

# A Functorial Recasting of Nonlocality and Entanglement

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## Abstract

Nonlocality and entanglement are cornerstones of quantum mechanics, exemplified by EPR correlations and Bell-inequality violations. Traditional Hilbert-space treatments, while phenomenally accurate, leave conceptual puzzles regarding how global entanglement emerges from local operational principles. In this paper, we propose a *functorial physics* approach to reformulate quantum nonlocality and entanglement within monoidal categories. By capturing quantum states as objects and processes as morphisms, we show that the mysterious aspects of nonlocal correlations can be viewed as a natural byproduct of composing objects in a higher-level categorical framework. The formalism also illustrates how measurement and classical data extraction can be coded as functors to a “classical category.” We provide explicit mathematical formulations, including how Bell-like states are represented in a monoidal category, and argue that functorial physics offers a more transparent interpretation of quantum correlations.

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## 1 Introduction

Quantum entanglement and nonlocal correlations challenge our classical intuition that separate systems have independent states. From the earliest discussions by Einstein, Podolsky, and Rosen (EPR) [1] to Bell’s theorem [2], it has become clear that quantum mechanics permits correlations that defy local classical realism. Despite its remarkable empirical success, the Hilbert-space formulation of quantum mechanics often leaves open interpretational questions about how, why, and where *entanglement* arises and how nonlocal influences (or the appearance thereof) remain consistent with relativistic no-faster-than-light signaling.

In recent years, *category theory* has played a significant role in recasting quantum theory in a compositional language. Categorical quantum mechanics, as pioneered by Abramsky, Coecke, and others [3,4], leverages the notion of *monoidal categories* to describe how quantum systems combine, transform, and interact. This paper focuses on how nonlocal correlations and entangled states naturally emerge (and become less mysterious) in a *functorial physics* perspective. Specifically, we:

- (i) Outline the standard Hilbert-space description of nonlocal correlations and the puzzle it presents,
- (ii) Introduce a monoidal category  $\mathcal{C}$  to represent quantum processes and states as objects and morphisms,
- (iii) Demonstrate how entangled states can be viewed as *morphisms* that do not factor through separate subsystems,
- (iv) Show how measurement processes can be encoded as functors from  $\mathcal{C}$  to a “classical data” category,
- (v) Provide explicit equations and diagrams to illustrate the new vantage, and
- (vi) Argue that these constructions clarify why nonlocality is a natural structural feature of quantum theory, not an add-on or paradox.

## 2 Nonlocal Correlations in Standard Quantum Mechanics

In the standard formalism, we associate each (finite-dimensional) quantum system with a Hilbert space  $\mathcal{H}$ . Composite systems are modeled by tensor products, so a bipartite system is  $\mathcal{H}_A \otimes \mathcal{H}_B$ . A *pure state*  $|\psi\rangle$  living in  $\mathcal{H}_A \otimes \mathcal{H}_B$  is *entangled* if it cannot be written as  $|\phi\rangle_A \otimes |\chi\rangle_B$ .

**Bell States.** A canonical example is the Bell pair (one of the four maximally entangled two-qubit states):

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (1)$$

Measuring either subsystem in the computational basis results in correlated outcomes (both 0 or both 1). Such correlations persist for any choice of measurement basis, violating Bell inequalities in certain settings [5], thus demonstrating the impossibility of local hidden-variable theories.

## 3 Categorical Structures

We now outline how quantum systems and processes can be *recast* in a functorial framework.

### 3.1 Monoidal Categories

A *monoidal category*  $(\mathcal{C}, \otimes, I)$  consists of:

- A collection of *objects*  $\text{Obj}(\mathcal{C})$ , which we interpret as physical systems,
- A collection of *morphisms*  $\text{Mor}(\mathcal{C})$ , representing physical processes (time evolution, transformations),
- A tensor product  $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$  that combines systems,
- A unit object  $I$  (akin to the “trivial” system) and natural isomorphisms for associativity and unitality.

In a quantum-mechanical model, we often take objects to be finite-dimensional Hilbert spaces, and morphisms to be linear operators.

### 3.2 States as Morphisms

An enlightening shift occurs when we think of a *state* of system  $A$  not as a vector  $|\psi\rangle \in \mathcal{H}_A$ , but as a morphism from the *monoidal unit*  $I$  to  $A$ . Concretely,

$$|\psi\rangle \longleftrightarrow \psi : I \rightarrow A. \quad (2)$$

Similarly, a *two-party* state (e.g., entangled) is a morphism  $\psi : I \rightarrow A \otimes B$ . If  $\psi$  factors as  $\psi_1 \otimes \psi_2$ , we say the state is *product*. If not,  $\psi$  encodes entanglement.

## 4 Nonlocality in a Functorial Light

### 4.1 Entangled States as Non-Factorizable Morphisms

Let  $\psi : I \rightarrow A \otimes B$  represent an entangled bipartite state. We say  $\psi$  is *non-factorizable* if there do *not* exist morphisms  $\psi_A : I \rightarrow A$  and  $\psi_B : I \rightarrow B$  such that  $\psi = (\psi_A \otimes \psi_B) \circ \alpha$ , for some identity (or invertible) morphism  $\alpha : I \rightarrow I$ . Non-factorizability is the categorical statement of entanglement.

**Diagrammatic Representation.** In string-diagram notation (common in monoidal categories), one can draw  $\psi$  as a “Y-shaped” wiring that merges the monoidal unit into  $A$  and  $B$  simultaneously. A product state would correspond to two parallel wires from  $I$  to  $A$  and  $I$  to  $B$ . The inability to separate these wires indicates entanglement.

### 4.2 Bell State Example

Referring to Eq. (1), in the Hilbert-space picture,  $|\beta_{00}\rangle$  is a superposition of  $|00\rangle$  and  $|11\rangle$ . In a categorical language, let  $A = B = \mathbb{C}^2$ , and let  $I$  be  $\mathbb{C}$  (the unit object). Then  $|\beta_{00}\rangle$  is a single morphism:

$$\beta_{00} : I \rightarrow A \otimes B \quad \text{with} \quad \beta_{00}(1) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (3)$$

The *nonlocal correlations* manifest in how  $\beta_{00}$  composes with measurement morphisms  $M : A \otimes B \rightarrow \text{ClassicalData}$ , as described below.

## 5 Measurements as Functors

In the functorial framework, a measurement process can be realized as a *functor*:

$$\mathcal{F} : \mathcal{C} \longrightarrow \mathcal{D}, \quad (4)$$

where  $\mathcal{C}$  is the category of quantum objects and morphisms, and  $\mathcal{D}$  is a category capturing classical data (e.g., probability distributions, outcomes). For instance, we might define:

- **Objects in  $\mathcal{C}$ :** Hilbert spaces,
- **Objects in  $\mathcal{D}$ :** sets or finite probability spaces,
- **Morphisms in  $\mathcal{C}$ :** linear maps (quantum channels, unitaries),
- **Morphisms in  $\mathcal{D}$ :** functions between sets or Markov kernels.

A key example is how a projective measurement on a system  $A$  becomes a morphism  $\mu : A \rightarrow X$  in  $\mathcal{C}$ , with  $X$  a classical register object. By collecting all such  $\mu$  into a single functor, one captures the universal principle that quantum processes can yield classical data in consistent ways. Nonlocal correlations then arise when measuring parts of entangled states  $A \otimes B$ , with the functor ensuring a global coherence condition.

## 6 Physical Interpretation

### 6.1 Why Nonlocality is “Natural” in a Categorical Sense

Because objects in a monoidal category compose via  $\otimes$  with universal properties (associativity up to coherent isomorphisms), the appearance of global correlations in a single morphism  $I \rightarrow A \otimes B$  is *structural*. There is no hidden mechanism transferring signals faster than light; rather, the functorial laws guarantee that local “slices” of this global map remain consistent with the entire diagram. If one tries to factor the morphism into independent local pieces, the mathematics tells us it cannot be done—mirroring Bell’s conclusion that no local hidden-variable factorization can replicate entangled statistics.

### 6.2 Bounding Bell Inequalities via Natural Transformations

Bell-type inequalities can be mapped to constraints on families of morphisms in  $\mathcal{D}$  (classical data), each arising from measuring different bases or settings in  $\mathcal{C}$ . The violation of these inequalities emerges when a single natural transformation cannot consistently factor across all measurement settings. In short, *monoidal coherence* outperforms the factorization assumptions underlying local hidden-variable theories [3].

## 7 Mathematical Formulations: An Illustrative Example

Let us consider a simplified bipartite scenario:

### 7.1 Setup

- Objects  $A, B \in \text{Obj}(\mathcal{C})$  correspond to finite-dimensional Hilbert spaces  $\mathcal{H}_A, \mathcal{H}_B$ .
- A state  $\psi : I \rightarrow A \otimes B$  is given by a morphism out of the monoidal unit  $I$ .
- Measurement morphisms  $m_A : A \rightarrow X, m_B : B \rightarrow Y$ , with  $X, Y \in \text{Obj}(\mathcal{D})$  classical outcome spaces.

### 7.2 Composition and Outcome Probabilities

Define a composite measurement as

$$m_A \otimes m_B : A \otimes B \longrightarrow X \otimes Y. \quad (5)$$

Then the probability distribution over outcomes  $(x, y) \in X \times Y$  is obtained by composing:

$$p(x, y) = (m_A \otimes m_B) \circ \psi(1). \quad (6)$$

In a standard Hilbert-space model, one recovers

$$p(x, y) = \langle \beta_{00} | P_x \otimes P_y | \beta_{00} \rangle,$$

where  $P_x, P_y$  are the projectors associated with measurement outcomes  $x, y$ .

### 7.3 Non-Factorizability Condition

If  $\psi$  is entangled, there exist measurement morphisms  $m_A, m_B$  such that  $p(x, y)$  cannot be written as  $p(x)q(y)$  or any locally correlated mixture. Categorically, the map

$$\psi \longrightarrow (m_A \otimes m_B) \circ \psi$$

cannot factor through separate images of  $A$  and  $B$ . This expresses the same condition as Bell's theorem, but with clearer compositional logic: entanglement is a universal phenomenon that arises in any monoidal category with the relevant structure.

## 8 Conclusions and Outlook

By reframing *nonlocality* and *entanglement* in terms of morphisms and functors in a monoidal category, we remove much of the puzzle around how separate measurement choices can yield inseparable correlations. The formalism naturally encodes global consistency conditions that yield “entangled” global morphisms from a single trivial source  $I$ .

This perspective does not disclaim the predictive content of quantum mechanics but rather reorganizes it into a compositional system where previously paradoxical or “spooky” phenomena become structurally expected. The approach scales beyond bipartite entanglement to multipartite settings, topological quantum field theories, and even experimental protocols in quantum information [6].

Future research aims to:

- Integrate *spacetime constraints* into a higher-categorical version of entanglement,
- Study *hypergraph categories* that capture multi-party quantum networks,
- Investigate new forms of *cryptographic* or *communication* protocols using the diagrammatic vantage,
- Explore the interplay with *general-relativistic spacetimes* and the extension of nonlocal correlations in curved geometries.

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## References

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