AI-Prompt-Driven Curriculum for Functorial Physics

A modular, self-guided curriculum using AI prompts to explore and internalize the framework. At each step, feed the prompt to your favorite LLM, refine its output, and compare against canonical references.

Module	**Learning Goals**	**AI Prompt**
1. Foundations of Category Theory	Understand objects, morphisms, functors, natural transformations	"Explain the basic concepts of category theory—objects, morphisms, functors, and natural transformations—with simple examples."
2. Classical Mechanics as a Category	Model symplectic manifolds and canonical maps categorically	"Describe the category whose objects are symplectic manifolds and whose morphisms are symplectomorphisms, including examples."
3. Quantum Mechanics as a Category	View Hilbert spaces and unitaries in categorical terms	"Define the category of Hilbert spaces with unitary morphisms. How do quantum channels generalize this?"
4. Building the Quantization Functor	Construct $\mathcal{Q} : \mathbf{C} \to \mathbf{Q}$ on simple systems	"Show how to quantize the 1D harmonic oscillator by defining a functor from its phase space to a Hilbert space and symplectic flow to the unitary propagator."
5. Semi-classical & De-Quantization Functor	Capture $\hbar \to 0$ limits as a functor	"Explain how to construct a functor from quantum systems back to classical phase spaces via WKB approximation, illustrating with coherent states."
6. Adjointness & Natural Transformations	Verify the adjoint relationship and correspondence principle	"Demonstrate the unit and counit natural transformations that witness an adjunction between quantization and semiclassical functors."
7. Applications & Extensions	Explore field theories, gauge symmetry, and topological functors	"Propose how higher-category or 2-functor structures can encode path integrals or gauge symmetry in Functorial Physics."
8. Research Project	Design a small research question in functorial unification	"Using Functorial Physics, outline a project to functorially quantize a simple gauge theory (e.g. U(1) electromagnetism) and compare classical and quantum structures."