Mathematical representations of the Earth

Earth materials and buried structures are complicated. Yet we need to be able to represent these mathematically so that responses from geophysical surveys can be numerically simulated. Ultimately, our goal is to find a mathematical representation of the earth, such that when simulated responses are generated, then the simulated responses are in agreement with the observations. The process of carrying this out is referred to as "inversion".

There are many ways to parameterize the earth mathematically. We can have discrete objects with boundaries or we can divide the earth into "cells", each of which has constant value of a physical property. The earth is 3-dimensional and to accommodate this the cells can be: layers, 2D cylinders, 3D prisms.

We generically refer to the parameterized earth as a "model". In the inverse problem we attempt to find values for each element in the model. In the application step, these values are interpreted to help solve the problem of interest. This means we need to know the relationship between values of a physical property and rock type, alteration, buried object etc.

Below we provide some examples of commonly used earth parameterizations.

**Uniform halfspace**

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1. **A "uniform halfspace"** means the earth beneath the surface has the same physical property value. No topography is permitted and the region above the surface is assumed to be air (sometimes called "free space"). In a "uniform wholespace" the entire volume has the same physical property value. This is useful in borehole studies.

**Typical geoscience tasks:** Mapping shallow apparent conductivity for site characterization, contaminated ground, or other shallow investigation projects

**Buried objects**

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1. When **buried objects** are the focus, the earth is usually considered to be uniform all around the object. The object itself may be represented with a more or less complicated set of parameters.

**Typical geoscience tasks:** Finding or characterizing buried utilities, tanks, UXOs, or other objects.

**1D (layered) models**

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1. In a **1D model**, the physical property is assumed to vary only in one direction (usually depth). The earth is commonly divided into layers (cells), each of which has a constant value of physical property. Surveys that are designed to yield 1D results are often called soundings. Results are often displayed in a way that resembles a drill core.

**Typical geoscience tasks:** Layered Earth problems, such as hydrology, overburden thickness, clay layer detection, etc.

**2D models**

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1. In a **2D model**, the physical property is assumed to vary in two directions, usually depth and the direction parallel to a survey line. Surveys that yield 2D results are interpreted as cross sections. The assumption is that the structures extend without change either side of the survey line.

**Typical geoscience tasks:** Detailed geologic structure characterization such as defining ore bodies or other geologic features

**3D models**

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1. In a **3D model**, the subsurface is divided into prismatic cells. Each cell is assumed to have a constant physical property. This is the most general parameterization and 3D inversion is computatinally intensive.

**Typical geoscience tasks:** Detailed geologic structure characterization such as defining ore bodies or other geologic features

For 2D and 3D models, structures within the earth may be considered as simple geometric shapes, or as continuously varying distributions of a physical property. Simple shapes (spheres, blocks, cylinders, etc.) are easy to describe - they require few parameters. For example a cylinder is fully described by a fixed radius, depth to top, length and density. For continuously varying physical property distributions, the Earth's structure must be described as a *function* with the physical property dependent upon position. Representing this mathematically requires that the earth be represented with many cells. In the next section, we expand upon the distinctions between discrete simple geometric shapes and continuously varying models.