The first assignment of Matrix Analysis

Slide2 Exercises 6

Solution: **(a)** 3-digit arithmetic without pivoting:

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right)_{R_2-10^3R_1} \longrightarrow \left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 0 & \boxed{10^3} & -10^3 \end{array}\right)$$

Cause when keeping three significant figures:

$$float(1+10^3) = float(0.1001\times 10^4) \overset{3-digit}{\longrightarrow} 0.100\times 10^4 = 10^3$$

Back substitution, then we get the answer:

$$x=0, y=-1$$

(b) 3-digit arithmetic with pivoting:

$$\left(\begin{array}{cc|c} 10^{-3} & -1 & 1 \\ 1 & 1 & 0 \end{array}\right)_{swap\ rows} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 10^{-3} & -1 & 1 \end{array}\right)_{R_2-10^{-3}R_1} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & \boxed{-1} & 1 \end{array}\right)$$

Cause when keeping three significant figures:

$$float(-1-10^{-3}) = float(-0.1001\times 10^1) \overset{3-digit}{\longrightarrow} -0.100\times 10^1 = -1$$

Back substitution, then we get the answer:

$$x=1, y=-1$$

Slide3 Exercises 4

Solution: **(a)** Transform the augmented matrix $[\mathbf{A}|b]$ as follows:

$$[\mathbf{A}|b] = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right)$$
$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

Obviously, the rank of $\mathbf{A}$ is equal to the rank of $[\mathbf{A}|b]$, so this system is consistent.

The fundamental set of solutions for the corresponding homogeneous system ($\mathbf{A}x=0$) is:

$$\xi_1 = (1,-1,-1,1)^T, \xi_2 = (2,-1,0,0)^T$$

A particular solution for the nonhomogeneous system is:

$$\eta = (0,0,1,1)^T$$

So the general solution for the nonhomogeneous system is:

$$x = \eta + k_1\xi_1 + k_2\xi_2, k_i \in R, i=1,2$$