# Cohomological Projections and Unified Functorial Physics:

Derived Hamiltonians as Translators Across Physics, Biology, and Computer Science

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**Abstract**

This paper explores the role of cohomological projections and derived functors as tools to unify disparate domains of physics, biology, and computer science. We introduce the concept of the *derived Hamiltonian* as a fundamental abstraction that translates between classical and quantum systems, encoding hidden symmetries and higher-dimensional topologies. By drawing parallels to biological systems and computational structures, we highlight how these mathematical frameworks reveal universal principles governing the complexity of the natural world.

## 1 Introduction

Modern science seeks to unify complex systems under a single framework, whether by integrating quantum mechanics with general relativity, modeling biological evolution, or constructing robust computational architectures. However, the language of classical physics and traditional computation often fails to capture the emergent, topological, and dynamic nature of these systems.

Cohomological projections and functorial physics provide an elegant way to navigate this landscape by treating physical forces, biological processes, and computational transitions as *projections of higher-dimensional structures*. This paper proposes that **derived Hamiltonians**—extensions of classical Hamiltonians into cohomological and categorical realms—serve as the backbone for this unification.

## 2 Derived Functors and Cohomological Projections

Derived functors generalize classical tools in algebraic topology and homological algebra by measuring the failure of exactness in mappings between objects. They are essential for extending local data (fields, particles, biological structures) to global properties (spacetime, ecosystems, distributed networks).

Let $F : \mathcal{A} \to \mathcal{B}$ be a left exact functor between abelian categories. The right derived functor $R^i F$ is defined as:

$$R^i F(A) = H^i(F(I^\bullet))$$

where $I^\bullet$ is an injective resolution of $A$. This formalism extends naturally to sheaf cohomology and physical systems.

### 2.1 Physics: Cohomological Spaces and Field Theory

In quantum field theory, derived functors describe the interactions of gauge fields and topological defects. The sheaf cohomology $H^i(X, \mathcal{F})$ represents the degrees of freedom associated with local fields $\mathcal{F}$ over a spacetime manifold $X$.

Derived functors reveal:

- **Gauge Anomalies:** Obstructions in the form of cohomology classes.

- **Quantum Corrections:** Higher-order interactions from derived categories.

- **Dualities:** Equivalences between apparently distinct field theories.

## 2.2 Biology: Cohomological Folding of Proteins and Ecosystems

In biology, derived categories model the evolution of complex systems from simple underlying rules. Protein folding, genetic transcription, and neural networks can be seen as derived functors mapping local information to global behavior.

$$H^i(\text{Gene Network}, \mathcal{P}) = \text{Functional Protein Output}$$

## 2.3 Computer Science: Computational Topology and Distributed Systems

Cohomology and derived categories are increasingly used in computer science to analyze distributed systems, data structures, and neural networks. Persistent homology tracks topological features of data, while derived categories describe phase transitions in computational processes.

$$H^i(\text{Algorithm}, \mathcal{C}) = \text{Emergent Complexity}$$

# 3 Derived Hamiltonians: Translators Between Domains

A **derived Hamiltonian** extends the classical Hamiltonian function $H(p, q)$ into a cohomological framework, allowing it to encode the dynamics of fields, particles, and higher symmetries.

$$\hat{H} = \sum_i R^i H(p, q) + \int_\Sigma \mathcal{F} \wedge dA$$

where $\Sigma$ is a cohomological surface, and $\mathcal{F}$ represents a higher-dimensional form encoding topological data.

## 3.1 Physical Interpretation

In physics, derived Hamiltonians generalize symplectic structures, capturing the dynamics of quantum states, black holes, and string interactions:

$$R^1 H = \text{Quantum Correction}, \quad R^2 H = \text{Topological Defect}$$

## 3.2 Biological Interpretation

For biological systems, the derived Hamiltonian governs energy landscapes for protein folding and genetic mutations, representing the "energy" of evolutionary pathways:

$$\hat{H}_{\text{bio}} = \sum_i R^i(\text{Genome})$$

## 3.3 Computational Interpretation

In computer science, derived Hamiltonians describe the optimization landscape of machine learning algorithms and the phase space of distributed networks.

# 4 Linking Derived Hamiltonians to Reality

## 4.1 Quantum Gravity and Spacetime Topology

Derived Hamiltonians encode the topology of spacetime by relating quantum fields to cohomological invariants. This resolves singularities and predicts new particles as higher-order cohomological projections.

## 4.2 Artificial Intelligence and Topos Theory

Topos theory provides a categorical framework for machine learning, with derived functors representing phase transitions in neural networks:

$$\text{AI Evolution} \sim D^b(Coh(X))$$

# 5 Applications and Future Directions

- **Physics:** Resolving black hole singularities and predicting dark matter through derived cohomological projections.

- **Biology:** Modeling protein folding pathways and evolutionary dynamics.

- **Computer Science:** Enhancing neural networks through topological optimization techniques.

# 6 Conclusion

Derived functors, cohomological projections, and derived Hamiltonians form the mathematical backbone of unified functorial physics. By expanding the role of these abstractions across disciplines, we pave the way for a deeper understanding of the universe, life, and computation.