

Measuring the Hyperfine Splitting of Rubidium

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The physical phenomenon which we sought to investigate in this three-week project is that of the hyperfine structure of atoms. This refers to the splitting of an atom's individual spectral lines into multiple components as a result of nuclear effects. We investigated the hyperfine structure of Rubidium-87, specifically that of the $780nm$ spectral line, using saturated absorption spectroscopy. This technique was employed due to the undesired effects of Doppler broadening. In addition to the main experiment, a calibration process was first necessary to determine the relation for time differences in the PHA output to differences in frequency between the Rubidium atoms' energy levels. We then constructed an arrangement to be able to measure the energy dips corresponding to the atoms' transition energies. We obtained three energies between the $F = 1$, $F = 2$, and $F = 3$ states of $5^2P_{3/2}$ of $150 \pm 20MHz$, $260 \pm 30MHz$, and $410 \pm 50MHz$ respectively. The best estimates lay within 6% of the literature values and their ranges of uncertainty were in accordance with the true energies.

I. INTRODUCTION

When a laser beam is directed at a gaseous Rubidium source, the photons cause energy transitions of the Rubidium atoms' electrons if their energy exactly corresponds to the energy difference of a transition. This results in narrow dips, known as Lamb dips[1], in the spectrum of the outgoing light where such energies are absorbed by the Rubidium atoms[2].

At absolute zero, each energy level has some associated uncertainty due to the Heisenberg Uncertainty Principle. Therefore, an atom can absorb photons over a certain small range of frequencies, known as the natural line width. At room temperature, however, this range of frequencies is substantially larger due to the constant random thermal motion of the atoms. Since a significant proportion of the atoms in the Rubidium chamber will come into contact with photons that are either red- or blue-shifted due to the Doppler effect, the line width will be caused to broaden[3]. This Doppler-shifted frequency is given by

$$\nu_{\gamma} = \nu_0(1 \pm \frac{v}{c}) \quad (1)$$

where ν_{γ} is the frequency of the photon, ν_0 is the resonance frequency at rest, and $\frac{v}{c}$ is the ratio of the atom's velocity to the speed of light. In our case, this Doppler broadening causes the range of frequencies for the $780nm$ spectral line to dwarf its natural line width.

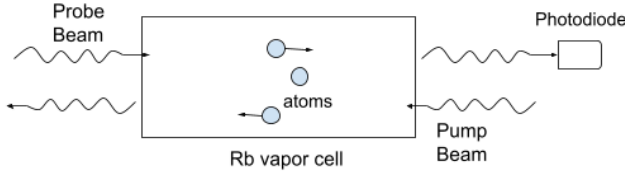


FIG. 1. experimental technique of Doppler-free saturated absorption spectroscopy

However, saturated absorption spectroscopy eliminates

this issue by the use of two laser beams (pump and probe) travelling in opposite directions, as shown in Figure 1. This arrangement necessitates that only atoms at rest relative to both beams be in resonance, and therefore be detected by the apparatus.

In this experiment we investigated the transitions between $5^2S_{1/2}$ and $5^2P_{3/2}$ and thus sought to obtain values for the energy differences between $F = 1$, $F = 2$, and $F = 3$ states of $5^2P_{3/2}$, resulting in three experimentally acquired figures.

II. EXPERIMENTAL SETUP

We had at our disposal the following instruments from which to construct an experimental setup: a Toptica DL-100 Laser, a linear polarizer for light of $780nm$ wavelength, a half-waveplate for $780nm$ light, a 50/50 beamsplitter which allowed 50% of incident light through and reflected the rest, a polarizing cube beamsplitter which reflected vertically polarized light only, Rubidium vapor cells, amplified photodiodes which produced a voltage proportional to the intensity of incident light, and several mirrors.

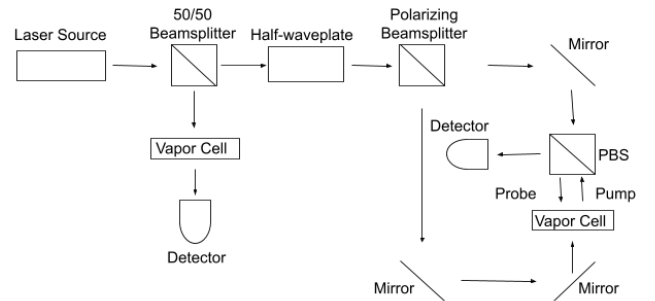


FIG. 2. Setup of apparatus

The position of the 50/50 beamsplitter after the laser source allowed observations of both the Doppler-

broadened peak, by having a detector after one Rubidium vapor cell with only a probe beam, as well as the Doppler-free peak, by directing the other half of the photons through a half-waveplate to make the pump beam more intense. This was needed to excite some of the atoms to higher energy states, so that the probe beam does not interact with as many atoms as it would if they were all in the ground state. Having both the Doppler-free and Doppler broadened peaks was important as it usefully allowed more accurate location of the voltage dips in later analysis. A second polarizing beamsplitter was placed after the half-waveplate to create two separate pump and probe beams, thus eliminating the issue of the atoms' random thermal motion in the vapor cell. Mirrors are used to angle the two probes such that they are parallel when entering the second Rubidium vapor cell, and the second detector is naturally placed after the polarizing beamsplitter and in line with the light reflected by it.

III. CALIBRATION

All data was collected remotely by laboratory technicians according to our proposal. In addition to collecting data based on the experimental setup described in the previous section, it was also necessary to make use of another arrangement including a Michelson interferometer. This calibration process was necessary due to the fact that the Doppler peaks alone cannot allow the frequency differences between the energy levels to be determined, as we would not have a way to relate the differences in time at which the various peaks are recorded with their corresponding frequency differences. We therefore proposed the construction of an additional setup, based on the theory of the superposition of light wave amplitudes, which took the form of a standard Michelson interferometer. From this arrangement we received voltage versus time data, which are plotted in Figure 3.

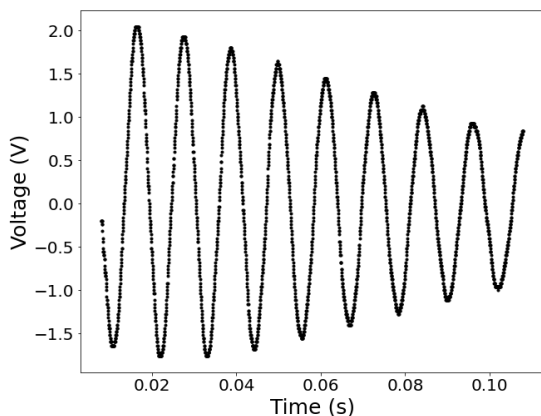


FIG. 3.

In order to obtain a relationship between the two variables frequency and time, it was necessary to make use of

the phase-difference relationship of the two beams produced. The beams had a phase difference given by

$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda}(2L_1 - 2L_2) = \frac{4\pi\nu}{c}(L_1 - L_2) \quad (2)$$

and since the intensity maxima must occur when the phase difference is a multiple of 2π , it can be seen that the frequency difference $\Delta\nu$ between successive maxima is given by

$$\Delta\nu = \frac{c}{2(L_1 - L_2)}. \quad (3)$$

From this result, it can be seen that given a certain value for the difference in the lengths from the beamsplitter to each mirror, the change in frequency between each successive peak can be found. Indeed, with the experimental values of $L_1 = 44.4 \pm 0.2\text{cm}$ and $L_2 = 12.7 \pm 0.2\text{cm}$, this came out to be $\Delta\nu = 4.73 \pm 0.12 \times 10^8\text{Hz}$. Once the frequency between the maxima was known, the time intervals between them needed to be calculated. This was achieved by simply locating times which corresponded to the maximum voltage values for each successive phase. The table below summarizes the voltage values and their corresponding times for the maximum of each peak.

Peak no.	Voltage (V)	Time (s)
1	2.04	0.0164
2	1.92	0.0275
3	1.80	0.0386
4	1.64	0.0497
5	1.44	0.0610
6	1.28	0.0725
7	1.12	0.0845
8	0.92	0.0959

TABLE I. Voltage and time values for the maximum of each peak

An uncertainty of 0.001s is given for all estimations of the maxima's times based on inspection of the calibration plot. From eight peak positions, seven estimates for the wave's time period were obtained. These were averaged to give a value of $\Delta t = 1.14 \pm 0.10 \times 10^{-2}\text{s}$. Now having both $\Delta\nu$ and Δt , we could obtain the factor relating the two variables: $m = \frac{\Delta\nu}{\Delta t} = \frac{4.73 \times 10^8}{1.14 \times 10^{-2}} = 4.16 \pm 0.47 \times 10^{10}\text{Hz s}^{-1}$, where the percentage uncertainty in m is the sum of the percentage uncertainties in $\Delta\nu$ and Δt .

IV. DATA ANALYSIS

The reason for obtaining the Doppler-broadened peak in addition to the Doppler-free peak was to be able to

subtract the broadened peak from the Doppler-free peak. However, as a result of the fact that the beam detected as the broadened peak was not passed through the half-waveplate, it was not as intense as the light used to obtain the Doppler-free peak. Therefore, we needed a measure of the scale factor between the two peaks. To do this, seven times were chosen outside the apparent range of photon absorption which caused signal dips, and the ratio of the Doppler-free peak's value to the Doppler broadened peak's value was calculated. The average of these ratios was determined to be 2.579. (The uncertainty in this figure is not given as the uncertainty in voltage was determined subsequently, but also has no bearing on the estimated time intervals between the dips.) This allowed a representation of the subtracted Doppler-free peak shown below in Figure 4.

For multiple datasets it would have been possible to estimate voltage uncertainty better by looking at the spread of points at same times after starting the recording of data. With only one dataset however, voltage uncertainty was estimated by determining the width of the curve. This can be justified as the signal is not changing vertically at the chosen points, so fluctuation in the voltage signal must be due to random variance. If the data were continuous, the standard deviation of the width would be taken, but the voltage only occupies certain discrete values due to the nature of the recording instrument. As a result, the uncertainty was simply estimated conservatively as $0.01V$, by looking at the width of the two peaks inspected where the derivative of the curve seemed to be zero.

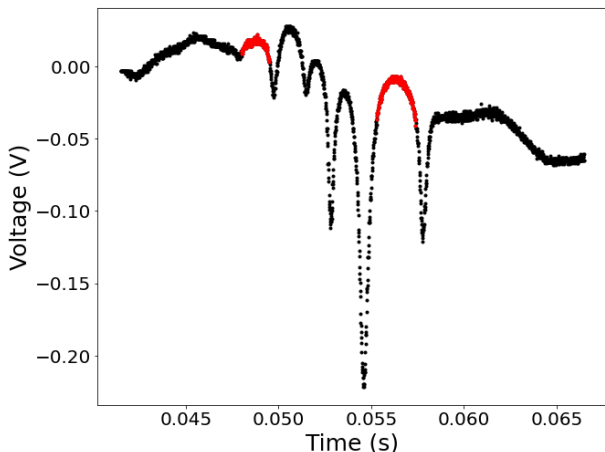


FIG. 4. subtracted Doppler-free peak with red-highlighted regions of interest, chosen due to the wide nature of their peaks, from which the uncertainty in voltage was estimated

Using the subtracted Doppler-broadened peak, each dip representing a resonance was able to be more clearly identified. The figure shows six dips, even though there are in fact only three energy resonances. The extra three dips are referred to as crossover resonances and are caused when two different transition frequencies are

within the Doppler-broadened frequency distribution of the photons in the laser beam[4]. Their frequencies are given by

$$f_{crossover} = \frac{f_1 + f_2}{2}, \quad (4)$$

where f_1 and f_2 are real resonances. This allows the nature of the dips, i.e. whether they are real or crossover resonances, to be determined. The first and last dips are of course necessarily real dips. It can be straightforwardly deduced that the second, fourth and fifth dips are crossover dips, leaving the third dip as representing a real resonance. To be sure, each of the six dips was fitted to a Lorentzian distribution plus a linear background. An example of these fits, fitting the third real peak, is documented in Figure 5.

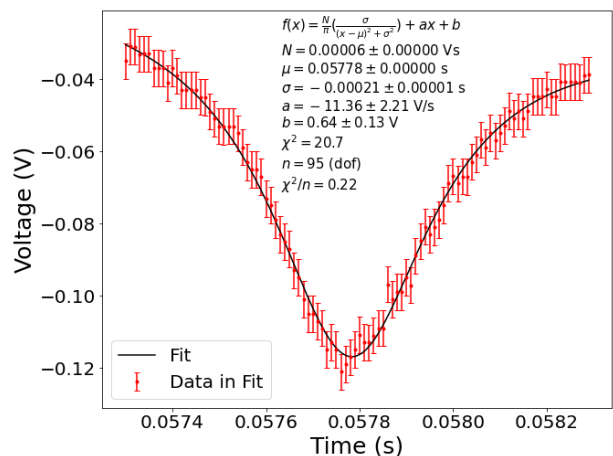


FIG. 5.

The fits allowed an accurate determination of the dips' minima, and thence of the three time differences which could be converted into frequency differences. Table II summarizes the fits' locations of the three real dips' minima.

Real Dip no.	Fit minimum (s)	Uncertainty (s)
1	4.786×10^{-2}	4×10^{-5}
2	5.147×10^{-2}	1×10^{-5}
3	5.778×10^{-2}	1×10^{-6}

TABLE II. Time values and uncertainties for the minima of each real dip

Since all the uncertainty in the fits' minima were very small as a fraction of the minima values, the uncertainties could be considered negligible. Since there are three minima, there will be three time intervals. These and the converted frequency difference (which correspond to the atoms' transition frequencies) are given in the Table III.

Transition	$F = 1 \rightarrow 2$	$F = 2 \rightarrow 3$	$F = 1 \rightarrow 3$
Δt (s)	3.61×10^{-3}	6.31×10^{-3}	9.92×10^{-3}
$\Delta\nu$ (Hz)	1.50×10^8	2.63×10^8	4.13×10^8
$\delta\Delta\nu$ (Hz)	1.69×10^7	2.97×10^7	4.67×10^7
ΔE (eV)	6.62×10^{-7}	1.09×10^{-6}	1.71×10^{-6}

TABLE III. Time intervals between minima, with associated calculated frequencies and energies

V. CONCLUSION

Our investigation finally resulted in the three transition frequencies: $150 \pm 20\text{MHz}$, $260 \pm 30\text{MHz}$, and $410 \pm 50\text{MHz}$. Compared with the respective literature values of 157MHz , 267MHz , and 424MHz [5], it can be seen that our experimentally obtained values for the three transition frequencies agree very well, as the literature values lie well within the ranges of uncertainty.

There were no overwhelming sources of systematic uncertainty, as is indicative of the corroboration of our results with the literature values. The largest source of systematic uncertainty was certainly that arising from the uncertainty in voltage, which we could only estimate

by inspection of the PHA output, as the precision of the instrument measuring the voltage was not given. In addition, this was a conservative estimate so as not to potentially underestimate it. The uncertainty in the scale factor m , coming primarily from the uncertainty in the time period of the interferometer calibration data and secondarily from the precision of the measurements of the interferometer arm lengths, was the other major source of uncertainty. The uncertainty in the aforementioned time period was also estimated by inspection of turning point widths in the interferometer output, and so was estimated conservatively lest the true uncertainty be less than the estimation. Despite the followingly somewhat large uncertainties in the final values, they do accurately describe the hyperfine splitting transition frequencies of the 780nm spectral line of Rubidium-87.

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