Discretizing the inverse problem

We have already shown how the problem

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| ./images/density\_mesh.jpg |

is

where the volume is divided into cells with the density constant in each cell.

is the response of the ith observation location due to a cell of constant density in the jth cell. (Remember the cell can be 1D (layer), 2D or 3D).

The above discretization allows the data to be written in matrix form:

The next task is to get a matrix representation of our model objective function. In 1D we had a combination

Suppose the region on which the model is defined is divided into cells

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| ./images/1D\_mesh.jpg |

and we assume the model is constant on each cell.

# Discretizing the data equations

Given data of the form

then there are a number of ways to write this in vector/matrix form.

## Method 1

Let be represented by "cells" where the model is constant in each cell.

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| ./images/1D\_mesh2.jpg |

For example on the kth cell ,

so

where is the integral of the jth kernel function over the kth cell. In matrix vector form this becomes

## Method 2: Quadrature formulation

where are known weights. For example, we could evaluate this with a midpoint rule

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| ./images/midpointrule.jpg |

Let be the width of the kth cell, and let denote the center of the kth cell, then

In our case

In this case the model vector

is generally written as so the model parameters are the values of the model at the cell centers. The elements of are

then

# Discretizing the model objective function

A general objective function in 1D is

We use the same discretization as we did in the forward problem. Divide the region on which the model is defined into cells and assume the model is constant in each cell.

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| ./images/Mcells.jpg |

Consider the first term

For convenience, let denote the length if the ith element. Then

where

The term that penalizes variation in the x-direction is similarly derived.

We want to find a numerical approximation

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| ./images/Xcells.jpg |

Let be the distance between the center f the cells. A discrete approximation to the integral is had by evaluating the derivative of the model based upon how much it changes between cell centers.

Note that there are only terms in the sum. The part represents the average gradient between the kth and k+1th cell. Now this can be written as

where

If is written as an matrix, then its last row is zero. The reason for a row to be zero is that there are only segments on which linear gradients have been defined. Effectively the two cells on each end have been neglected.

So we have:

If we discretize a combination of these with a reference model, then

where is an matrix.

So our inverse problem in which we minimize

becomes

Now we only need how to solve this (see notes on the UBCGIF website). Before I reproduce only the basic equation, first, take the gradient:

so

and

This is an system of equations solved for . Solve this for many values of and model that reproduces the data to the desired value.

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| ./images/tikhonov\_curve.jpg |

# Vector differentiation

Consider

Similarly,

Consider

If is symmetric then so

Now do the procedure on