Beyond Coordinates: Conceptual Advantages of a Functorial, Coordinate–Free Formulation of Gravitation

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Abstract

We articulate the conceptual gains achieved by replacing the coordinate–heavy formalism of General Relativity (GR) with a functorial, category–theoretic framework—termed *Functorial Physics*. By treating gravitational curvature as the limit of a diagram in a suitable higher category, we obtain a coordinate–free language that unifies local and global geometric data, eliminates gauge redundancies, and interfaces naturally with quantum topological structures. This paper offers a systematic comparison between the two paradigms, demonstrating how functorial methods clarify foundational issues, simplify certain calculations, and open new avenues for unification with quantum field theory.

Contents

1 Introduction

Coordinate systems are indispensable computational tools in classical geometry but often obscure the intrinsic properties of spacetime. General Relativity, formulated in terms of tensor fields on a manifold, remains coordinate–dependent at the level of explicit calculations. Functorial Physics proposes a shift: model spacetime and its curvature as objects and morphisms in higher categories, extracting curvature via universal properties. We examine why this shift is conceptually—and potentially empirically—superior.

2 Background

2.1 Coordinate–Dependent Formalism in GR

Brief survey: manifolds, local charts, metric tensor $g_{\mu\nu}$, Christoffel symbols $\Gamma^{\mu}_{\nu\rho}$, Riemann tensor $R^{\mu}_{\nu\rho\sigma}$.

2.2 Category Theory Essentials

Objects, morphisms, functors, natural transformations, limits/colimits. Higher categories and $(\infty, 1)$–categories.

2.3 Functorial Physics Primer

Definition of the curvature functor $\mathcal{R} : \mathrm{Man} \to \mathrm{VBund}$; curvature as a limit.

3 Coordinate vs. Functorial Descriptions

3.1 Locality and Universality

Coordinate patches require atlases; functorial description encodes locality via diagrams and universality via limits.

3.2 Gauge Redundancy

How coordinate transformations induce redundancy; functorial framework avoids spurious degrees of freedom.

3.3 Global Topological Data

Holonomy and monodromy captured naturally via functor composition; contrast with stitching local tensors.

4 Conceptual Advantages

4.1 Axiomatic Clarity

Universal properties replace coordinate calculations, leading to shorter proofs (e.g., Gauss–Bonnet) in categorical language.

4.2 Computational Modularity

Composable functors permit automatic code generation and parallel computation pipelines.

4.3 Quantum Compatibility

Functorial language aligns with TQFTs and state–sum models, smoothing the interface with quantum gravity.

4.4 Higher–Dimensional Extensions

Generalizes gracefully to supergeometry and noncommutative geometry via enriched categories.

4.5 Unification Potential

Functorial framework provides common language for gauge theories, topological phases, and gravity.

5 Case Studies

5.1 Gravitational Lensing Revisited

Derive lensing deflection functorially; compare clarity and error propagation.

5.2 Black Hole Thermodynamics

Express surface gravity as categorical invariant.

5.3 Cosmic String Spacetimes

Compute deficit angle via limit construction.

6 Challenges and Criticisms

6.1 Accessibility and Learning Curve

Higher category theory barrier.

6.2 Tooling Maturity

Proof assistants and numerical libraries still under development.

6.3 Empirical Anchoring

Need for concrete, testable predictions (cf. companion roadmap paper).

7 Discussion

Synthesis of advantages versus challenges; roadmap for adoption in theoretical and experimental communities.

8 Conclusion

Functorial Physics offers a conceptually cleaner, more unified language for gravitation. By eliminating coordinate baggage and leveraging universal constructions, it not only clarifies existing results but also paves a path toward quantum–gravitational unification.

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