

# Integrity (Crypto Lecture #1)



# Administrivia

Office hours

SSD forms

Project one: released next week, watch Piazza for announcement  
Find your partner!

Homework 1 due on Friday, September 14, 6 PM  
Don't be late!  
Submit w/ Canvas

Submission is through Gradescope, **must be a PDF**

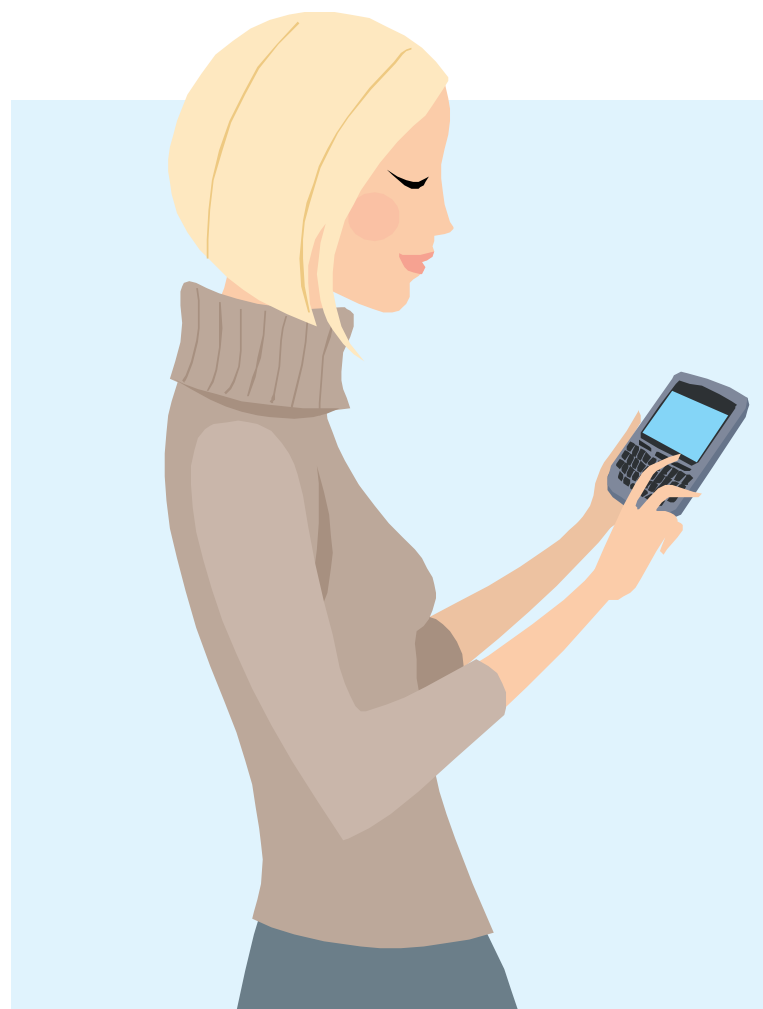
# Cryptography?

What is cryptography?  $f: \{0,1\}^n \rightarrow \{0,1\}^m$   
*invertible*

Why study cryptography?

What do we want to use cryptography to defend against?

What properties does cryptography provide / what properties do we want from the cryptography that we use?



*Alice*

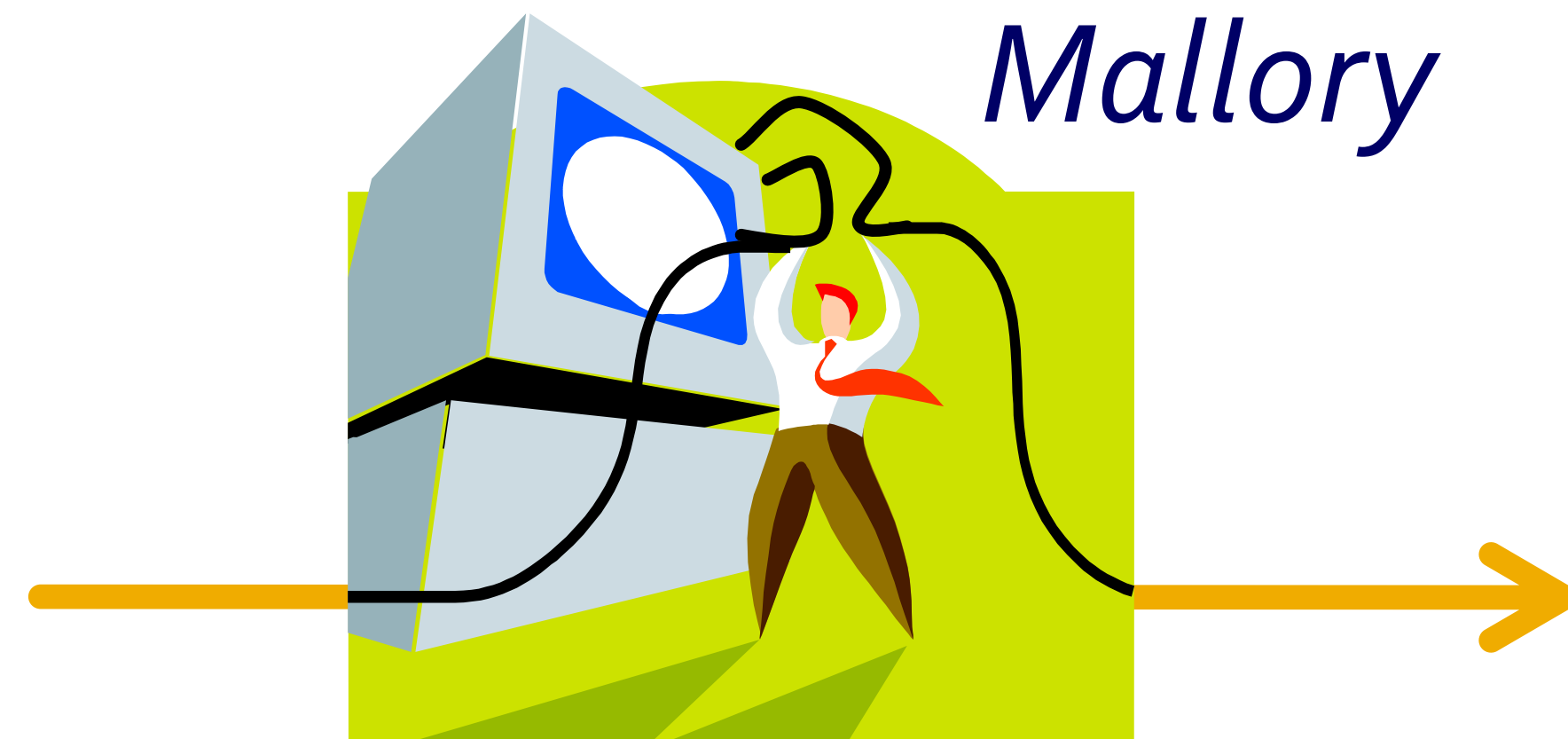
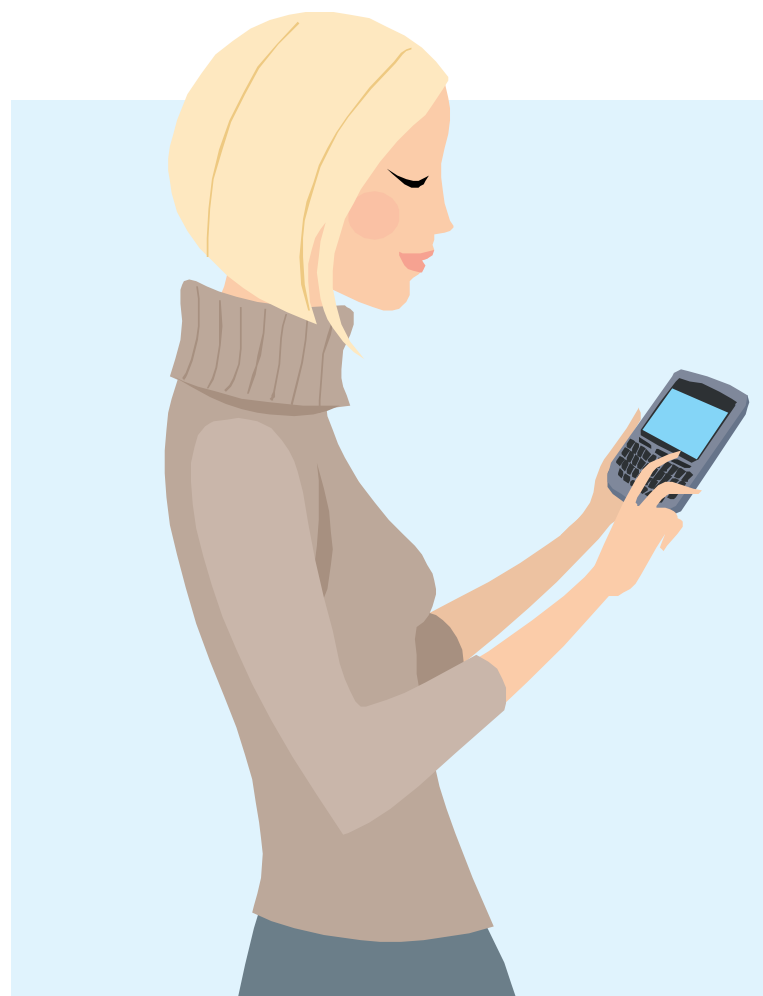


*Eve*

Passive Eavesdropper



*Bob*



*Mallory*

Man-in-the-Middle



# Properties of a Secure Channel

Confidentiality *defeat Eve*

Integrity  *defeat Mallory*

Authentication *how does Bob know the message from Alice*

# Message Integrity

An attacker cannot modify messages without being detected.

# Goal: Message Integrity

Alice wants to send message ***m*** to Bob, *but...*

- We don't trust the messenger
- I.e., we don't trust the network



Want to be sure **what Bob receives** is actually **what Alice sent**.

# Goal: Message Integrity

## Threat Model

- Mallory can see, modify, and forge messages
  - Mallory wants to trick Bob into accepting a message Alice didn't send
- man-in-the-middle





# One Approach...

1. Alice computes  $\mathbf{v} := f(\mathbf{m})$

2. E.g.  $\mathbf{m}$  = "Attack at dawn",  $f(\mathbf{m})$  = 628369867...



3. Bob verifies that  $\mathbf{v}' = f(\mathbf{m}')$ , and accepts the message if and only if this is true.

# Function $f$ ?

Bob accepts the message iff  $\mathbf{v}' = \mathbf{f}(\mathbf{m}')$ .

We want  $\mathbf{f}$  to be easily computable by Alice and Bob, but **not** computable by Mallory.

(As we shall see) we lose if Mallory can learn  $\mathbf{f}(\mathbf{x})$  for any  $\mathbf{x} \neq \mathbf{m}$ !



# Candidate: Random Function

Input: Any size up to huge maximum

Output: Fixed size (e.g. 256 bits)

Defined by a giant lookup table that's filled in by flipping coins.

0	→	0011111001010001...
1	→	1110011010010100...
2	→	0101010001010000...

# Random Function

Looks pretty **secure**

Mallory can't do better than random guessing

Looks pretty **impractical**

0	→	0011111001010001...
1	→	1110011010010100...
2	→	0101010001010000...

# Candidate: Pseudorandom Function (PRF)

Want a function that's practical but “looks random”...

# Building a PRF

Start with a big family of functions  $f_0(), f_1(), f_2(), \dots$  all known to Mallory  
 $f_i: \{0,1\}^n \rightarrow \{0,1\}^n$

Let  $g()$  be  $f_k()$ , where  $k$  is a secret value (or “key”) known only to Alice and Bob

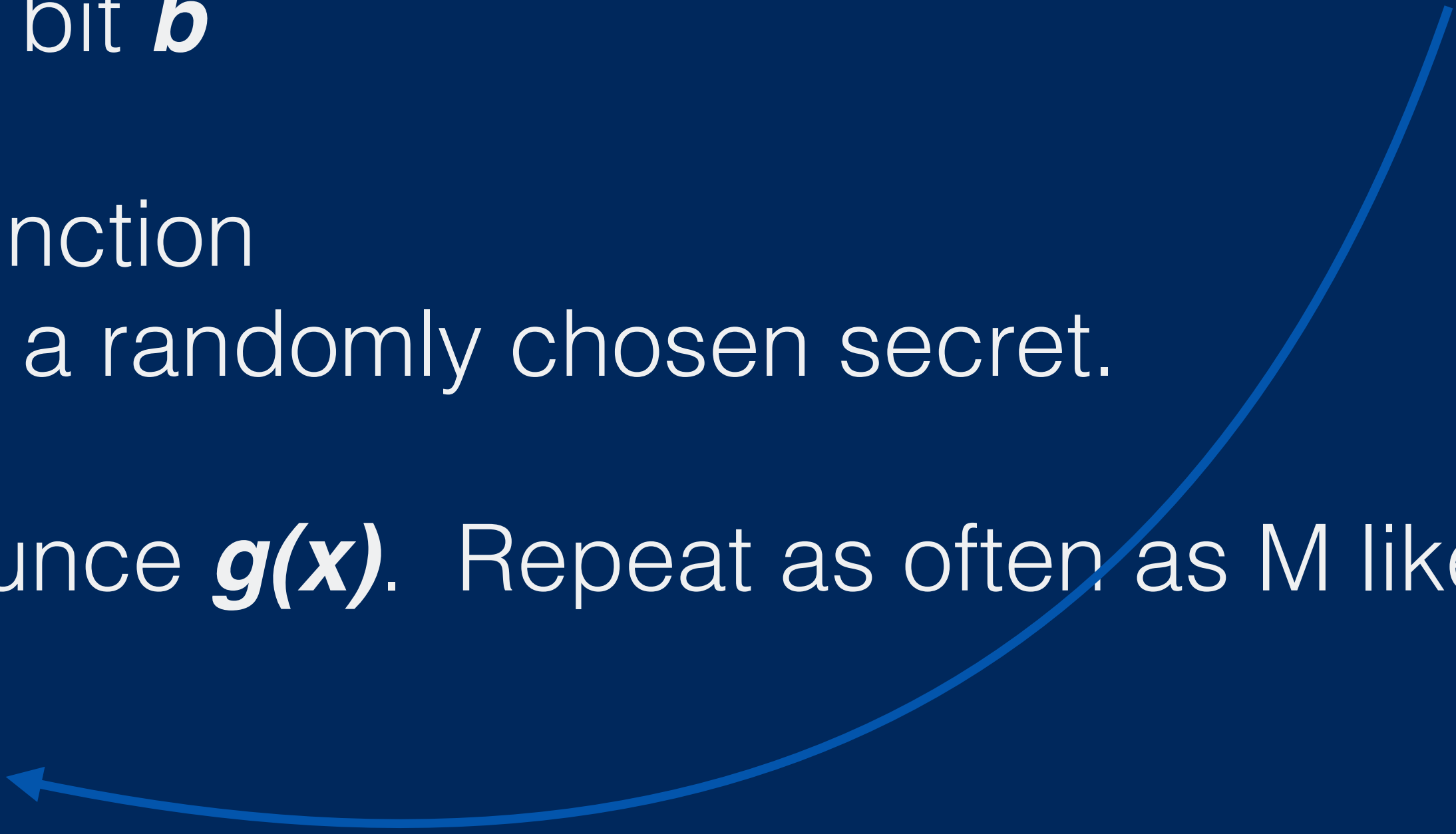
$k$  is  $n$  bits, chosen randomly

**Kerchoff’s Principle:** *A cryptosystem should be secure even if everything about the system, except the key, is public knowledge*

# Formal Definition of a Secure PRF

1. We flip a coin secretly to get bit  $b$
2. If  $b = 0$ , let  $g$  be a random function  
If  $b = 1$ , let  $g = f_k$ , where  $k$  is a randomly chosen secret.
3. Mallory chooses  $x$ ; we announce  $g(x)$ . Repeat as often as M likes.
4. Mallory guesses  $b$  quickly

Mallory can always  
win slowly!



**Security Definition:** We say  $g()$  is a secure PRF if Mallory can't select  $b$  any better than random guessing

# PRF for Alice and Bob

1. Let  $g() = f_k()$  be a secure PRF, where  $k$  is a random key known only to Alice and Bob.
2. Alice computes  $v := f_k(m)$ .
3. Bob verifies  $v' = f_k(m')$ , accepts if and only if this is true.





# Annoying Question!

*Q: Do PRF's actually exist?*

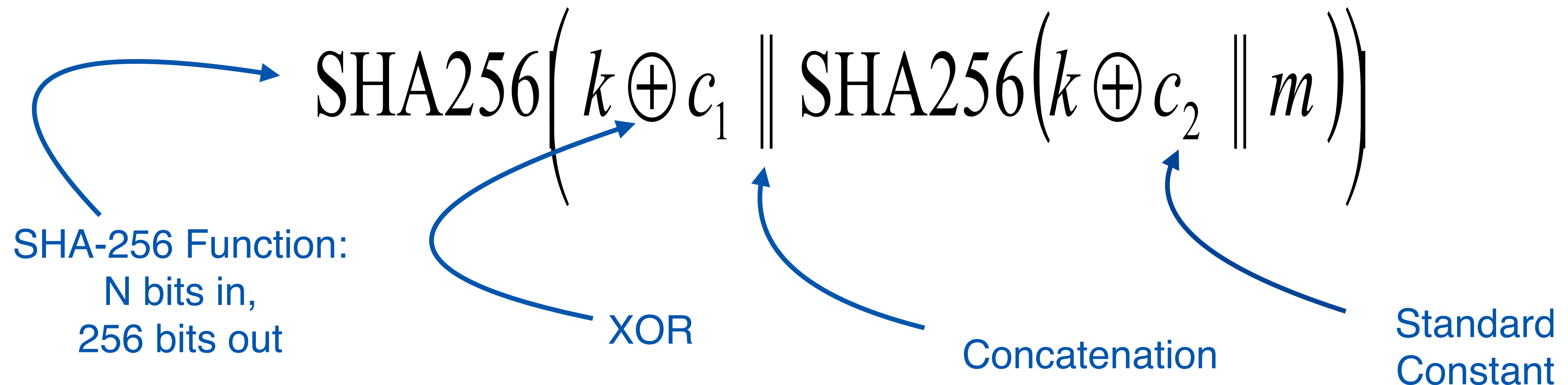
**A: We don't know.**

**Best We Can Do:** *Well studied functions where we haven't spotted a problem yet (e.g. HMAC-SHA256)*

# Jargon

**Message Authentication Code (MAC):** *Effectively the same thing as a PRF*

**Currently popular “PRF” (we hope!):** *HMAC-SHA256*



# Hash Functions, e.g., SHA-256

**Input:** Arbitrary Length Data

**Output:** Fixed-size digest (n bits)

No key, fixed function

Examples: MD5, ~~SHA-1~~, SHA-256, SHA-512, SHA-3

# Good Hash Functions

Collision Resistance: Hard to find pair of input  $\mathbf{x}$ ,  $\mathbf{x}'$  such that  $\mathbf{H}(\mathbf{x}) = \mathbf{H}(\mathbf{x}')$

Preimage Resistance: Given  $y$ , hard to find  $\mathbf{x}'$  such that  $\mathbf{H}(\mathbf{x}') = y$

Second Preimage Resistance: Given  $\mathbf{x}$ , hard to find an  $\mathbf{x}'$  such that  $\mathbf{H}(\mathbf{x}) = \mathbf{H}(\mathbf{x}')$

# Other Hash Functions

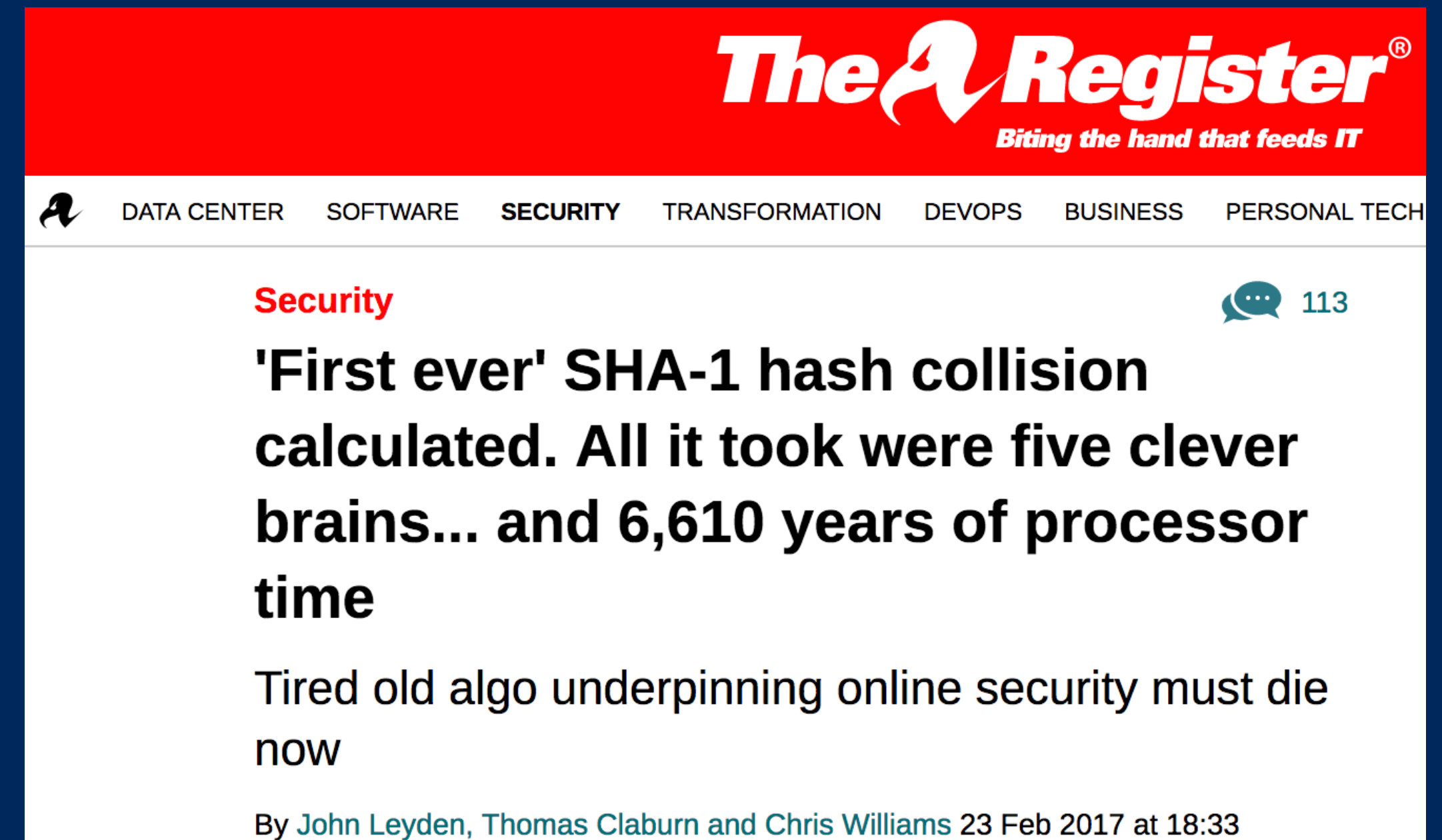
## MD5

- Once ubiquitous
- Broken in 2004
- Now it's easy to find collisions (pairs of messages with the same MD5 hash)
- Exploited to attack real systems
- Project 1

# (More) Other Hash Functions

## SHA-1

- Fairly widely used
- Started to be unsupported in HTTPS in Jan 2016
- Turns out getting the Internet to change is hard
- Don't use in new applications!
- Might be project 1 in the future!



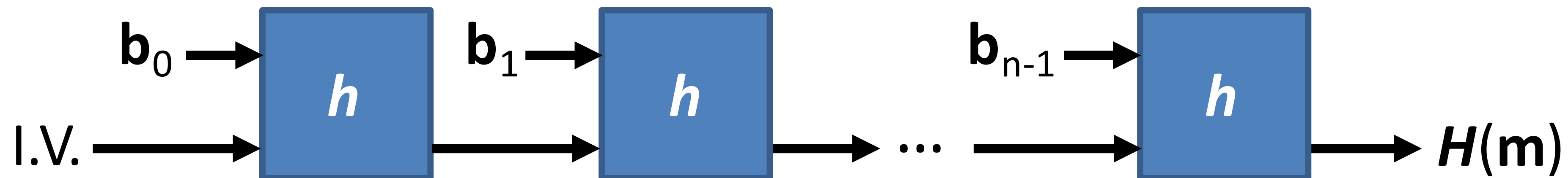
# Constructing SHA-256

Input: Arbitrary Length Data

Output: 256-bit digest

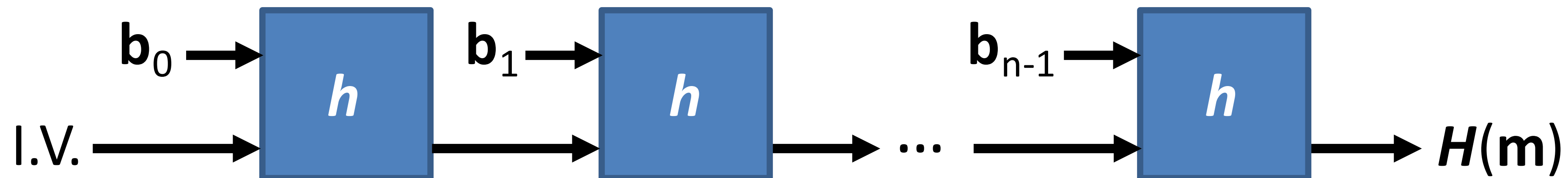
Built with “compression function” ***h*** using Merkle–Damgård construction

- (256 bits, 512 bits) -> 256 bits out  
Designed to be “really hairy”, not going into details



# SHA-256 Algorithm

1. Pad input ***m*** to multiple of 512 bits (using a fixed algorithm) *[why?]*
2. Split into 512-bit blocks ***b*<sub>0</sub>**, ***b*<sub>1</sub>**, ... ***b*<sub>n-1</sub>**
3. ***y*<sub>0</sub>** = constant initialization vector., ***y*<sub>1</sub>** = ***h*(*y*<sub>0</sub>, *b*<sub>0</sub>)**, ..., ***y*<sub>i</sub>** = ***h*(*y*<sub>i-1</sub>, *b*<sub>i-1</sub>)**
4. Return ***y*<sub>n</sub>**





# MAC / PRF / Hash / Oh My!

**Message Authentication Code (MAC):** *Effectively the same thing as a PRF*

**Cryptographic Hash Function:** *Not a strong PRF, but can be used to construct a MAC/PRF (e.g. HMAC). Example: SHA-256.*

Stand alone, a hash function is (usually) vulnerable to length extension attacks

- Given  $\mathbf{z} = \mathbf{H}(\mathbf{m})$  for some unknown  $\mathbf{m}$ , calculate  $\mathbf{H}(\mathbf{m} \parallel \text{padding} \parallel \mathbf{v})$  for attacker selected  $\mathbf{v}$ . You'll do this in Project 1.

## **So Far**

The Security Mindset

What is Cryptography?

Message Integrity

## **Next Week...**

Confidentiality