TMR4243 - Marine Control Systems II

Homework assignment 6 - Observer design

1 Observability

1.1 Observability and detectability

1. For the following system, check if the system is *observable* and/or *detectable*.
   Dynamics:
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
   Outputs:
   (a) $y = x_1$
   (b) $y = x_2$

2. For the above system, calculate the observability Gramian matrix Q_∞ for $t = \infty$, calculate its rank, and conclude again on observability for the two cases:
   (a) $y = x_1$
   (b) $y = x_2$

3. For the above system with $y = x_1$, design a Luenberger observer, and:
   (a) Show how to use the `place.m` command to design the injection gain L of the observer, that places the closed-loop observer poles at $p_{obs} = \{-10, -15\}$.
   (b) Show how to use the `place.m` command to design a state feedback control gain K, that renders $x = 0$ UGES and places the closed-loop control feedback poles at $p_{ctrl} = \{-1, -3\}$.
   (c) Show that the *separation principle* holds.

4. For the following system, check if the system is *observable* and/or *detectable*.
   Dynamics:
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
   Outputs:
   (a) $y = x_1$
   (b) $y = x_2$
   (c) $y = x_3$
   (d) $y = x_1 + x_2$
   (e) $y_1 = x_1$, $y_2 = x_3$

5. For the above system with $y = x_1 + x_2$, design a Luenberger observer, and:
   (a) Show how to use the `place.m` command to design the injection gain L of the observer, that places the closed-loop observer poles at $p_{obs} = \{-10, -15, -12\}$.

(b) Show how to use the `place.m` command to design a state feedback control gain K, that renders $x = 0$ UGES and places the control feedback poles so that the closed-loop characteristic polynomial becomes $(s+2)\left(s^2 + 2\xi\omega s + \omega^2\right)$ with damping $\xi = 0.7$ and natural frequency $\omega = 2$.

(c) Show that the *separation principle* holds.

6. For the time-varying system:

$$\begin{aligned} \dot{x} &= A(t)x + u \\ y &= C(t)x \end{aligned}$$

with

$$C(t) := \begin{bmatrix} \cos t & \sin 2t \end{bmatrix}, \qquad A(t) := w(t)C(t)^\top C(t),$$

show that the pair $[A(t), C(t)]$ is UCO.

7. Consider a vessel that has a GPS antenna and receiver. Considering only the horizontal positions, we assume we know the lever arm $l_1 \in \mathbb{R}^2$ from the vessel origin (VO) to GPS antenna 1. Then the vessel owner decides to buy an additional GPS for redundancy reasons, and installs the new antenna at a new location. But we do not know exactly the lever arm $l_2 \in \mathbb{R}^2$ for this new GPS 2 antenna. We assume we have a good measurement of the heading $\psi(t)$ of the vessel from the gyrocompass.

(a) Show that the two position measurements from the two GPS, given in terms of the VO position $p(t)$ in NED and the respective lever arms l_1 and l_2 in the body-frame, can be expressed by:

$$\begin{aligned} p_1(t) &= p(t) + R(\psi(t))l_1 \\ p_2(t) &= p(t) + R(\psi(t))l_2, \end{aligned}$$

where $R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$.

(b) Show that $R \in SO(2)$, that is, R is a 2×2 rotation matrix.

(c) Let the state vector be $x := l_2 \in \mathbb{R}^2$ and output $y(t) := p_2(t) - p_1(t) + R(\psi(t))l_1$, and set up the resulting time-varying linear state-space model.

(d) Show that this system is UCO.

(e) Propose an observer that based on real-time measurements of $p_1(t)$, $p_2(t)$, and $\psi(t)$ online estimates the lever arm l_2.

8. Suppose in MC-Lab we will do a towing test with C/S Enterprise I in pure surge direction, to identify the drag in surge. To this end we have connected the model ship to the towing carriage with a tension sensor at the towing line. We are then able to tow the model ship at various speeds and measure the reulting forces.
The drag force is given by the model

$$\tau_u = X_{|u|u}\,|u|\,u + X_u u$$

where u is the surge velocity, $X_{|u|u}$ and X_u are the nonlinear and linear drag coefficients, and τ_u is the measured towing force. We now consider an observer design to estimate the drag parameters, and we parameterize the system as

$$\begin{aligned} \dot{x} &= 0, & x &= \begin{bmatrix} X_{|u|u} & X_u \end{bmatrix}^\top \\ y &= C(t)x, & y &= \tau_u, \qquad C(t) = \begin{bmatrix} |u(t)|\,u(t) & u(t) \end{bmatrix}. \end{aligned}$$

The surge speed $u(t)$ is given by the measured towing speed of the carriage. The above system is then a time-varying linear system.

(a) Suppose in the interval $[t_0, t_1]$ we tow the model at a constant speed $u(t) = u_0$. Show that the LTV observability Gramian matrix (see Definition UCO in lecture)

$$Q_{[t_0,t_1]} = \int_{t_0}^{t_1} \Phi(\tau, t)^\top C(\tau)^\top C(\tau)\Phi(\tau, t)d\tau$$

is rank deficient in this case.

(b) Suppose in the interval $[t_1, t_2]$ we stepped up the speed to $u(t) = u_1 > u_0$. Calculate again the observability Gramian matrix $Q_{[t_0,t_2]}$ over to total interval $[t_0, t_2]$, by splitting the integral between the two subintervals $[t_0, t_1]$ and $[t_1, t_2]$. Is the Gramian $Q_{[t_0,t_2]}$ still rank deficient?

(c) If it still is rank deficient, repeat again for a third speed $u(t) = u_2 > u_1$ for the period $[t_2, t_3]$. Eventually you will find that the rank is built up over time with sufficient variation in speed u.

(d) Propose an observer that can be implemented to online estimate the drag coefficients while running the experiment.

1.2 Nonlinear observer designs

1.2.1 Slow-speed surface vessel

Consider the low-speed vessel model

$$\begin{aligned}
\dot{\eta} &= R(\psi)\nu \\
\dot{b} &= 0 \\
M\dot{\nu} + D\nu &= \tau + b
\end{aligned}$$

where $\eta = col(x, y, \psi)$, $\nu = col(u, v, r)$, $D = D^\top > 0$ is the damping matrix, $M = M^\top > 0$ is the mass matrix, $b \in \mathbb{R}^3$ is a bias load vector, τ is the thruster loads, and

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with properties

$$\begin{aligned}
R(\psi)^\top R(\psi) &= R(\psi)R(\psi)^\top = I \\
\det(R(\psi)) &= 1 \\
R(\psi)^{-1} &= R(\psi)^\top \\
\dot{R} &= R(\psi)S(r), \qquad S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad r = \dot{\psi}.
\end{aligned}$$

Let $\eta_b := R^\top \eta$, $x := col(\eta_b, b, \nu)$, and $u := \tau$, and assume we have the measurements $y_1 = \eta_b$ and $y_2 = r$.

1. Assume D is a constant matrix.

   (a) Show that the slow speed dynamics can be written

$$\begin{aligned}
\dot{x} &= Ax + \rho(u, y_1, y_2) \\
y_1 &= C_1 x \\
y_2 &= C_2 x.
\end{aligned}$$

   (b) Show that the pair (A, C_1) is observable.

(c) Propose a nonlinear observer with state estimate $\hat{x}$ that estimates the state vector x. Show that the estimation error $\tilde{x} := x - \hat{x}$ is rendered UGES.

2. Assume now that $D = D(\nu)$ is indeed a nonlinear damping matrix, and let $d(\nu) := D(\nu)\nu$ be the corresponding monotonically nondecreasing damping load as function of ν.

   (a) Show that the system then can be written on the form

$$\dot{x} = Ax + G\gamma(Hx) + \rho(u, y_1, y_2), \qquad y_1 = C_1 x$$

   (b) Propose a new nonlinear observer for this plant, and give design conditions on the injection gains for the observer error dynamics to be GES.

1.2.2 Example in lecture

In the lecture we looked at the example system

$$\begin{aligned} \dot{x}_1 &= x_2 - k\sin x_1 \\ \dot{x}_2 &= -k\cos x_2 + u \\ y &= x_1, \end{aligned}$$

and we showed that for $k = 0.1$ we could do the "Observer with global Lipschitz condition" design to render the observer error $\tilde{x} = x - \hat{x} = 0$ GES.

Redo this example and see if you can design the injection gain L that renders $\tilde{x} = 0$ GES for the cases $k = 0.175$ and $k = 0.25$.

1.2.3 DP observer with bias

Work through the proof of the theorem on the "DP observer with bias" at the end of the lecture presentation; see (Værnø and Skjetne, 2017).

References

Værnø, S. A. and Skjetne, R. (2017). Observer for simplified dp model: Design and proof. Lecture note, Norwegian Univ. Sci. & Tech., Trondheim, Norway.