TMR4243 - Marine Control Systems II

Homework assignment 7

1 Maneuvering a scalar plant

Consider the plant

$$\dot{p} = u, \qquad p = col(x, y) \in \mathbb{R}^2,$$

and let the control objective be maneuvering Skjetne (2005), with geometric task to control p to and along a path $p_d(s)$ parameterized by a path variable s, and a dynamic task to satisfy a speed assignment $\upsilon_s(s,t)$ for $\dot{s}$.

Task 1 *Propose a parametrization for $p_d(s)$ that gives a straight-line path.*

Task 2 *Propose a parametrization for $p_d(s)$ that gives an ellipsoidal path, centered in (x_c, y_c) and with radius r_x in the x-direction and radius r_y in the y-direction.*

Let υ_0 be a desired constant speed along the path for $\dot{p}(t)$.

Task 3 *Propose a speed assignment $\upsilon_s(t,s)$ for $\dot{s}(t)$, that ensures path-following at constant speed υ_0* [m/s], *along:*

1. *The straight-line path.*
2. *The ellipsoidal path.*

Hint: This shall ensure that $|\dot{p}| = |\dot{p}_d| = \upsilon_0$ when p is tracking $p_d(s)$.

Task 4 *Show that with the control law and CLF*

$$u = -K\left(p - p_d(s)\right) + p_d^s(s)\upsilon_s(t,s), \qquad K = K^\top > 0$$

$$V(p,s) = \frac{1}{2}\left(p - p_d(s)\right)^\top \left(p - p_d(s)\right)$$

we get

$$\dot{V} = -\left(p - p_d(s)\right)^\top K \left(p - p_d(s)\right) - V^s(p,s)\left(\upsilon_s(t,s) - \dot{s}\right).$$

What is the expression for $V^s(p,s)$?

Task 5 *Show that the update law for $\dot{s}$ given by*

$$\dot{s} = \upsilon_s(t,s)$$

*solves the Maneuvering Problem. Why do we call this a **Tracking update law?***

Task 6 *Show that the update law for $\dot{s}$ given by*

$$\dot{s} = \upsilon_s(t,s) - \mu V^s(p,s), \qquad \mu \geq 0$$

*solves the Maneuvering Problem. Why do we call this a **Gradient update law?***

Note that the vector $p_d^s(s)$ is for any value s the tangent vector along the path at $p_d(s)$. A modified version of the gradient update law is to ensure that the tangent vector $p_d^s(s)$ is normalized. This will avoid a varying gain from $V^s(p,s)$ along the path according to the parametrization.

Task 7 *Show that the modified gradient update law for $\dot{s}$ given by*

$$\dot{s} = \upsilon_s(t,s) - \frac{\mu}{|p_d^s(s)|}V^s(p,s), \qquad \mu \geq 0$$

*solves the Maneuvering Problem. Why do we call this a **Unit-tangent gradient update law?***

Task 8 *Let the path be the straight-line, and determine path coefficients so that the path becomes the x-axis. Consider $V(p,s)$ to be a cost function, where you fix the position p to be constant and let s be a free optimization variable.*

1. *Let p be located at $p = col(5,0)$. What is the value of s that minimizes $s \mapsto V(p,s)$?*
2. *Let p be located at $p = col(15,7)$. What is the value of s that minimizes $s \mapsto V(p,s)$?*

Task 9 *Let $\omega_s = \upsilon_s(t,s) - \dot{s}$ be the path-variable speed error, and*

$$V_2(p,s,\omega_s) := V(p,s) + \frac{1}{2\lambda\mu}\omega_s^2$$

(considering ω_s as an additional state in the system). Show that the update law for $\dot{s}$ given by

$$\begin{aligned} \dot{s} &= \upsilon_s(t,s) - \omega_s \\ \dot{\omega}_s &= -\lambda\left(\omega_s - \mu V^s(p,s)\right), \qquad \mu \geq 0 \end{aligned}$$

gives

$$\dot{V}_2 = -\left(p - p_d(s)\right)^\top K \left(p - p_d(s)\right) - \frac{1}{\mu}\omega_s^2,$$

and solves the Maneuvering Problem. Why do we call this a ***Filtered gradient update law?***

You shall now implement the closed-loop system in Matlab/Simulink with the unit-tangent gradient update law, and the two alternative paths and speed assignments:

- The straight-line path, going through the points $(2,0)$ and $(10,4)$.
- The ellipsoidal path, centered at $(x_c, y_c) = (6,0)$ with radii $(r_x, r_y) = (5,3)$.

Set the other variables to:

$$\begin{aligned} K &= 1.0 \cdot I \\ \upsilon_0 &= 1.0 \text{ [m/s]} \end{aligned}$$

Test different initial conditions for $p(0) = (x_0, y_0)$ and $s(0)$ for the two paths and speed assignments, and observe the fast transient response for $s(t)$ the first second of simulation and thereby the overall behavior of the trajectories.

Simulate also with $\mu = 0$ and discuss the differences in the responses.

In particular, test and report the following:

Task 10 *For the straight-line path, start with initial condition $s(0) = 0$ and $p(0) = (6,5)$.*

1. *Set $\mu = 0$ and simulate for 3 seconds, and store the data.*
2. *Set $\mu = 10$ and simulate for 3 seconds, and store the data.*

For both cases, plot the 2D position responses of $(p(t), p_d(s(t)))$ in the same figure during the two seconds. Plot also the time-plots of $s(t)$ in another figure. Add the figures to your report, using the provided 'plotpdftex.m' script. Discuss the behavior of the tracking update law ($\mu = 0$) versus the gradient update law ($\mu = 10$).

Task 11 *For the ellipsoidal path, start with initial condition $s(0) = 0$ and $p(0) = (7,2)$.*

1. *Set $\mu = 0$ and simulate for 25 seconds, and store the data.*
2. *Set $\mu = 10$ and simulate for 25 seconds, and store the data.*

For both cases, plot the 2D position responses of $(p(t), p_d(s(t)))$ in the same figure during the five seconds. Plot also the time-plots of $s(t)$ in another figure. Add the figures to your report, using the provided 'plotpdftex.m' script. Discuss the behavior of the tracking update law ($\mu = 0$) versus the gradient update law ($\mu = 10$).

References

Skjetne, R. (2005). *The Maneuvering Problem.* PhD thesis, Norwegian Univ. Sci. & Tech., Trondheim, Norway.