TMR4243 - MARINE CONTROL SYSTEMS II

Homework assignment 5

# 1 Backstepping

## 1.1 Task: Integrator backstepping

For the following systems, determine the relative degree and perform an integrator backstepping design:

1. Objective: Control $x_1 \to x_d(t)$. Plant:

$$\dot{x}_1 = x_2 \tag{1}$$
$$\dot{x}_2 = \left(1 + x_1^2\right) u - 2x_1 - 3\,|x_2|\,x_2 + e^{1-t} x_1 x_2 \tag{2}$$

2. Objective: Control $x_1 \to 0$. Plant:

$$\dot{x}_1 = x_2 \tag{3}$$
$$\dot{x}_2 = x_3 \tag{4}$$
$$\dot{x}_3 = u - x_1 - 3x_2 - 2x_3 \tag{5}$$

Draw the block-diagrams for the closed-loop systems in the $z$-dynamics.

## 1.2 Task: Vectorial backstepping

For the Euler-Lagrange system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D(q,\dot{q})\dot{q} + g(q) = \tau \tag{6}$$

where $q \in \mathbb{R}^n$ and $\dot{q}$ are generalized coordinates and velocities in an inertial frame, $M(q) \in \mathbb{R}^{n\times n}$ is an invertible inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is a Coriolis matriix, $D(q,\dot{q}) \in \mathbb{R}^{n\times n}$ model damping/friction coefficients, $g(q)$ model restoring/spring forces, and $\tau \in \mathbb{R}^n$ is an actuated control input force vector. The objective is to control the coordinate vector $q(t)$ to track a desired vector $q_d(t) \in \mathbb{R}^n$.

Choose state variables, and set the system up on state space form.

Perform a vectorial backstepping design that solves the tracking objective and ensures that the closed-loop error system is UGES.

## 1.3 Task: LgV backstepping

With the objective to control $x_1 \to 0$, design a feedback control law based on LgV-backstepping (Arcak and Kokotović, 2001) for the plant

$$\dot{x}_1 = x_2 \tag{7}$$
$$\dot{x}_2 = x_3 \tag{8}$$
$$\dot{x}_3 = u - x_1 - 3x_2 - 2x_3 \tag{9}$$

Draw the block-diagram for the closed-loop system in the $z$-dynamics, compare and contrast this to your answer above.

### 1.4 Task: Integral action

For the simplified DP plant

$$
\begin{aligned}
\dot{\xi} &= \eta &\quad (10)\\
\dot{\eta} &= R(\psi)\nu &\quad (11)\\
M\dot{\nu} &= \tau &\quad (12)
\end{aligned}
$$

where $\eta \in \mathbb{R}^3$ is the position/heading of the vessel in an inertial frame, $\nu \in \mathbb{R}^3$ is the body-fixed velocity vector, $R(\psi)$ is the rotation matrix, $M = M^\top > 0$ is the mass matrix, and $\xi$ is an integral action state. The control objective is to control $\eta \to 0$ and include integral action in the control law.

Instead of a typical 3-step design, you shall perform a 2-step design that deviates from the traditional backstepping procedure. You will then design a PI control law in the 1st step that renders the CLF negative semidefinite and satisfies the LaSalle-Yoshizawa theorem. Then you shall backstep this in a 2nd step leading to the control law for $\tau \in \mathbb{R}^3$.

Correspondingly, let $z_1 := \eta$, $z_2 := \nu - \alpha_1$, and

$$
\begin{aligned}
V_1 \;&:= \frac{1}{2} z_1^\top z_1 + \frac{1}{2} \xi^\top K_i \xi, \qquad K_i = K_i^\top > 0 &\quad (13)\\
V_2 \;&:= V_1 + \frac{1}{2} z_2^\top M z_2. &\quad (14)
\end{aligned}
$$

Perform a 2-step backstepping design, where you use $V_1$ in the 1st step and $V_2$ in the 2nd step, to design a control law with integral action, and conclude on stability and convergence properties.

Expect that the derivative of the Lyapunov function will only be negative semidefinite, but by application of the LaSalle-Yoshizawa theorem you should get the result you need.

## References

Arcak, M. and Kokotović, P. (2001). Redesign of backstepping for robustness against unmodelled dynamics. *Int. J. Robust Nonlinear Contr.*, 11(7):633–643. Robustness in identification and control.