Functorial Mechanics and Derived Hamiltonians: Strengthening Unified Theories with Yoneda, CHL, and Kolmogorov–Arnold

Magneton Labs

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Abstract

This paper explores the application of functorial mathematics to derived Hamiltonians, leveraging the Yoneda Lemma, the Curry–Howard–Lambek (CHL) correspondence, and the Kolmogorov–Arnold Representation Theorem. We propose a unified framework that models quantum fields, molecular systems, and transport dynamics using category theory and higher-order Hamiltonian functions.

1 Introduction

A central challenge in unifying physics, biophysics, and quantum mechanics lies in expressing complex systems in a consistent framework. Derived Hamiltonians $R^n H$ offer a pathway for evolving systems, introducing higher-order corrections to classical Hamiltonians H. By integrating the Yoneda Lemma, CHL, and Kolmogorov–Arnold, we strengthen the hypothesis of a unified theory that spans multiple domains.

2 Yoneda Lemma and Hamiltonians

The Yoneda Lemma states:
$$X \cong \mathrm{Nat}(\mathrm{Hom}(X, -), F)$$

Where X is fully characterized by its morphisms to other objects. Applying this to Hamiltonians:

$$H_n \sim \mathrm{Hom}(X, R^n H)$$

This implies that the Hamiltonian H can be described functorially by the transformations between state spaces, leading to:
$$H \vdash R^n H$$

where $R^n H$ denotes the derived n^{th}-order correction to H.

3 Curry–Howard–Lambek (CHL) and Proofs of Energy Evolution

The CHL correspondence establishes a deep link between proofs, programs, and categories:

$$H \vdash R^n H$$

This formalism interprets Hamiltonian evolution as a type-theoretic proof transformation. A state evolving through time t reflects a derivable proof step:

$$H(t) \vdash H(t + \Delta t)$$

where each proof corresponds to energy conservation and physical transitions.

4   Kolmogorov–Arnold and Decomposition of Hamiltonians

The Kolmogorov–Arnold theorem states that any multivariate function can be decomposed as:

$$H(q_1, q_2, \ldots, q_n) = \sum_i \phi_i(\psi_i(q_i))$$

Thus, higher-order Hamiltonians can be expressed as sums of single-variable functions:

$$R^n H = \sum_{i=1}^{n} \phi_i(H_i)$$

This decomposition simplifies modeling complex systems such as quantum fields or protein folding pathways by reducing the computational burden.

5   Conclusion

Integrating functorial mathematics with Hamiltonian dynamics provides a robust framework for modeling systems at various scales, from quantum mechanics to biophysical systems. By applying the Yoneda Lemma, CHL, and Kolmogorov–Arnold, we enhance the predictive power of derived Hamiltonians, reinforcing the foundation of a unified theory.