From Categorical Limits to Empirical Validation: A Roadmap for Testing Functorial Curvature in Gravitational Physics

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Abstract

Functorial Physics proposes that gravitational curvature arises as a categorical limit in an appropriate fibration of smooth manifolds and vector bundles. Translating this abstract claim into falsifiable predictions requires a multi-layered toolchain—from formal proof assistants through numerical relativity and precision experiments to statistical inference. We outline a five-stage roadmap and specify the software, hardware, and theoretical innovations needed at each stage to close the loop between category-theoretic formulation and empirical validation.

Contents

1 Introduction

Recent advances in categorical and homotopical methods suggest that classical geometric quantities—such as the Riemann curvature tensor—can be reconstructed as universal limits in higher categories. While mathematically compelling, the physical legitimacy of such constructions hinges on whether they yield novel, testable predictions beyond General Relativity (GR). This paper details a staged programme to assess that legitimacy, structuring the workflow into five successive transformations:

1. **Abstract Categorical Limit $\Rightarrow$ Formal Proof Artefacts**

2. **Formal Artefacts $\Rightarrow$ Concrete Curvature Tensors**

3. **Curvature Tensors $\Rightarrow$ Simulated Observables**

4. **Simulated Observables $\Rightarrow$ Experimental Measurement**

5. **Experimental Data $\Rightarrow$ Data-Driven Validation**

Each stage imposes distinct technical requirements; we delineate them in dedicated sections.

2 Stage 1: Formalization of Categorical Limits

2.1 Objective

Rigourously encode the functorial curvature construction in a proof assistant, ensuring that all coherence laws and universal properties are machine-verified.

2.2 Required Tools

- **Proof Assistant**: Lean 4 with mathlib4 extensions for synthetic differential geometry.

- **Domain-Specific Language (DSL)**: An embedded language `FPCat` (Functorial Physics Category Theory) supporting objects for manifolds, morphisms for smooth maps, and 2-morphisms for bundle morphisms.

- **Automated Theorem Provers**: Integration with SMT solvers (Z3) for first-order obligations.

2.3 Deliverables

1. Lean definitions of the curvature functor $\mathcal{R} : \mathrm{Man} \to \mathrm{VBund}$.

2. Proof scripts establishing that $\mathcal{R}$ preserves limits.

3. Export facilities to serialize proofs in OpenTheory or Dedukti for interoperability.

3 Stage 2: Translation to Differential-Geometric Constructs

3.1 Objective

Compile categorical artefacts into coordinate-level tensor quantities suitable for numerical relativity.

3.2 Compiler Architecture

Front-End Parses `FPCat` signatures to extract limit diagrams.

Intermediate Representation A typed term language capturing natural transformations.

Back-End Emits symbolic expressions for Christoffel symbols, $R^{\mu}{}_{\nu\rho\sigma}$, and geodesic deviation equations.

3.3 Dependencies

- **SymPy** for symbolic algebra.

- **Julia** (`DifferentialGeometry.jl`) for high-order tensor calculus.

- **LLVM** or **MLIR** for generating optimized kernel code targeting CUDA/ROCm.

4 Stage 3: Numerical Simulation of Observables

4.1 Objective

Evolve the curvature tensors in physically plausible scenarios and compute observable signatures.

4.2 Simulation Pipeline

1. Discretize spacetime via $3+1$ ADM decomposition.

2. Solve Einstein–like field equations augmented by categorical corrections using **Einstein Toolkit** or **GRChombo**.

3. Post-process waveforms, redshift distributions, and lensing maps.

4.3 Computational Resources

High-performance GPU clusters (e.g. NVIDIA H100) with InfiniBand networking; expected job size $\sim 10^4$ GPU-hours per parameter sweep.

5 Stage 4: Experimental Measurement Strategies

5.1 Laboratory-Scale Experiments

- **Bose–Einstein Condensate Horizons**: Measure phonon Hawking spectra for deviations.

- **Atom Interferometry**: Dual-species interferometers sensitive to $\leq 10^{-13}\,g$ curvature gradients.

- **Micro-Torsion Balances**: Detect short-range curvature corrections below $0.1\,\mathrm{mm}$.

5.2 Astrophysical Observatories

- **Pulsar Timing Arrays**: Extend observation baselines to $30\,\mathrm{yr}$ for nano-Hz GW background.

- **Satellite Geodesy**: Propose *GRACE-III* with cold-atom accelerometers.

- **Space-based Interferometers**: Tune *LISA-Plus* arm lengths for mid-frequency GWs.

6 Stage 5: Data-Driven Validation

6.1 Statistical Inference Framework

1. Construct Bayesian hierarchical models comparing categorical vs. GR predictions.

2. Employ Markov Chain Monte Carlo (e.g. `PyMC`) on HPC nodes.

3. Quantify evidence via Bayes factors and information criteria (WAIC, LOO).

6.2 Topological Data Analysis

Use persistent homology on simulated vs. observed datasets to detect qualitative shape differences in gravitational-wave manifold reconstructions.

7 Integration and Milestones

Year	Milestone	Key Deliverables
1	Stage 1 Complete	Lean library, verification reports
2	Stage 2 Compiler MVP	SymPy/LLVM backend
3	Stage 3 Pilot Sims	Waveform catalog $\leq 1\%$ numerical error
4	Stage 4 Lab Prototypes	BEC horizon data, torsion balance sensitivity curves
5	Stage 5 Full Analysis	Bayes factor publication, arXiv preprint v1

Table 1: Indicative five-year roadmap.

8 Discussion and Future Work

We highlight open challenges: categorical renormalization, coupling to quantum fields, and scalability of proof automation. Synergistic collaboration between category theorists, numerical relativists, and experimentalists will be essential.

Acknowledgements

References

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