We can make use of the following to find the associated eigenvalue and eigenvector of A:

1. $\boldsymbol{Av} = \boldsymbol{\lambda v}$. A represents the matrix, v the eigenvector, and λ the eigenvalue associated with A.
2. $\boldsymbol{det(\lambda I - A) = 0}$. Useful for finding eigenvalues of A.
3. $\boldsymbol{(\lambda I - A)v = 0}$. Useful for finding eigenvectors associated with the eigenvalue (ie. $\lambda_1$).

Initialize matrices and solve for eigenvalues via **det(λI-A)=0**.

$$A: \begin{matrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{matrix}, \lambda I: \begin{matrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{matrix} \rightarrow$$

$$\text{det}(\lambda I\text{-}A): \text{det}\left(\begin{matrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{matrix} - \begin{matrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{matrix}\right) \rightarrow \text{det}\left(\begin{matrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{matrix}\right)$$

$(\lambda - 1)[(\lambda - 4)(\lambda - 6) - (0)(-5)] - (-2)\,[(0)(\lambda - 6) - (0)(-5)] + (-3)[(0)(0) - (0)(\lambda - 4)] \rightarrow$

$(\lambda - 1)[(\lambda - 4)(\lambda - 6)] + 2[0] - 3[0]$

$(\lambda - 1)(\lambda - 4)(\lambda - 6) = 0$

With **characteristics polynomial**: $\boldsymbol{\lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0}$,

and corresponding **eigenvalues** of $\boldsymbol{\lambda_1}$**=1,** $\boldsymbol{\lambda_2}$**=4,** $\boldsymbol{\lambda_3}$**=6**.

At this point we have to make use of $\boldsymbol{(\lambda I - A)v = 0}$ to solve for associated eigenvectors.

$$\text{Taking } \begin{matrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{matrix} \text{ and substituting } \boldsymbol{\lambda_1=1} \rightarrow \begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Leading to $v_1$ = anything, $v_2$= -(5/3)$v_3$, $v_3$=0 …

For eigenvalue $\boldsymbol{\lambda_1}$**=1,** the associated **eigenvector** is $\boldsymbol{v_1} = \begin{matrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{matrix}$.

$$\text{Taking } \begin{matrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{matrix} \text{ and substituting } \boldsymbol{\lambda_2=4} \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Leading to $v_1$ = (2/3)$v_2$+ $v_3$ and $v_3$=0 …

For eigenvalue $\boldsymbol{\lambda_2}$**=4,** the associated **eigenvector** is $\boldsymbol{v_2} = \begin{matrix} \mathbf{2/3} \\ \mathbf{1} \\ \mathbf{0} \end{matrix}$.

$$\text{Taking } \begin{matrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{matrix} \text{ and substituting } \boldsymbol{\lambda_3=6} \rightarrow \begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

Leading to $v_1$ = (2/5)$v_2$+ (3/5)$v_3$ and $v_2$=(5/2) $v_3$

Thus leading to our plugging $v_2$into $v_1$= (2/5)(5/2) $v_3$ + (3/5)$v_3$ = (8/5)$v_3$

For eigenvalue $\boldsymbol{\lambda_3}$**=6,** the associated **eigenvector** is $\boldsymbol{v_3}$**=** $\begin{matrix} \mathbf{8/5} \\ \mathbf{5/2} \\ \mathbf{1} \end{matrix}$ .

To summarize, the eigenvectors for matrix A are: $\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}$, $\begin{matrix} 2/3 \\ 1 \\ 0 \end{matrix}$ , and $\begin{matrix} 8/5 \\ 5/2 \\ 1 \end{matrix}$.