Overview of linear inverse theory

* Forward problem
* Inverse problem
* Fundamental challenges for inverse problems

# Overview of linear inverse theory

# The geophysical experiment

Geophysicists are continually faced with a genuine type of problem. Our goal is to extract quantitative information about the earth (or a physical system) without directly sampling. To carry the task out, the geophysicist sets up an experiment (sensitive to particular physical properties that are diagnostic of the study).

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The experiment that is designed depends upon what information is sought

* large scale (whole earth, find )
* exploration (oil, gas, minerals, find )
* environmental (utilities, salt water intrusion, )

# Experiment

* energy is input into the earth
* propagation of the energy depends upon physical laws.
* initial primary energy interacts with substructure to generate scattered (secondary) energy which is returned to a receiver.
* receiver outputs a (set) if numbers which constitute our data.

Knowledge of (1) the data, (2) physics about how energy propagates through the earth, and (3) the physical experiment (sources/receivers geometry, ...) must somehow be combined to reveal information about the earth. This is the goal of inverse theory. It provides a set of mathematical tools by which to extract quantitative answers to specific questions.

# Examples

**Cross-well tomography**. We measure the travel time and want to know the velocity .

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**DC Surveys**. We measure voltage and want and need many sources.

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**Medical imaging**:

* CAT
* X-ray
* Detectors record amplitude
* find (effectively an absorption coefficient)
* need to rotate source/receivers to get good coverage
* note the difference with the geophysics problem where we only have sources and receivers on the surface.

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**Optics: Shaw problem.** Each ray (incident angle) scatters into a region of . The goal is to measure the intensity on the right hand side and determine the strength of the source on the left hand side.

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