

# Integration of Automorphic and Spectral Data into Universal Categorical Algebraic Topology (UCAT)

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## Abstract

The integration of automorphic data and spectral data into the Universal Categorical Algebraic Topology (UCAT) framework represents a significant step toward creating a unified foundation for advanced mathematical theories. By incorporating the Langlands Functor  $LG$ , which establishes an equivalence between automorphic and spectral data, we extend UCAT's capabilities to include higher categorical constructs and dualities from the Geometric Langlands Program.

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## 1 Introduction

The Universal Categorical Algebraic Topology (UCAT) framework aims to provide a unifying foundation for mathematics by combining universal algebra, homotopy type theory (HoTT), and topos theory. In this paper, we integrate automorphic data and spectral data into UCAT, mediated by the Langlands Functor  $LG$ . This integration enhances UCAT's capacity to model sophisticated dualities from the Geometric Langlands Program.

## 2 UCAT Framework Recap

The UCAT framework is based on three core mathematical pillars:

- **Universal Algebra:** Provides an abstract basis for defining algebraic structures through operations and identities.
- **Homotopy Type Theory (HoTT):** Introduces homotopy concepts into type theory, allowing for reasoning about higher-dimensional types and providing a constructive foundation.
- **Topos Theory:** Generalizes the universe of mathematical discourse using category theory, providing an internal logic that supports both classical and intuitionistic reasoning.

These elements are unified in UCAT, forming a comprehensive framework for describing various mathematical structures and logical systems.

### 3 Automorphic Data in UCAT

Automorphic data, represented by D-modules on the moduli stack  $\text{Bun}_G$ , naturally fits into the UCAT framework:

- **Algebraic Perspective:** Differential equations on  $G$ -bundles can be modeled using universal algebra within UCAT. Operations on D-modules correspond to morphisms in a category defined by UCAT's algebraic structures.
- **Topological Perspective:** The moduli stack  $\text{Bun}_G$  is interpreted as a higher categorical space in the language of HoTT. UCAT's type-theoretic framework treats  $\text{Bun}_G$  as a homotopy type, where solutions to differential equations are represented as path types.
- **Internal Logic:** The internal logic of UCAT's topos  $\mathcal{T}$  is used to reason about the properties of D-modules, aligning with the intuitionistic logic of the Brouwer–Heyting–Kolmogorov (BHK) interpretation.

In UCAT, automorphic data is formalized as a category:

$$\mathcal{C}_{\text{auto}} = D\text{-mod}^{1/2}(\text{Bun}_G).$$

### 4 Spectral Data in UCAT

Spectral data, represented by ind-coherent sheaves on the moduli stack of local systems  $\text{LS}_{G^\vee}$ , relates closely to the topological and categorical aspects of UCAT:

- **Homotopy Type Theory:** Local systems are modeled as higher-dimensional path types, capturing continuity and flat connections. Spectral data fits into UCAT's type-theoretic framework as homotopy types.
- **Topos Theory:** The moduli stack  $\text{LS}_{G^\vee}$  is treated as an object within a topos  $\mathcal{T}$  in UCAT. The category of ind-coherent sheaves is interpreted using UCAT's internal logic, where nilpotent singular support conditions are managed within the topos.
- **Universal Algebra:** Spectral data involves the dual group  $G^\vee$ , defined through the root data of  $G$ . The representation theory of  $G^\vee$  aligns with the universal algebraic structures in UCAT.

In UCAT, spectral data is formalized as a category:

$$\mathcal{C}_{\text{spec}} = \text{IndCohNilp}(\text{LS}_{G^\vee}).$$

### 5 Role of the Langlands Functor in UCAT

The Langlands Functor  $LG$  integrates automorphic and spectral data within UCAT as a universal morphism between enriched categories:

$$LG : \mathcal{C}_{\text{auto}} \longrightarrow \mathcal{C}_{\text{spec}}.$$

- **Categorical Equivalence:** The functor  $LG$  establishes an equivalence between  $\mathcal{C}_{\text{auto}}$  and  $\mathcal{C}_{\text{spec}}$ , mediated by the 2-Fourier-Mukai transform. This transform acts as a higher-dimensional analog of the classical Fourier-Mukai transform, preserving the structure of the data.

- **Internal Logic and Equivalence:** UCAT’s internal logic, derived from the topos  $\mathcal{T}$ , provides a framework for reasoning about this equivalence. The constructive nature of UCAT, via the BHK interpretation, ensures the equivalence holds in both classical and constructive settings.

The Langlands Functor  $LG$  embodies the deep duality between automorphic and spectral data, highlighting UCAT’s unifying power.

## 6 Implications for UCAT

The integration of automorphic and spectral data into UCAT, facilitated by the Langlands Functor  $LG$ , has several significant implications:

- **Enhanced Expressiveness:** UCAT can now model complex dualities found in the Geometric Langlands Program, demonstrating its strength as a unifying framework.
- **Constructive Foundations:** The constructive aspects of UCAT align with the geometric Langlands correspondence, providing a natural setting for intuitionistic reasoning.
- **Higher Categorical Integration:** The use of the 2-Fourier-Mukai transform extends UCAT’s ability to handle higher-dimensional transformations, applicable to advanced problems in algebraic geometry and representation theory.

## 7 Conclusion

The integration of the Langlands Functor and its associated data into UCAT showcases the framework’s versatility and depth. By unifying automorphic and spectral data, UCAT extends its foundational capabilities to encompass advanced dualities in modern mathematical theory, bridging gaps between algebra, topology, and logic.

## References

## References

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