# The second assignment of Matrix Analysis

## Slide4 Exercises11

**Solution: (a)** Suppose $PA = LU$, $P$ is the row permutation matrix.

$$[A|p] = \left(\begin{array}{cccc|c} 1 & 2 & 4 & 17 & 1 \\ 3 & 6 & -12 & 3 & 2 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \to \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ 1 & 2 & 4 & 17 & 1 \\ 2 & 3 & -3 & 2 & 3 \\ 0 & 2 & -2 & 6 & 4 \end{array}\right) \to$$

$$\left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{1/3} & 0 & 8 & 16 & 1 \\ \mathbf{2/3} & -1 & 5 & 0 & 3 \\ \mathbf{0} & 2 & -2 & 6 & 4 \end{array}\right) \to \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{2/3} & -1 & 5 & 0 & 3 \\ \mathbf{1/3} & 0 & 8 & 16 & 1 \end{array}\right) \to$$

$$\left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{2/3} & \mathbf{-1/2} & 4 & 3 & 3 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \end{array}\right) \to \left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \\ \mathbf{2/3} & \mathbf{-1/2} & 4 & 3 & 3 \end{array}\right) \to$$

$$\left(\begin{array}{cccc|c} 3 & 6 & -12 & 3 & 2 \\ \mathbf{0} & 2 & -2 & 6 & 4 \\ \mathbf{1/3} & \mathbf{0} & 8 & 16 & 1 \\ \mathbf{2/3} & \mathbf{-1/2} & \mathbf{1/2} & -5 & 3 \end{array}\right)$$

Therefore,

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 2/3 & -1/2 & 1/2 & 1 \end{pmatrix}, U = \begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix},$$

**(b)** The system $Ax = b$ is equivalent to $PAx = Pb$. In **(a)** we have already performed the factorization $PA = LU$, then we can solve $Ly = Pb$ for $y$ by forward substitution:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 1 & 0 \\ 2/3 & -1/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 17 \\ 3 \end{pmatrix} \Longrightarrow y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix}$$

and then solve $Ux = y$ by back substitution:

$$\begin{pmatrix} 3 & 6 & -12 & 3 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & 16 \\ 0 & 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 16 \\ -5 \end{pmatrix} \Longrightarrow x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$