Functorial Physics and Derived Hamiltonians: A Unified Framework for Quantum-Relativistic Systems

Matthew Long

Magneton Labs

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Abstract

This paper introduces a unified framework integrating quantum mechanics and general relativity through the lens of functorial physics and derived Hamiltonians. Building on higher category theory, topological quantum field theory (TQFT), noncommutative geometry, and the Unified Evolution Equation (UEE), we propose a categorical formalism that models physical systems as functors between structured categories. Derived Hamiltonians capture gauge symmetries, constraints, and higher-order corrections, providing insights into quantum gravity and emergent topological phenomena. This work addresses limitations in canonical quantization, introduces conceptual bridges to loop quantum gravity and spin-foam models, and demonstrates potential applications in quantum computation.

Contents

1 Introduction

Bridging quantum mechanics and general relativity remains an enduring challenge in physics. The incompatibility of quantum field theory (QFT) with general relativity (GR) suggests the need for a more flexible, mathematically rigorous framework capable of encoding gravitational dynamics and quantum behavior simultaneously. This paper proposes that *functorial physics*—rooted in category theory and enriched by derived Hamiltonians—serves as a promising candidate for such unification.

2　Functorial Physics: A Categorical Approach

Functorial physics interprets physical systems as categories, with transformations between them modeled as functors. Consider a category of phase spaces $\mathcal{C}$ and a category of observables $\mathcal{D}$. A functor

$$F : \mathcal{C} \to \mathcal{D} \tag{1}$$

maps physical states to measurable quantities, preserving the structure of the system. This abstraction naturally generalizes classical canonical transformations and quantization procedures.

3　Derived Hamiltonians and the UEE

The *derived Hamiltonian* extends the classical Hamiltonian to account for topological, gauge, and quantum corrections. Formally,

$$H_{\text{derived}} = R^1 H \tag{2}$$

where R^1 denotes the first right-derived functor applied to the classical Hamiltonian H. This generalization allows the inclusion of gauge constraints, BRST symmetries, and gravitational curvature.

The **Unified Evolution Equation (UEE)** is a key formulation that governs the evolution of quantum states under curved spacetime:

$$\frac{d}{dt}\Psi(t) = \frac{\hbar c}{l_p^2}[D_\mu, D_\nu]\Psi(t) \oplus \ldots \tag{3}$$

where D_μ are covariant derivatives encoding gravitational effects.

4　Topological Quantum Field Theory (TQFT)

TQFT plays a significant role in functorial physics by incorporating topological invariants into physical models. Derived Hamiltonians, when extended to TQFT, yield corrections of the form:

$$H_{\text{derived}} = H + \int_{\mathcal{M}} \omega \tag{4}$$

where ω is a topological invariant defined over a manifold $\mathcal{M}$. This reflects how non-trivial spacetime topology influences quantum dynamics.

5 Noncommutative Geometry and Gauge Theory

Noncommutative geometry provides a natural extension of Hamiltonian mechanics to account for quantum gravitational corrections. The commutation relation

$$[X^i, X^j] = i\theta^{ij} \tag{5}$$

introduces noncommutative coordinates, where θ^{ij} represents deformation parameters.

6 Applications in Quantum Gravity and Computation

6.1 Loop Quantum Gravity and Spin-Foams

Derived Hamiltonians provide new insights into loop quantum gravity by encoding spin-foam structures through categorical adjunctions. This enhances the understanding of discrete spacetime and its relation to quantum states.

6.2 Quantum Computing and TDA

Topological Data Analysis (TDA) and quantum computation benefit from functorial mappings between data structures and state spaces. Functorial physics lays the groundwork for fault-tolerant quantum circuits by leveraging topological invariants.

7 Future Directions and Experimental Considerations

The framework proposed in this paper necessitates experimental validation through gravitational wave observations and high-energy particle experiments. We suggest further exploration of derived Hamiltonians in the context of holographic dualities and emergent spacetime.

8 Conclusion

Functorial physics and derived Hamiltonians present a transformative approach to unifying quantum mechanics and general relativity. By leveraging category theory, TQFT, and noncommutative geometry, we propose a novel framework capable of addressing longstanding challenges in quantum gravity and emergent physical phenomena.

References

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