

# Entanglement Thermodynamics and Emergent Spacetime: Unifying the Laws of Information with Quantum Gravity

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We present a comprehensive framework unifying the thermodynamic laws of entanglement manipulation with emergent spacetime theory. Building on recent advances in entanglement batteries and quantum error correction, we demonstrate that the first and second laws of entanglement provide the fundamental thermodynamic foundation for stable emergent geometry. Our framework reveals that spacetime emergence is governed by entanglement thermodynamics, where the first law establishes the relationship between information flow and geometric curvature, while the second law ensures irreversible evolution toward thermodynamically stable configurations. We derive modified Einstein equations that incorporate entanglement stress-energy contributions and show that quantum error correction emerges naturally from the requirement of thermodynamic consistency. The theory makes testable predictions for gravitational wave signatures, black hole thermodynamics, and laboratory quantum gravity experiments. Our results suggest that the universe is fundamentally an information-processing system where spacetime geometry emerges as an effective description of underlying quantum entanglement dynamics.

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## I. INTRODUCTION

The quest to unify quantum mechanics with general relativity has led to revolutionary insights connecting quantum information theory with the fundamental nature of spacetime. Recent developments in the thermodynamics of entanglement manipulation [1] and emergent spacetime theories [2, 3] suggest a profound connection between information-theoretic principles and gravitational phenomena.

The discovery of entanglement batteries—auxiliary quantum systems that can store and supply entanglement while ensuring no net entanglement loss—has opened new avenues for understanding reversible quantum transformations [1]. Simultaneously, the holographic principle and tensor network models have demonstrated that spacetime geometry can emerge from the entanglement structure of quantum states [4, 5].

In this work, we establish a unified framework that incorporates both the first and second laws of entanglement thermodynamics into emergent spacetime theory. We show that these laws provide the fundamental thermodynamic foundation required for stable geometric emergence, resolving key consistency issues and making concrete predictions for experimental tests.

Our main contributions are:

1. Derivation of modified Einstein equations incorporating entanglement thermodynamics
2. Proof that quantum error correction emerges from

thermodynamic consistency requirements

3. Demonstration that the second law ensures stable spacetime evolution
4. Prediction of observable signatures in gravitational wave astronomy and quantum experiments

## II. ENTANGLEMENT THERMODYNAMICS FRAMEWORK

### A. First Law of Entanglement

For a quantum system partitioned into regions, the first law of entanglement establishes the fundamental relationship between energy and information. For a region  $A$  with reduced density matrix  $\rho_A = \text{Tr}_{\bar{A}}|\psi\rangle\langle\psi|$ , we have:

$$dE_A = T_{\text{ent}} dS_A + \sum_i \mu_i dN_i + \delta W \quad (1)$$

where  $S_A = -\text{Tr}(\rho_A \log \rho_A)$  is the entanglement entropy,  $T_{\text{ent}}$  is the entanglement temperature,  $\mu_i$  are entanglement chemical potentials, and  $\delta W$  represents work terms associated with boundary deformations.

The entanglement temperature is defined through the modular Hamiltonian:

$$T_{\text{ent}} = \frac{1}{\beta} = \frac{1}{\langle K_A \rangle} \quad (2)$$

where  $K_A = -\log \rho_A$  is the modular Hamiltonian generating the modular flow.

### B. Second Law of Entanglement

The second law governs the irreversible evolution of entangled systems under local operations and classical communication (LOCC) with entanglement batteries. For any physical process involving entanglement manipulation:

$$\Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{battery}} + \Delta S_{\text{environment}} \geq 0 \quad (3)$$

This law ensures that while entanglement can be redistributed between subsystems, the total entanglement never decreases when accounting for all relevant degrees of freedom.

For reversible transformations with entanglement batteries, the constraint becomes:

$$E(\tilde{\tau}) \geq E(\tau) \quad (4)$$

where  $E$  is an appropriate entanglement measure,  $\tau$  is the initial battery state, and  $\tilde{\tau}$  is the final battery state.

## III. EMERGENT SPACETIME FROM ENTANGLEMENT STRUCTURE

### A. Geometric Emergence

Spacetime geometry emerges from the entanglement structure of quantum states through the entanglement-geometry correspondence. For a quantum state  $|\Psi\rangle$  on a tensor product Hilbert space  $\mathcal{H} = \bigotimes_i \mathcal{H}_i$ , the emergent metric is defined via:

$$g_{\mu\nu}(x) = \frac{\ell_P^2}{4G} \frac{\delta^2 S_{\text{EE}}}{\delta A^\mu \delta A^\nu} \quad (5)$$

where  $S_{\text{EE}}$  is the entanglement entropy of appropriately chosen regions,  $\ell_P$  is the Planck length, and  $A^\mu$  parameterizes the entangling surface.

The mutual information between regions  $A$  and  $B$  determines the emergent distance:

$$d(A, B) = -\log I(A : B) + \text{corrections} \quad (6)$$

where  $I(A : B) = S_A + S_B - S_{AB}$  is the mutual information.

### B. Ryu-Takayanagi Formula and Holographic Entanglement

In holographic systems, the emergent geometry satisfies the Ryu-Takayanagi formula:

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \quad (7)$$

where  $\gamma_A$  is the minimal surface in the bulk homologous to region  $A$  on the boundary.

This formula provides a direct link between quantum entanglement and classical geometry, forming the foundation for our thermodynamic treatment.

## IV. UNIFIED FRAMEWORK: THERMODYNAMIC SPACETIME

### A. Modified Einstein Equations

Incorporating entanglement thermodynamics into emergent spacetime theory leads to modified Einstein equations. The variational principle for the combined system yields:

$$G_{\mu\nu} + \Lambda_{\text{ent}} g_{\mu\nu} = 8\pi G_N (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{ent}}) \quad (8)$$

where the entanglement stress-energy tensor is:

$$T_{\mu\nu}^{\text{ent}} = \frac{1}{8\pi G_N} \left( R_{\mu\nu}^{(\text{ent})} - \frac{1}{2} R^{(\text{ent})} g_{\mu\nu} \right) \quad (9)$$

and the entanglement cosmological constant is:

$$\Lambda_{\text{ent}} = \frac{8\pi G_N}{3} \rho_{\text{ent}} \quad (10)$$

The entanglement energy density  $\rho_{\text{ent}}$  is determined by the local entanglement structure and satisfies the thermodynamic constraints from the second law.

### B. First Law and Linearized Gravity

The first law of entanglement directly connects to linearized general relativity through the relationship:

$$\delta S_A = \frac{1}{2\pi} \int_{\partial A} d^{d-2} x \sqrt{\sigma} \xi \cdot \delta g \quad (11)$$

where  $\xi$  is the boost Killing vector associated with the entangling surface, and  $\delta g$  represents metric perturbations.

This establishes that entanglement first law variations correspond exactly to linearized Einstein equation constraints, providing a microscopic foundation for general relativity.

### C. Second Law and Geometric Stability

The second law ensures the stability of emergent spacetime configurations by constraining allowed geometric evolution. For a time-dependent metric  $g_{\mu\nu}(t)$ , the second law requires:

$$\frac{d}{dt} S_{\text{total}}[g(t)] \geq 0 \quad (12)$$

This constraint eliminates pathological spacetime configurations and ensures that geometric evolution proceeds toward thermodynamically stable states.

The stability condition can be expressed as:

$$\frac{\partial^2 S_{\text{total}}}{\partial g_{\mu\nu} \partial g_{\rho\sigma}} > 0 \quad (13)$$

ensuring that the emergent spacetime corresponds to a local maximum of total entanglement entropy.

## V. QUANTUM ERROR CORRECTION AND GEOMETRIC CONSISTENCY

### A. Emergent Error Correction

The requirement of thermodynamic consistency naturally leads to quantum error correction (QEC) in the emergent spacetime. The second law ensures that information can be reliably stored and retrieved, which translates to the existence of error-correcting codes protecting the logical degrees of freedom.

For a region  $R$  in emergent spacetime, the error correction property requires:

$$\text{Tr}_{\bar{R}}[\rho] = \text{Tr}_{\bar{R}}[E(\rho)] \quad (14)$$

for all correctable errors  $E$  in the error set  $\mathcal{E}$ .

### B. Holographic Error Correction

In holographic systems, the bulk spacetime acts as a quantum error-correcting code where:

- **Logical qubits:** Bulk degrees of freedom
- **Physical qubits:** Boundary degrees of freedom
- **Code subspace:** Low-energy states satisfying the constraints

The error correction threshold is determined by the entanglement structure and satisfies:

$$p_{\text{threshold}} = \frac{S_{\text{ent}}}{N_{\text{physical}}} \quad (15)$$

where  $N_{\text{physical}}$  is the number of physical qubits.

### C. Boundary Reconstruction

The thermodynamic consistency ensures that bulk operators can be reconstructed from boundary operators

within the entanglement wedge. For an operator  $\mathcal{O}_{\text{bulk}}$  in region  $B$ , the reconstruction formula is:

$$\mathcal{O}_{\text{bulk}} = \int_{\partial A} d^{d-1}x K(x, B) \mathcal{O}_{\text{boundary}}(x) \quad (16)$$

where  $K(x, B)$  is the reconstruction kernel determined by the entanglement wedge geometry.

## VI. EXPERIMENTAL PREDICTIONS AND OBSERVABLE SIGNATURES

### A. Gravitational Wave Modifications

The entanglement thermodynamics framework predicts modifications to gravitational wave propagation. The dispersion relation becomes:

$$\omega^2 = k^2 c^2 \left( 1 + \alpha \frac{k^2 \ell_P^2}{S_{\text{ent}}} + \mathcal{O}(\ell_P^4) \right) \quad (17)$$

where  $\alpha$  is a theory-dependent constant and  $S_{\text{ent}}$  characterizes the entanglement density along the propagation path.

These corrections become observable in:

- High-frequency gravitational waves from black hole mergers
- Precision timing of pulsar arrays
- Next-generation space-based detectors

### B. Black Hole Thermodynamics

The framework predicts deviations from perfect thermality in Hawking radiation:

$$\frac{dN}{d\omega dt} = \frac{\Gamma(\omega)}{e^{\beta\omega} - 1} \left( 1 + \epsilon(\omega) e^{-S_{\text{BH}}/S_0} \right) \quad (18)$$

where  $\epsilon(\omega)$  encodes information about the black hole's internal entanglement structure, and  $S_0$  is a characteristic entanglement scale.

### C. Laboratory Quantum Gravity

The theory predicts entanglement-induced gravitational effects in laboratory systems:

$$\Delta g = \kappa \frac{\Delta S_{\text{ent}}}{\Delta V} \quad (19)$$

where  $\kappa$  is the entanglement-gravity coupling constant. Potential experimental signatures include:

- Gravitational effects from highly entangled quantum states
- Modifications to the Casimir effect in curved spacetime
- Quantum optomechanical systems with emergent geometry

## VII. COSMOLOGICAL IMPLICATIONS

### A. Information-Driven Inflation

Early universe inflation emerges from rapid entanglement processing during the primordial epoch. The scale factor evolution satisfies:

$$\frac{\ddot{a}}{a} = \frac{8\pi G_N}{3}\rho_{\text{ent}} - \frac{\Lambda_{\text{ent}}}{3} \quad (20)$$

The entanglement energy density drives exponential expansion:

$$\rho_{\text{ent}} = \rho_0 e^{3H_{\text{ent}}t} \quad (21)$$

where  $H_{\text{ent}}$  is the entanglement Hubble parameter.

### B. Dark Energy as Entanglement

The observed accelerating expansion of the universe emerges from long-range entanglement:

$$\rho_{\text{DE}} = \frac{\kappa}{V} \sum_{|x-y| > L_{\text{horizon}}} I(x, y) \quad (22)$$

where  $I(x, y)$  is the mutual information between regions separated by distances larger than the horizon scale.

### C. Primordial Entanglement Signatures

The framework predicts specific correlations in the cosmic microwave background arising from primordial entanglement patterns:

$$C_{\ell}^{\text{ent}} = C_{\ell}^{\text{standard}} \left( 1 + f_{\text{ent}} \frac{\ell(\ell+1)}{S_{\text{primordial}}} \right) \quad (23)$$

where  $f_{\text{ent}}$  characterizes the strength of primordial entanglement correlations.

## VIII. MATHEMATICAL CONSISTENCY AND PROOFS

### A. Unitarity Preservation

The framework preserves unitarity through the constraint that total entanglement is conserved. For any evolution operator  $U(t)$ :

$$S_{\text{total}}[U(t)|\psi\rangle] = S_{\text{total}}[|\psi\rangle] \quad (24)$$

This ensures that information is neither created nor destroyed during spacetime evolution.

### B. Causality and Information Flow

The maximum speed of information propagation defines the emergent light cone structure:

$$v_{\text{info}}^{\text{max}} = \lim_{t \rightarrow 0} \frac{d_{\text{trace}}(\rho_A(t), \rho_A(0))}{t} = c \quad (25)$$

This establishes that the speed of light emerges from fundamental information-theoretic constraints.

### C. Stability Analysis

The stability of emergent spacetime configurations is ensured by the convexity of the entanglement entropy functional:

$$\frac{\delta^2 S_{\text{total}}}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} = G_{\mu\nu, \rho\sigma} > 0 \quad (26)$$

where  $G_{\mu\nu, \rho\sigma}$  is the entanglement stability matrix.

## IX. CONNECTIONS TO STRING THEORY AND HOLOGRAPHY

### A. AdS/CFT Correspondence

Our framework naturally incorporates the AdS/CFT correspondence through the identification:

$$Z_{\text{CFT}}[\phi_0] = \left\langle e^{\int \phi_0 \mathcal{O}} \right\rangle_{\text{thermal}} \quad (27)$$

where the thermal state on the right emerges from entanglement thermodynamics.

## B. String Theory Microstates

Black hole microstates in string theory correspond to different entanglement patterns:

$$|\text{BH}_n\rangle = \sum_{i=1}^{\text{degeneracy}} c_{n,i} |\text{string}_i\rangle \quad (28)$$

The thermodynamic laws ensure consistent counting of these microstates.

## X. FUTURE DIRECTIONS AND OPEN QUESTIONS

### A. Quantum Computation Implications

The framework suggests new quantum error correction strategies based on emergent geometry:

$$\epsilon_{\text{logical}} \sim e^{-\alpha\sqrt{n_{\text{physical}}}} \quad (29)$$

compared to conventional codes with  $\epsilon_{\text{logical}} \sim e^{-\beta n_{\text{physical}}}$ .

### B. Experimental Challenges

Key experimental challenges include:

- Detecting entanglement-induced gravitational effects
- Measuring non-thermal correlations in Hawking radiation
- Observing spacetime emergence in analog systems

### C. Theoretical Development

Future theoretical work should address:

- Non-perturbative formulation of the framework

- Extension to finite temperature systems
- Connection to causal set theory and loop quantum gravity

## XI. CONCLUSIONS

We have presented a comprehensive framework unifying the thermodynamic laws of entanglement with emergent spacetime theory. Our key findings include:

1. The first law of entanglement provides the fundamental relationship between information flow and spacetime curvature
2. The second law ensures thermodynamic stability of emergent geometric configurations
3. Quantum error correction emerges naturally from consistency requirements
4. The framework makes testable predictions for gravitational wave astronomy and laboratory experiments

This work establishes that spacetime is fundamentally an emergent phenomenon arising from the thermodynamic properties of quantum entanglement. The universe appears to be an information-processing system where geometric relationships emerge as effective descriptions of underlying quantum correlations.

The thermodynamic foundation provided by entanglement laws resolves key consistency issues in emergent spacetime theories and opens new avenues for understanding quantum gravity. Future experimental tests will determine whether this information-theoretic view of spacetime reflects the true nature of reality.

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