

Quantum Computation and Quantum Information

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and ISAAC L. CHUANG**

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Quantum Computation and Quantum Information

10th Anniversary Edition

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