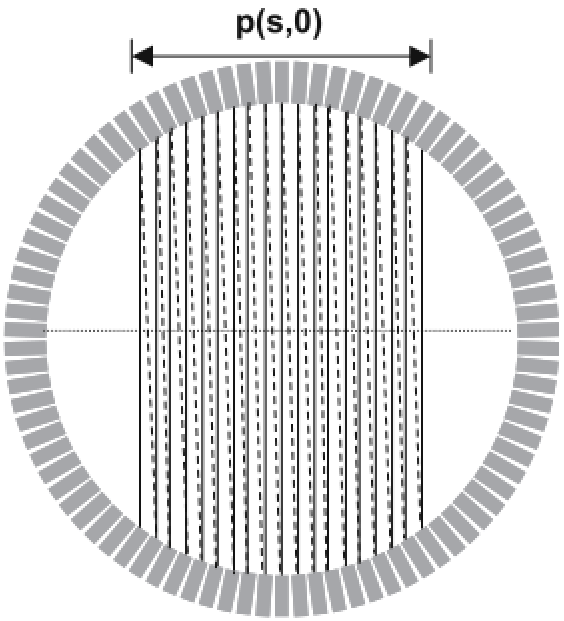


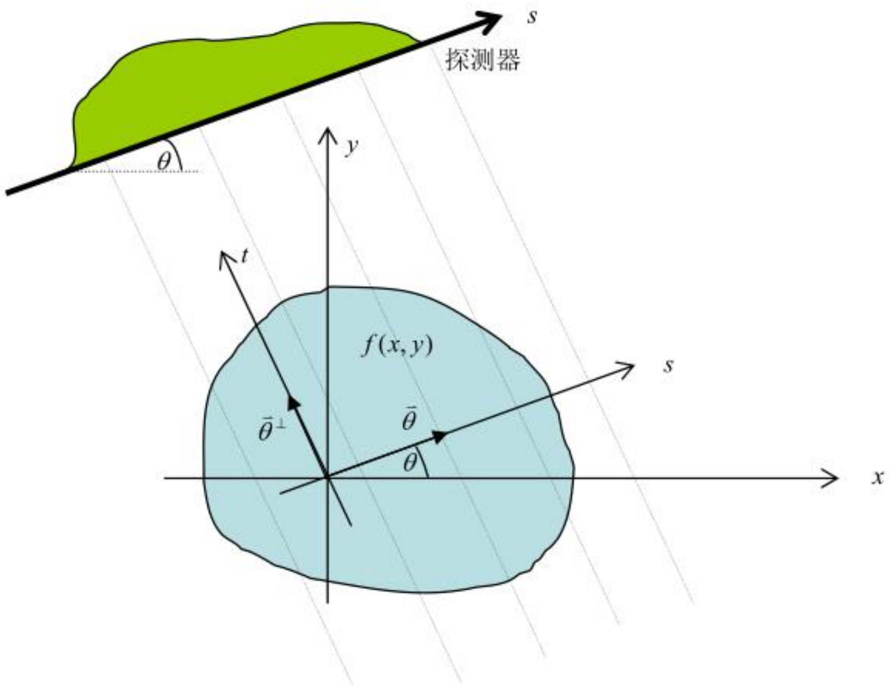


断层成像的基本数学问题

$$
\left\{
\begin{array}{c}
x_1 + x_2 + x_3 = p_1 \\
x_4 + x_5 + x_6 = p_2 \\
x_7 + x_8 + x_9 = p_3 \\
x_3 + x_6 + x_9 = p_4 \\
x_2 + x_5 + x_8 = p_5 \\
x_1 + x_4 + x_7 = p_6 \\
2(\sqrt{2}-1)x_4 + (2-\sqrt{2})x_7 + 2(\sqrt{2}-1)x_8 = p_7 \\
\sqrt{2}x_1 + \sqrt{2}x_5 + \sqrt{2}x_9 = p_8 \\
2(\sqrt{2}-1)x_2 + (2-\sqrt{2})x_3 + 2(\sqrt{2}-1)x_6 = p_9
\end{array}
\right.
$$
。

基本数学问题





$$p(s,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta+y\sin\theta-s)dxdy\text{，}$$

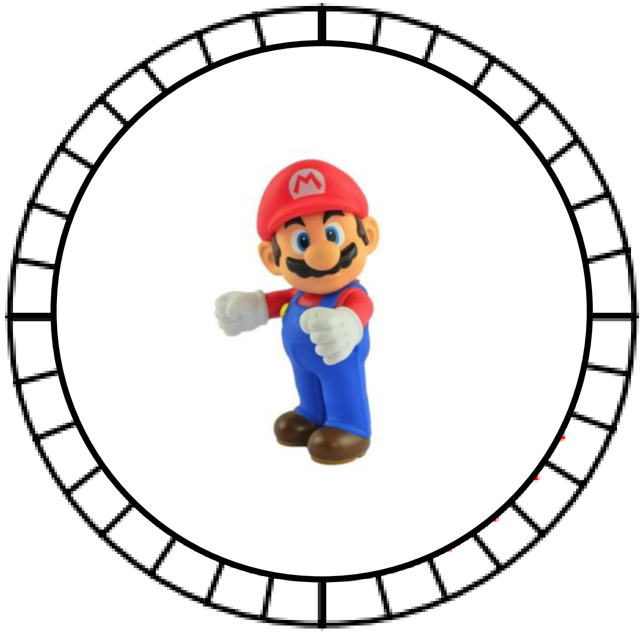
$$p(s,\theta)=\int_{-\infty}^{\infty} f(s\vec{\theta}+t\vec{\theta}^{\perp})dt\text{，}$$

投影

逆问题：已知投影数据，求图像信息。

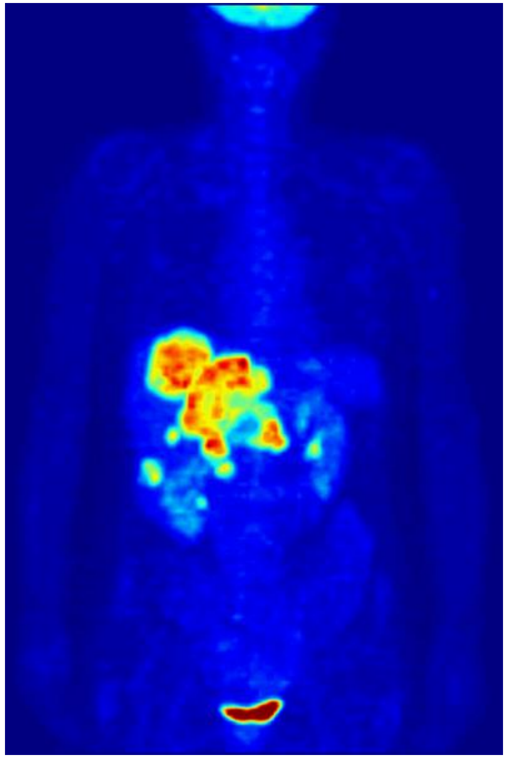
Question 1

- What is image reconstruction ?



$x_j = ?$

reconstruction

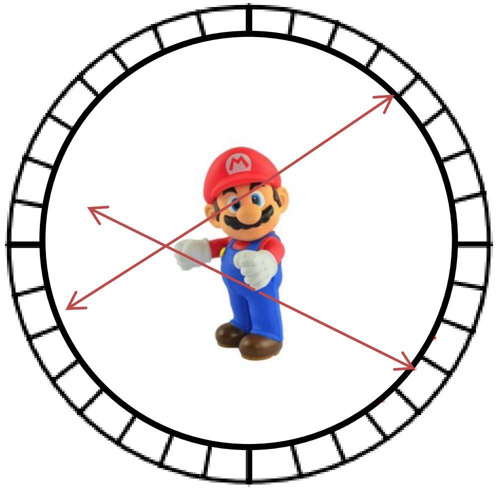


We want to know the distribution of the tracer in human body.

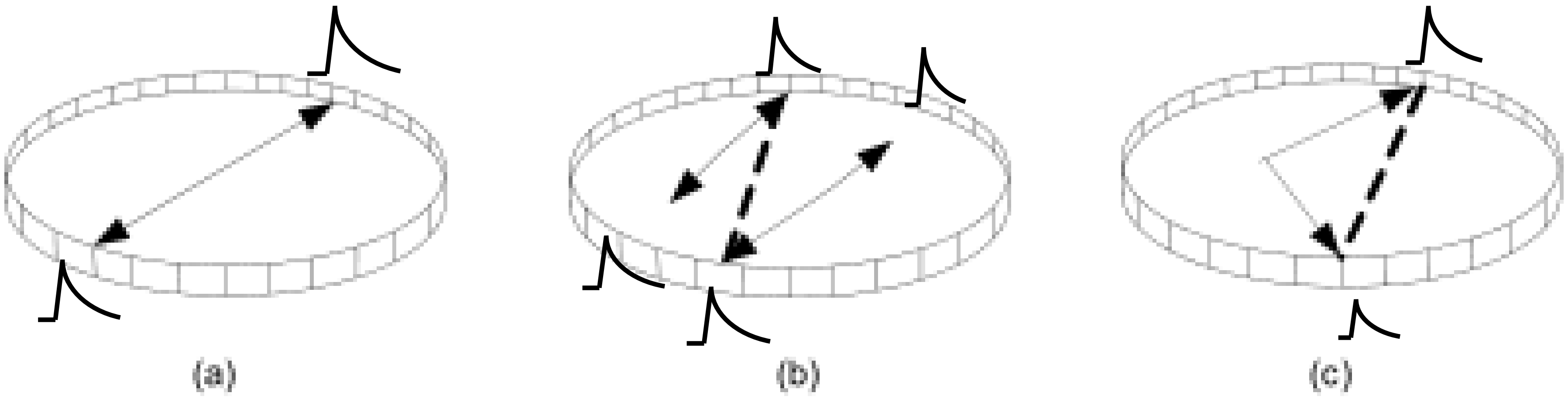
Question 2

- What can we detect by the machine ?(why)

Singles → Coincidence



many gammas



True

Random

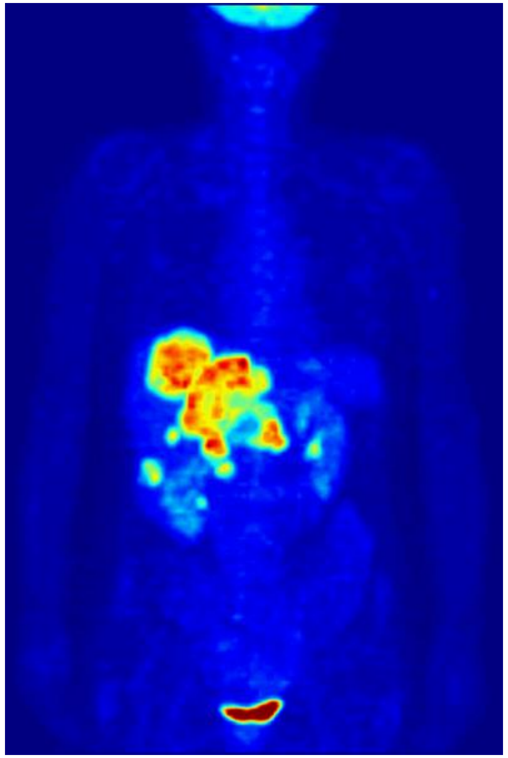
Scatter

Question 3

- How to do the reconstruction ?

A classic Possion model : ML-EM

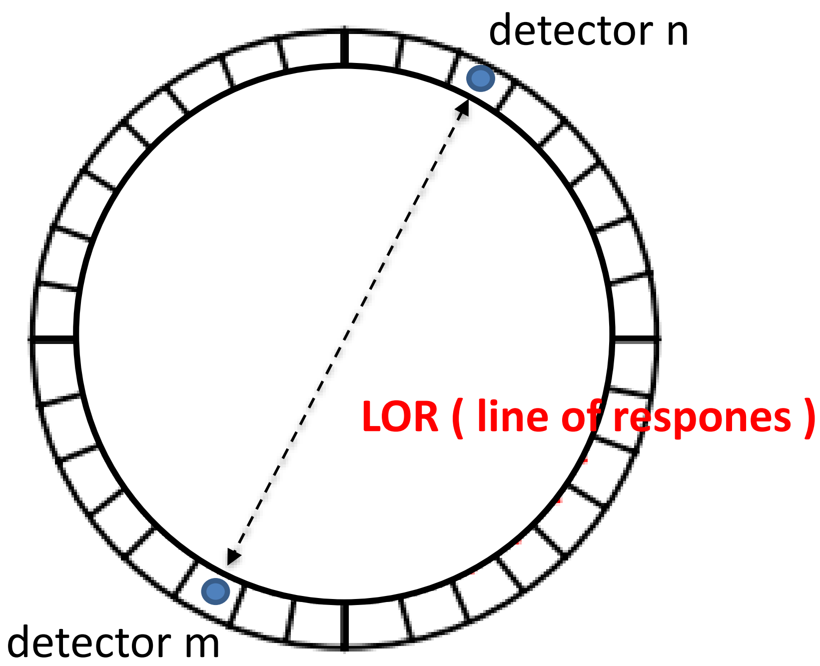
Coincendence →

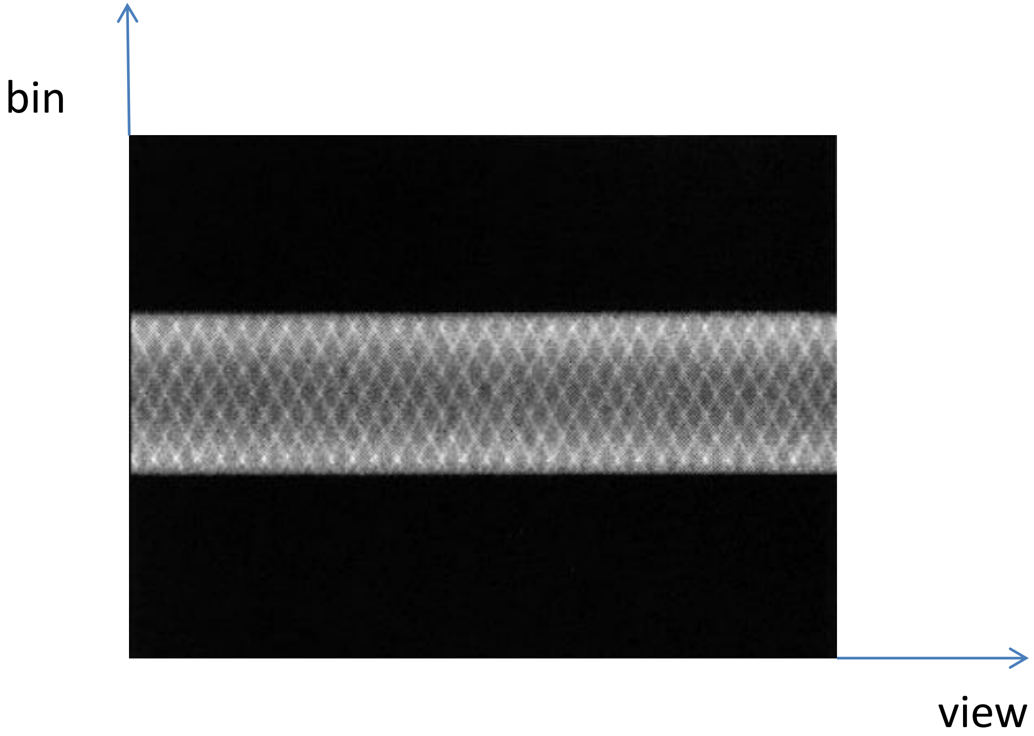


ML-EM : Maximum-Likelihood Expectation-Maximization

Ollinger J M. Maximum-likelihood reconstruction of transmission images in emission computed tomography via the EM algorithm[J]. IEEE Transactions on Medical Imaging, 1994, 13(1):89-101.

LOR & sinogram





sinogram

ML-EM

- Possion model :

$$Y_i \sim P\left(\sum_j A_{ij} X_j\right)$$

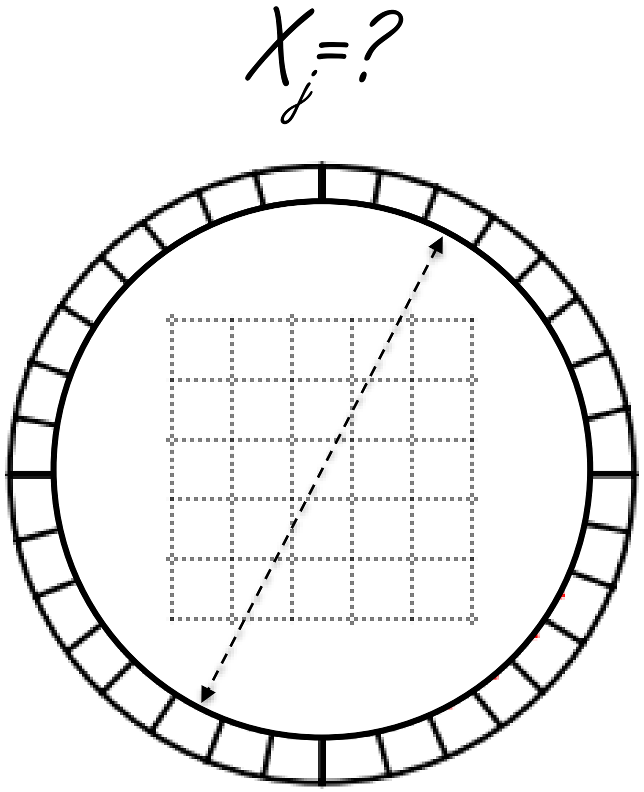
Y is the coincidence counts of LOR_i.
The elements, A_{ij}, of the system response matrix A denote the probability of detecting an emission from pixel site j at LOR_i.

We define a random variable C_{ij}. It denote the counts of detecting an emission from pixel site j at LOR_i.

$$Z_{ij} \sim P\left(A_{ij} X_j\right)$$

and

$$Y_i = \sum_j Z_{ij}$$



ML (Maximum-Likelihood)

- Constructing objective function by likelihood function.

$$Z_{ij} \sim P\left(A_{ij} X_j\right) \qquad P\left(Z_{ij} = k\right) = \frac{\left(A_{ij} X_j\right)^k e^{-A_{ij} X_j}}{k!}$$

- We assume that we have known the Z_{ij}. And construct a likelyhood function by Z_{ij}.

$$L(X \mid Z) = \frac{\left(A_{ij} X_j\right)^{Z_{ij}} e^{-A_{ij} X_j}}{Z_{ij}!}$$

- Logarithm and simplification.

$$\ln\left(L(X \mid Z)\right) = \sum_i \sum_j \left(-A_{ij} X_j + Z_{ij} \ln\left(A_{ij} X_j\right)\right) + const$$

ML (Maximum-Likelihood)

- Object function of Possion model constructed by likelyhood function:

$$\ln\left(L(X \mid Z)\right) = \sum_{i}\sum_{j}\left(-A_{ij}X_{j} + Z_{ij}\ln\left(A_{ij}X_{j}\right)\right) + const$$

- Object function is :

$$\boxed{\Phi(X) = \sum_{i}\sum_{j}\left(-A_{ij}X_{j} + Z_{ij}\ln\left(A_{ij}X_{j}\right)\right)}$$

EM (Expectation-Maximization)

$$\Phi(X)=\sum_i\sum_j\left(-A_{ij}X_j+Z_{ij}\ln\left(A_{ij}X_j\right)\right)$$

- We still don't know about the Z_{ij}.
- Using EM algorithm to solve the object function.
- Use conditional expectations of Zij instead of itself.

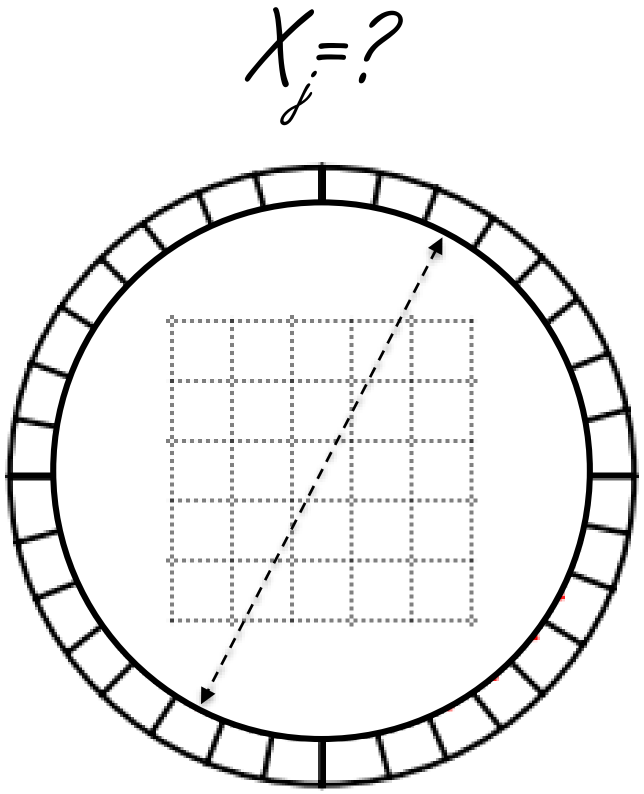
$$Z_{ij}=E\left(Z_{ij}\mid X^k\right)=Y_i\frac{A_{ij}X^k}{\sum_j A_{ij}X^k}$$

- X^k is the image of the k iteration.

EM (Expectation-Maximization)

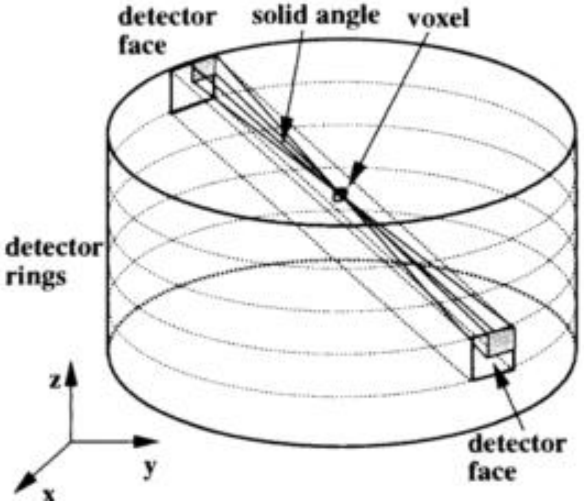
- The image of k+1 iteration is :

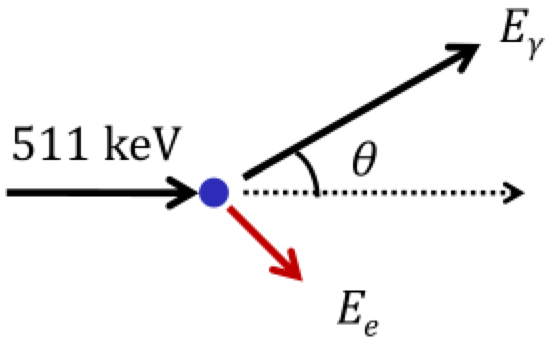
$$X^{k+1} = X = X^{k} \frac{\sum_i A_{ij} \frac{Y_i}{\sum_j A_{ij} X_j^k}}{\sum_i A_{ij}}$$

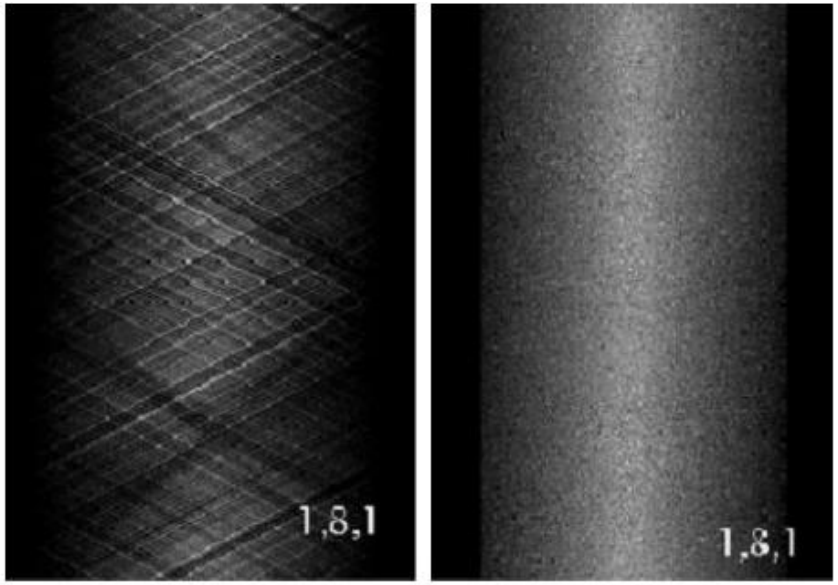


What's more

- physical factor of system response :
  - geometry
  - scatter
  - random coincidence
  - detector effection
  - ......

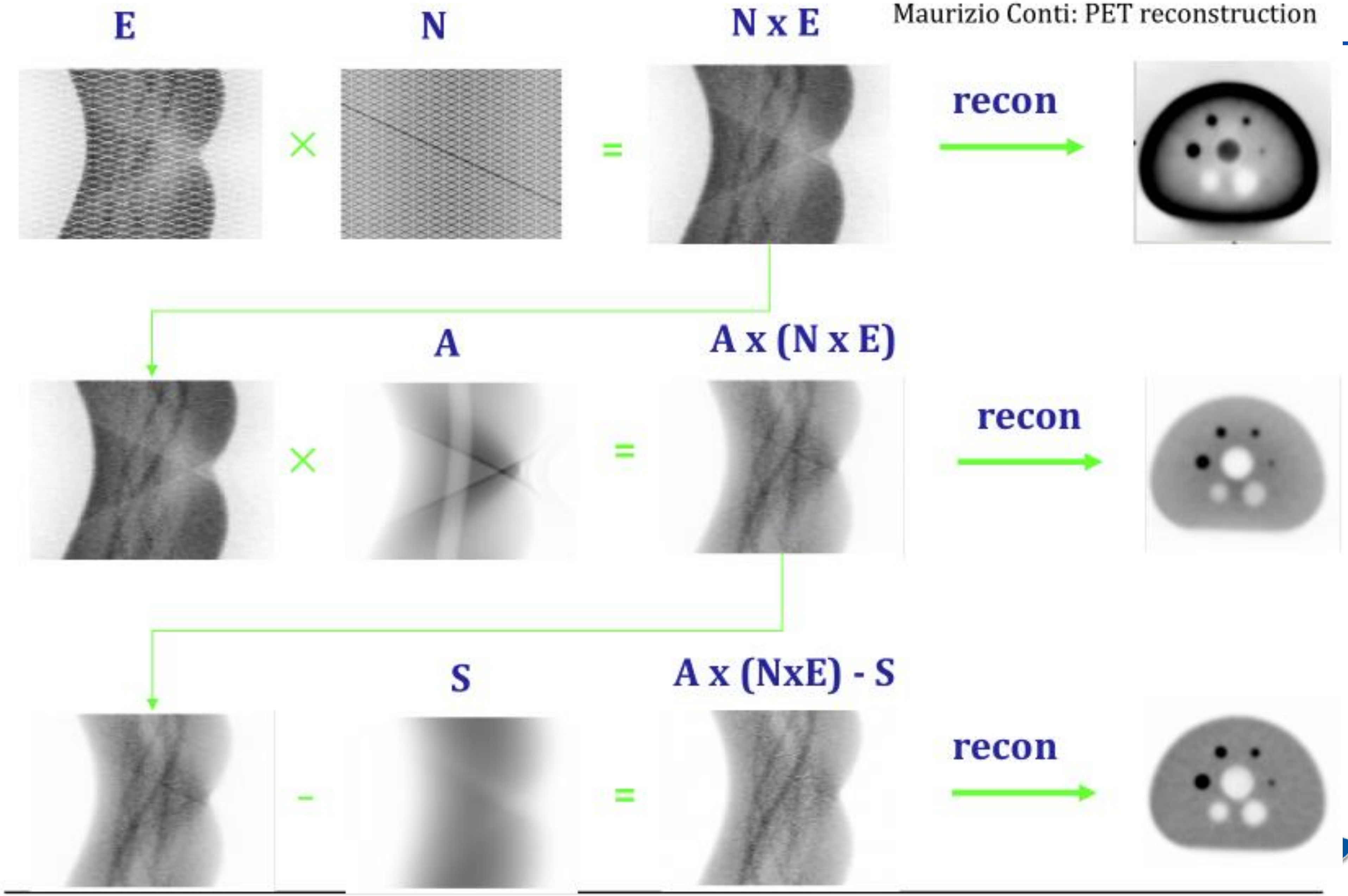






Uncorrected Corrected

Maurizio Conti: PET reconstruction



Thank you !

题目

培训时讲到了两种PET扫描的泊松模型$Y_i \sim P\left(\sum_j A_{ij} X_j\right)$和 $Z_{ij} \sim P(A_{ij} X_j)$

1）请分别解释各符号的含义。

2）在ML-EM算法中使用的是哪个模型构造似然函数并通过极大似然法构造目标函数？

3）根据极大似然法推倒出ML-EM算法的目标函数。（选做）

提示：泊松分布概率公式为：$P(Z_{ij} = k) = \dfrac{(A_{ij} X_j)^k e^{-A_{ij} X_j}}{k!}$