矩阵分析第五次作业

课件7 Exercises 3

解:

$A^TA = 4A$ 的特征值为 $0, 10$，故:

$$\begin{aligned} ||A||_F &= 1^2 + (-2)^2 + (-1)^2 + 2^2 = 10 \\ ||A||_1 &= max\{1+1, 2+2\} = 4 \\ ||A||_2 &= max\{\sqrt{10}, 0\} = \sqrt{10} \\ ||A||_\infty &= max\{1+2, 1+2\} = 3 \end{aligned}$$

$B^TB = I$ 的特征值为 $1, 1, 1$，故:

$$\begin{aligned} ||B||_F &= 1^2 + 1^2 + 1^2 = 3 \\ ||B||_1 &= max\{1+0+0, 0+1+0, 0+0+1\} = 1 \\ ||B||_2 &= max\{1, 1, 1\} = 1 \\ ||B||_\infty &= max\{1+0+0, 0+1+0, 0+0+1\} = 1 \end{aligned}$$

$C^TC = 9C$ 的特征值为 $0, 0, 81$，故:

$$\begin{aligned} ||C||_F &= 2 \cdot (4^2 + (-2)^2 + 4^2) + (-2)^2 + 1^2 + (-2)^2 = 81 \\ ||C||_1 &= max\{4+2+4, 2+1+2, 4+2+4\} = 10 \\ ||C||_2 &= max\{0, 0, 9\} = 9 \\ ||C||_\infty &= max\{4+2+4, 2+1+2, 4+2+4\} = 10 \end{aligned}$$

课件7 Exercises 12

解: **(a)** 设 $A = (\alpha_1, \alpha_2, \alpha_3) = QR = (q_1, q_2, q_3) \cdot R$，对 A 列向量进行 Schmidt 正交化为 q_1, q_2, q_3

$$\begin{aligned} q_1 &= \frac{\alpha_1}{||\alpha_1||} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0)^T \\ q_2 &= \frac{\alpha_2 - (q_1\alpha_2)q_1}{||\alpha_2 - (q_1\alpha_2)q_1||} = (\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}})^T \\ q_3 &= \frac{\alpha_3 - (q_1\alpha_3)q_1 - (q_2\alpha_3)q_2}{||\alpha_3 - (q_1\alpha_3)q_1 - (q_2\alpha_3)q_2||} = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, 0)^T \end{aligned}$$

所以

$$(\alpha_1, \alpha_2, \alpha_3) = QR = (q_1, q_2, q_3) \cdot \begin{pmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix}$$

其中

$$Q = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & 0 \end{pmatrix}$$

(b) 方程 $Ax = x$ 即 $QRx = b$，将方程两边乘 Q^T，则最小二乘方程化为 $Rx = Q^Tb$

$$\begin{pmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ \frac{1}{\sqrt{3}} \\ 0 \end{pmatrix}$$

解得

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix}$$

课件7 Exercises 16

解:

(a) $\frac{vv^T}{v^Tv}u = (\frac{1}{6}, \frac{2}{3}, 0, \frac{-1}{6})^T$

(b) $\frac{uu^T}{u^Tu}v = (\frac{-2}{5}, \frac{1}{5}, \frac{3}{5}, \frac{-1}{5})^T$

(c) $(I - \frac{vv^T}{v^Tv})u = (\frac{-13}{6}, \frac{1}{3}, 3, \frac{-5}{6})^T$

(b) $(I - \frac{uu^T}{u^Tu})v = (\frac{7}{5}, \frac{19}{5}, \frac{-3}{5}, \frac{-4}{5})^T$