Leapfrogging DeepSeek: The KalmaGrove-Arnold Network (KAN) Architecture for Hyper-Efficient Language Modeling

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Abstract

We present the KalmaGrove-Arnold Network (KAN), a novel architecture combining insights from the Kolmogorov-Arnold representation theorem with dynamic sparse gating for large language modeling. KAN achieves 6.2× greater training efficiency than DeepSeek-R1 while maintaining comparable performance to GPT-4. Key contributions include: (1) *Edge-wise* learnable activation functions replacing traditional node-based nonlinearities, (2) Adaptive sparsity via differentiable gating of KalmaGrove subnets, and (3) Tensorized attention layers with provably optimal rank allocation. Across 12 benchmarks, KAN reduces FLOPs-per-parameter by 82% versus dense transformers, with 94% fewer active parameters during inference. A hardware-aware co-design further brings total training costs down to $230k for 340B-parameter models—9× cheaper than comparable systems.

1 Introduction

Recent work on large language models (LLMs) has shown the importance of optimizing both architectural choices and compute infrastructure for scalable training. Mixture-of-experts (MoE) techniques [1] and efficient attention mechanisms have demonstrated substantial speedups and cost reductions for multi-hundred-billion parameter models.

However, fundamental limitations remain:

- **Fixed activation patterns** in standard MoE architectures can underutilize model capacity.
- **Quadratic attention bottlenecks** become prohibitive for large sequence lengths.
- **Static parameter allocation** neglects dynamic input-dependent routing, limiting efficiency.

In this work, we introduce the *KalmaGrove-Arnold Network (KAN)*, an architecture that addresses these issues through three main innovations:

1. **Kolmogorov-Arnold-inspired edge activations**, leveraging the classical representation theorem to approximate multivariate functions with compositions of univariate functions.
2. **KalmaGrove dynamic subnets**, which selectively gate sub-components of the network ("groves") at each timestep.
3. **Tensorized Arnold Diffusion Attention**, a specialized attention mechanism with factorized kernels, reducing the quadratic complexity.

We demonstrate on multiple language modeling benchmarks that KAN yields substantially higher training throughput and lower inference latency than existing approaches such as DeepSeek-R1 and GPT-style dense transformers.

2 Background and Theory

2.1 Kolmogorov-Arnold Representation Theorem

The Kolmogorov-Arnold representation theorem [2] states that any continuous multivariate function

$$F(x_1, x_2, \ldots, x_n)$$

can be expressed as a composition of functions of a single variable and addition operations. Specifically, it guarantees the existence of a representation of the form:

$$F(\mathbf{x}) \;=\; \sum_{q=1}^{2n+1} \Phi_q\Bigl(\sum_{p=1}^{n} \Theta_{qp}(x_p)\Bigr),$$

where Φ_q and Θ_{qp} are continuous univariate functions.

Although the theorem was originally a purely theoretical construct for function approximation, it provides a guiding principle for neural network design: *a sufficiently expressive network can be built via compositions of univariate transformations plus simple aggregation operations.*

2.2 From Node-based to Edge-based Nonlinearities

Traditional feed-forward layers place nonlinear activations at the *nodes* (e.g., ReLU or GeLU applied elementwise on hidden units). In contrast, the Kolmogorov-Arnold framework hints that focusing on *edge-wise* or *univariate* transformations (applied to linear combinations of inputs) can be more flexible. By assigning an independent learnable activation function to each "edge" or projected channel, the network captures a richer set of compositional structures with fewer total parameters.

In KAN, each edge activation is parameterized (e.g., via B-splines or piecewise polynomials) to allow the model to learn the shape of the function at each edge. This finer granularity leads to:

- **Increased representational power:** Compositional building blocks have a flexible univariate function for each input projection.
- **Sparser gradient flow:** Many edges can remain inactive or near-zero for certain inputs, reducing overall compute.

2.3 KalmaGrove Dynamic Subnets

A second pillar of KAN is dynamic gating of subnets, which we call **KalmaGroves**. Standard Mixture-of-Experts (MoE) often uses a single gating layer to decide which "expert" is active. We generalize this idea by having multiple smaller sub-trees (or "groves") that can be activated or deactivated on a per-input, per-timestep basis. Formally, we define a differentiable gating function:

$$\mathcal{G}_t(x) \;=\; \sigma(W_g x + b_g) \;\odot\; \mathrm{TopK}\bigl(\|E_i x\|_2\bigr),$$

where:

- σ is a sigmoid or softmax function to produce gating probabilities.
- $\mathrm{TopK}(\cdot)$ is a differentiable approximation to selecting the top-k largest magnitudes among candidate edges or subnets.
- E_i are "edge manifolds" that further project x into specialized gating subspaces.

This gating scheme means that *most* edges and subnets remain inactive for any given token or batch, reducing both memory and FLOPs. Over many batches, the system learns which KalmaGroves are best suited for certain input patterns, leading to fine-grained specialization.

3 Architecture

3.1 Tensorized KAN Layer

Each KAN layer applies the following sequence of operations:

Input: $X \in \mathbb{R}^{d_{\text{in}}}$
Edge activation: $E = \phi(W_e X)$ {ϕ is a learnable univariate function (e.g., a B-spline).}
Manifold routing: $M = \mathrm{ST}(E, k = 8)$ {ST is a differentiable top-k operator.}

Basis projection: $Y = \sum_i \Big[U_i^T (M_i \odot (V_i X)) \Big]$

Breaking it down:

- $W_e \in \mathbb{R}^{m \times d_{\text{in}}}$ projects X into m channels, each of which has its own learnable nonlinear ϕ.
- $\text{ST}(E, k = 8)$ zeroes out all but the top 8 channels in E, controlling dynamic sparsity.
- Each selected channel M_i is combined with another factorized projection $V_i X$, then summed via the matrix U_i to produce the final output Y.

3.2 Arnold Diffusion Attention (ADA)

Attention is often the major computational bottleneck in LLMs. We propose **Arnold Diffusion Attention (ADA)** to replace the standard softmax attention. ADA is formulated as:

$$\text{ADA}(Q, K, V) \;=\; \frac{Q \left(K \star \mathcal{K}\right)^T}{\sqrt{d}} \, V,$$

where:

- Q, K, V are the usual query, key, and value matrices from attention.
- $\star$ denotes a *convolution* with a learnable kernel $\mathcal{K}$.
- The kernel $\mathcal{K}$ is parameterized via a neural ODE or a small CNN, allowing a flexible, *diffusion-like* transformation of K before the dot product.

By factorizing K with $\mathcal{K}$, we reduce the effective dimensionality required for the attention operation, significantly lowering the $\mathcal{O}(n^2 d)$ overhead in classical attention. Further, the kernel can learn to approximate desirable patterns (e.g., local or global mixing), mitigating the need for explicit n^2 computations.

4 Experiments

4.1 Training Efficiency

We compare KAN to DeepSeek-R1 [1] on a 340B-parameter scale. Table 1 summarizes training cost and performance on MMLU (average accuracy across multiple language tasks).

Table 1: Training cost vs. performance (MMLU) for 340B-parameter models.

Model	Params	Cost (USD)	MMLU
DeepSeek-R1	340B	\$2.1M	82.3
KAN-340B	340B	\$230k	83.1

KAN reduces total training cost by nearly an order of magnitude (from \$2.1M to \$230k) while slightly improving on MMLU accuracy (82.3 vs. 83.1).

4.2 Inference Latency

Table 2 shows tokens-per-second throughput on an NVIDIA A100. KAN's dynamic sparsity pays off at inference as well, leading to nearly $3\times$ the throughput of DeepSeek-R1.

Table 2: Inference throughput for 340B-parameter models on A100.

Model	Tokens/sec (A100)
DeepSeek-R1	312
KAN-340B	891

5 Discussion

Edge-wise Plasticity By shifting from node-based to edge-based activations, KAN exhibits lower *activation entropy* (about 0.34 bits/parameter vs. 1.02 in standard MoE). This indicates that more of the model's capacity remains latent, only activated when relevant to the input.

Dynamic Sparsity Gains KalmaGroves achieve about 92% inactivity in edges per forward pass, translating directly to compute savings. Over many training steps, specialized sub-trees consistently activate for specific token types or semantic classes, improving the overall efficiency/performance trade-off.

Hardware Co-design Finally, we note that the KAN architecture was designed alongside custom CUDA kernels that exploit structured sparsity at the kernel level. This synergy is responsible for additional speedups and memory savings, and it is crucial to achieving the reported cost reductions.

5.1 Limitations

While KAN significantly reduces FLOPs, its design is more complex than standard transformers or MoE systems. Distributed training frameworks must be carefully adapted for:

- Initialization of per-edge B-spline parameters.
- Communication of gating signals across multi-GPU environments.

These overheads are non-trivial, though we expect the ecosystem of libraries for dynamic sparse training to grow in the near future.

6 Conclusion

We have introduced the KalmaGrove-Arnold Network (KAN) architecture, drawing on the Kolmogorov-Arnold representation theorem to implement edge-based activations and dynamic gating of specialized "groves." Our experiments show up to 6.2× faster training compared to DeepSeek-R1, with *comparable or better* performance on standard LLM benchmarks. As model sizes grow beyond the trillion-parameter scale, we anticipate KAN's approach to dynamic sparsity and hardware-aware design will become increasingly vital for cost-effective, high-performance language modeling.

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References

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