DATA 604 HW5

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These exercises and their solutions were made with reference to *Modeling and Simulation in Python* (version 3) authored by Allen B. Downey. This week's exercises focused on **Case Studies**, chapters 10 and 11 of the text.

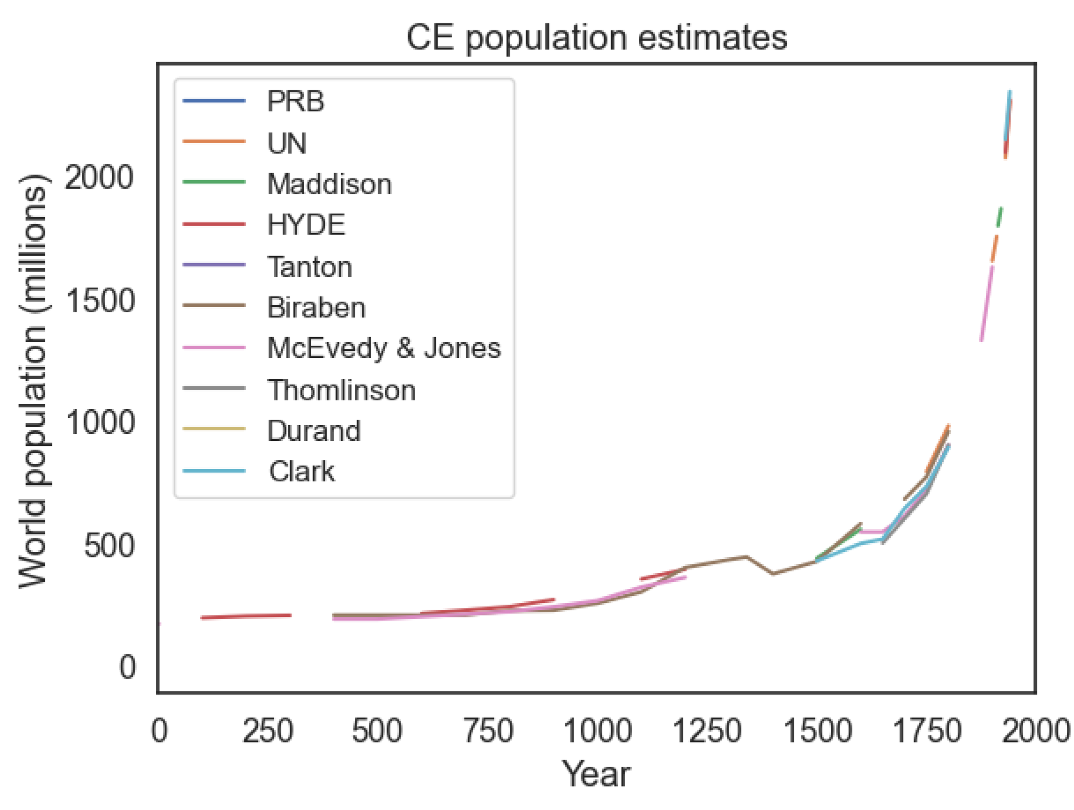
In [1]:
```
# Configure Jupyter so figures appear in the notebook
%matplotlib inline

# Configure Jupyter to display the assigned value after an assignment
%config InteractiveShell.ast_node_interactivity='last_expr_or_assign'

# import functions from the modsim.py module
from modsim import *

from pandas import read_html
```

Chapter 10

Background

On the Wikipedia page about world population estimates, the first table contains estimates for prehistoric populations. The following cells process this table and plot some of the results.

In [2]:
```
#read in data from Wikipedia
filename = 'data/World_population_estimates.html'
tables = read_html(filename, header=0, index_col=0, decimal='M')
table1 = tables[1]

#replace long column names with shortened form:
table1.columns = ['PRB', 'UN', 'Maddison', 'HYDE', 'Tanton',
                  'Biraben', 'McEvedy & Jones', 'Thomlinson', 'Durand', 'Clark']

#Coerce column entries to numeric
for col in table1.columns:
    table1[col] = pd.to_numeric(table1[col], errors='coerce')

#Plot our table
#table1.plot()
#decorate(xlim=[-10000, 2000], xlabel='Year',
#          ylabel='World population (millions)',
#          title='Prehistoric population estimates')
#plt.legend(fontsize='small');

#Use `xlim` to zoom in on everything after Year 0.
table1.plot()
decorate(xlim=[0, 2000], xlabel='Year',
         ylabel='World population (millions)',
         title='CE population estimates')
plt.legend(fontsize='small');
```

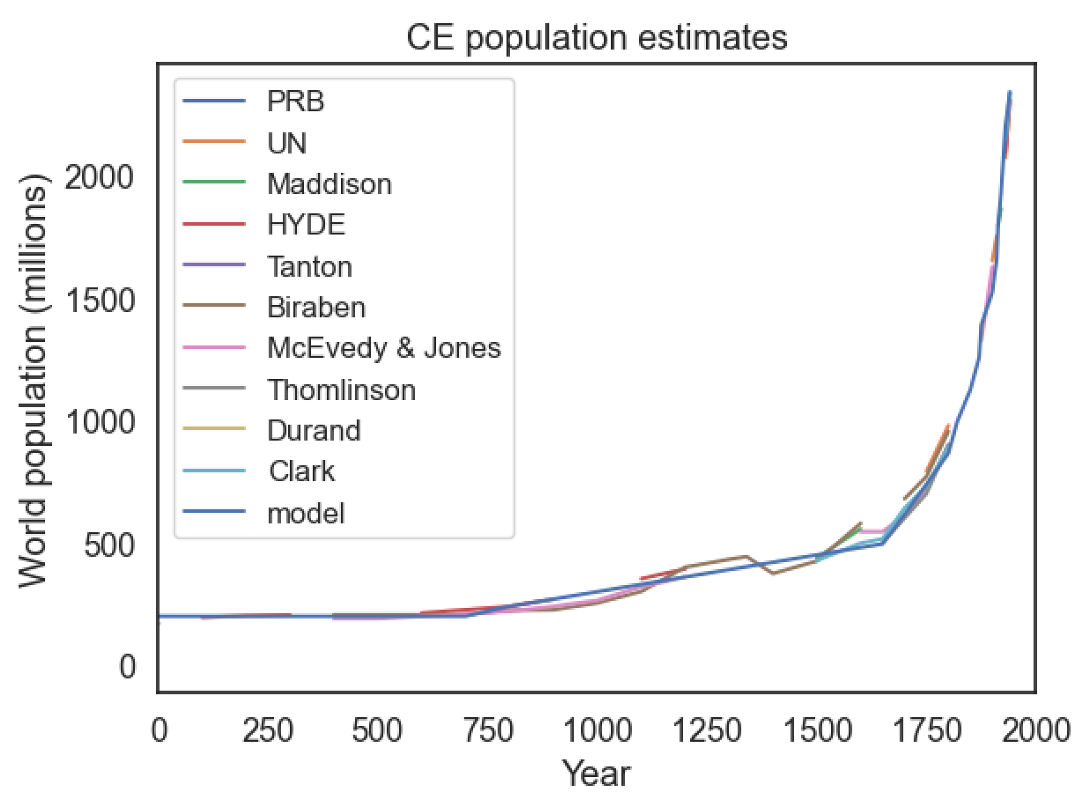


Optional Exercise

See if you can find a model that fits these data well from Year 0 to 1950.

How well does your best model predict actual population growth from 1950 to the present?

In [3]:
```
#Pt1: See if you can find a model that fits these data well from Year 0 to 1950.

#declare variables
alpha1 = 0.3
alpha2 = 3.525 / 50 #slope from 1700 thru 1900 divided by constant varied until output matched desire
model_list = []

#iterate through each year entry
for i in table1.index:
    if i <= 700:
        model_list.append(200)
    elif i > 700 and i < 1700:
        model_list.append(alpha1 * i ) #slight growth - linear
    else: #beyond 1700
        model_list.append(alpha2 * i + model_list[-1])

table1['model'] = model_list

table1.plot()
decorate(xlim=[0, 2000], xlabel='Year',
         ylabel='World population (millions)',
         title='CE population estimates')
plt.legend(fontsize='small');
```



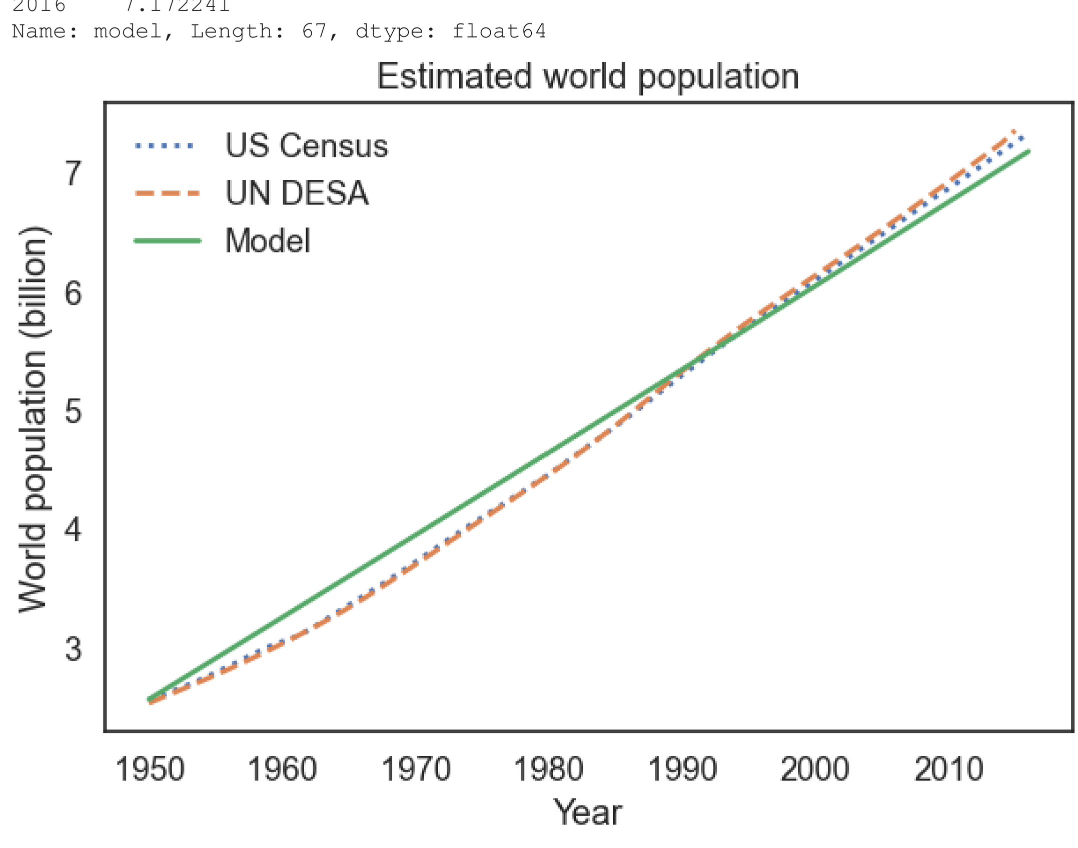
I observed that the population seemed to grow at varied rates during different time periods:

- upto the year 700 the population was stable / constant about 200 mil
- from 700 to 1700 the population grew at a slight rate (`alpha1`)
- from 1700 onward the population grew at a faster rate and so we added the population from the entry prior and reduced the value of our multiplier (`alpha2`)

I then observed how this model performed on data from 1950 onward:

In [4]:
```
# How well does your best model predict actual population growth from 1950 to the present?

#extract proper data
table2 = tables[2]

#replace long column names with shortened form:
table2.columns = ['census', 'PRB', 'un', 'Maddison', 'HYDE', 'Tanton',
                  'Biraben', 'McEvedy & Jones', 'Thomlinson', 'Durand', 'Clark']

#declare variables
alpha2 = 3.525 / 50 #slope from 1700 thru 1900 divided by constant varied until output matched desire
model_list = []
un = table2.un / 1e9
census = table2.census / 1e9

#iterate through each year entry
for i in table2.index:
    if i == 1950:
        model_list.append(get_first_value(census))
    else:
        model_list.append(model_list[-1] + (alpha2 * i) / 2000) #convert millions to billions

table2['model'] = model_list


model = table2.model

plot(census, ':', label='US Census')
plot(un, '--', label='UN DESA')
plot(model, '-', label='Model')

decorate(xlabel='Year',
         ylabel='World population (billion)',
         title='Estimated world population')

#model

#Kernel>Restart & Run All (to update/run)
```

Out[4]:
```
Year
1950    2.557629
1951    2.626401
1952    2.695209
1953    2.764053
1954    2.832931
          ...
2012    6.888197
2013    6.959155
2014    7.030149
2015    7.101177
2016    7.172241
Name: model, Length: 67, dtype: float64
```



The same code snippet used upto the year 1950 was applied post-1950 (with a conversion factor) and the result above shows that, while limited and certainly general, the equation (`model_list[-1] + (alpha2 * i) / 2000`) generates a relatively accurate model.

Chapter 11

SIR Implementation

Exercise Suppose the time between contacts is 4 days and the recovery time is 5 days. After 14 weeks, how many students, total, have been infected?

Hint: what is the change in S between the beginning and the end of the simulation?

In [19]:
```
init = State(S=89, I=1, R=0) #generate state variable
init /= sum(init) #convert from number to fraction

def make_system(beta, gamma):
    """Make a system object for the SIR model.

    beta: contact rate in days
    gamma: recovery rate in days

    returns: System object
    """
    init = State(S=89, I=1, R=0)
    init /= sum(init)

    t0 = 0
    t_end = 7 * 14 #14 weeks

    return System(init=init, t0=t0, t_end=t_end,
                  beta=beta, gamma=gamma)

tc = 4      # time between contacts in days
tr = 5      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)

#take the state during the current time step and return the state during the next time step
def update_func(state, t, system):
    """Update the SIR model.

    state: State with variables S, I, R
    t: time step
    system: System with beta and gamma

    returns: State object
    """
    s, i, r = state

    infected = system.beta * i * s
    recovered = system.gamma * i

    s -= infected
    i += infected - recovered
    r += recovered

    return State(S=s, I=i, R=r)

def run_simulation(system, update_func): #call update function for each time step
    """Runs a simulation of the system.

    system: System object
    update_func: function that updates state

    returns: State object for final state
    """
    state = system.init

    for t in linrange(system.t0, system.t_end):
        state = update_func(state, t, system)

    return state

final = run_simulation(system, update_func) #the state of the system at t_end

#How many students have been infected?
(init.S - final.S) * 90 #the opposite of how many haven't
```

Out[19]: 34.08459698173313

Provided a time between contacts of 4 days and recovery time of 5 days, after 14 weeks, 34 students will have been infected.

Using a DataFrame

Exercise Suppose the time between contacts is 4 days and the recovery time is 5 days. Simulate this scenario for 14 weeks and plot the results.

In [21]:
```
def run_simulation(system, update_func):
    """Runs a simulation of the system.

    system: System object
    update_func: function that updates state

    returns: TimeFrame
    """
    frame = TimeFrame(columns=system.init.index)
    frame.row[system.t0] = system.init

    for t in linrange(system.t0, system.t_end):
        frame.row[t+1] = update_func(frame.row[t], t, system)

    return frame

def plot_results(S, I, R):
    """Plot the results of a SIR model.

    S: TimeSeries
    I: TimeSeries
    R: TimeSeries
    """
    plot(S, '--', label='Susceptible')
    plot(I, '-', label='Infected')
    plot(R, ':', label='Recovered')
    decorate(xlabel='Time (days)',
             ylabel='Fraction of population')

tc = 4      # time between contacts in days
tr = 5      # recovery time in days

beta = 1 / tc      # contact rate in per day
gamma = 1 / tr     # recovery rate in per day

system = make_system(beta, gamma)
results = run_simulation(system, update_func)
results.head()

plot_results(results.S, results.I, results.R)
```