Marine Control Systems II

Lecture 10: Adaptive control

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Goals of lecture

- To understand the difference between *Direct adaptive control* and *Indirect adaptive control*.
- To be able to explain the concept of *Persistency of excitation* (PE).
- To understand the equivalence between UCO and PE
  - Show that UCO and PE implies UGES.
- Estimation by adaptive control.

Literature

Adaptive control:

- Lecture presentation.
- "Work note: Stability lemma"
- "Extract from Stability of Adaptive Systems (1977)" – for deeper insight (technical)

Scalar plant

We consider the scalar linear plant:

$$\begin{aligned} \dot{x} &= ax + bu + c \\ &= bu + \varphi(x)^\top \theta \\ \theta^\top &= \begin{bmatrix} a & c \end{bmatrix}, \qquad \varphi(x)^\top = \begin{bmatrix} x & 1 \end{bmatrix} \end{aligned}$$

where a and c are unknown constant parameters, and b is known.
Control objective: To regulate $x(t) \to 0$.

Indirect adaptive control

- In INDIRECT adaptive control, the estimated (adapted) parameters are used indirectly to calculate the control parameters.
- Typically this is designed by adaptively estimating model parameters, which are thereby used in a model-based control design.

An example follows for our scalar linear plant...

Direct adaptive control

- In DIRECT adaptive control, the feedback control gains are directly estimated (adapted).

An example follows for our scalar linear plant...

2.3.15

Indirect adaptive control scalar plant

$$\dot{x} = bu + ax + c \qquad \text{Control } x \to 0$$

Let $\hat{a}, \hat{c}$ be estimates of a, c, and $\tilde{a} = a - \hat{a}$, $\tilde{c} = c - \hat{c}$

Let a CLF be

$$V = \frac{1}{2}x^2 + \frac{1}{2\gamma_1}\tilde{a}^2 + \frac{1}{2\gamma_2}\tilde{c}^2$$

$$\dot{V} = bux + \underset{\uparrow \atop \tilde{a}+\hat{a}}{a}x^2 + \underset{\uparrow \atop \tilde{c}+\hat{c}}{c}x - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{c}\dot{\hat{c}}$$

$$= (bu + \hat{a}x + \hat{c})x + \tilde{a}x^2 + \tilde{c}x - \frac{1}{\gamma_1}\tilde{a}\dot{\hat{a}} - \frac{1}{\gamma_2}\tilde{c}\dot{\hat{c}}$$

Feedback control law:

$$\boxed{u = \frac{1}{b}\left[-k_p x - \hat{a}x - \hat{c}\right]}$$

$$\Rightarrow \dot{V} = -k_p x^2 - \tilde{a}\left[\frac{1}{\gamma_1}\dot{\hat{a}} - x^2\right] - \tilde{c}\left[\frac{1}{\gamma_2}\dot{\hat{c}} - x\right]$$

Adaptive update laws:

$$\boxed{\dot{\hat{a}} = \gamma_1 x^2 \qquad \dot{\hat{c}} = \gamma_2 x}$$

$$\dot{V} = -k_p x^2 \le 0 \quad \Leftrightarrow \quad \text{UGS or UGAS?}$$

Closed loop (error) system:

$$\left.\begin{aligned} \dot{x} &= -k_p x + \tilde{a}x + \tilde{c} \\ \dot{\tilde{a}} &= -\gamma_1 x^2 \\ \dot{\tilde{c}} &= -\gamma_2 x \end{aligned}\right\}$$

Under what conditions can we guarantee that $\tilde{a} \to 0$ and $\tilde{c} \to 0$?

PE!

Indirect adaptive because we adapt the model parameters a, c that the control gains depend upon.

2.3.15

Direct adaptive control scalar plant

$\dot{x} = ax + bu + c$ Control $x \to 0$

Since c is a constant matched bias, we know integral action is needed and proposes a PI control law.

An ideal control law is then

$$\dot{\xi} = x$$
$$u = -k_p x - k_i \xi$$ (where sign of k_p, k_i depends on $\mathrm{sgn}(b)$)

Ideal closed-loop:

$\Rightarrow$
$$\dot{\xi} = x$$
$$\dot{x} = -(bk_p - a)x - bk_i\xi + c$$

Let $k = bk_p - a > 0$ and $l = bk_i > 0$ be the desired gains, and

$$A = \begin{bmatrix} 0 & 1 \\ -k & -l \end{bmatrix}$$ be Hurwitz

Let $\tilde{\xi} = \xi - \frac{c}{l} = \xi - \frac{c}{bk_i}$ $\Rightarrow \dot{x} = -kx - l\tilde{\xi}$

$$z = \begin{bmatrix} \tilde{\xi} \\ x \end{bmatrix} \Rightarrow \dot{z} = Az$$

Problem: a is unknown, implying we cannot calculate k_p.

Thus, Let $\hat{k}_p$ be an estimate of k_p

and $\tilde{k}_p = k_p - \hat{k}_p$

... Direct adaptive control

Also, select k_i s.t. $bk_i = l > 0$

We now apply the adaptive "PI" control law

$$\boxed{u = -\hat{k}_p x - k_i z}$$

Note:
$$\tilde{z} = z - \frac{c}{l}$$
$l = bk_i > 0$ known
$k = bk_p - a > 0$ unknown

$$\Rightarrow \quad \dot{\tilde{z}} = x$$

$$\begin{aligned}\dot{x} &= ax - b\hat{k}_p x - bk_i z + c \quad + bk_p x - bk_p x \\ &= -(bk_p - a)x - l\tilde{z} + b\tilde{k}_p x \\ &= -kx - l\tilde{z} + b\tilde{k}_p x\end{aligned}$$

CLF: $V = \frac{l}{2}\tilde{z}^2 + \frac{1}{2}x^2 + \frac{1}{2\gamma}\tilde{k}_p^2$

$$\begin{aligned}\Rightarrow \dot{V} &= \cancel{l\tilde{z}x} - kx^2 - \cancel{l\tilde{z}x} + b\tilde{k}_p x^2 - \frac{1}{\gamma}\tilde{k}_p \dot{\hat{k}}_p \\ &= -kx^2 + \left[bx^2 - \frac{1}{\gamma}\dot{\hat{k}}_p\right]\tilde{k}_p\end{aligned}$$

Adaptive update law:

$$\boxed{\dot{\hat{k}}_p = \gamma b x^2}$$

$\Rightarrow \dot{V} = -kx^2 \leq 0$

UGS

LaSalle-Yoshizawa:

$x(t) \to 0$

Closed-loop:

$$\boxed{\begin{aligned}\dot{\tilde{k}}_p &= -\gamma b x^2 \\ \dot{\tilde{z}} &= x \\ \dot{x} &= -kx - l\tilde{z} + b\tilde{k}_p x\end{aligned}}$$

Direct adaptive because we directly adapt the feedback control gain $\hat{k}_p$.

Definition UCO

Definition

(Anderson et al., 1986) For a dynamic linear system

$$\begin{aligned} \dot{x} &= F(t)x \\ y &= N(t)^\top x, \qquad t \geq 0 \end{aligned}$$

the pair $[F(t), N(t)]$ is said to be Uniformly Completely Observable (UCO) if there exist positive constants α, β, and T such that

$$\alpha I \leq \int_t^{t+T} \Phi(\tau, t)^\top N(\tau) N(\tau)^\top \Phi(\tau, t) d\tau \leq \beta I$$

holds for all $t \geq 0$, where $\Phi(t, t_0)$ is the transition matrix for the above system, that is, $x(t) = \Phi(t, t_0)x(t_0)$.

Definition PE

Definition

(Anderson et al., 1986) Consider a matrix $Q(t) \in \mathbb{R}^{m \times n}$ for each $t \geq 0$. If there exist positive constants α, β, and T such that

$$\alpha I \leq \int_t^{t+T} Q(\tau) Q(\tau)^\top d\tau \leq \beta I$$

holds for all $t \geq 0$, then $Q(t)$ is said to be Persistently Exciting (PE).

UCO implies UGES

Lemma

Consider the dynamic linear system

$$\dot{x} = F(t)x$$

where the matrix function $F(\,\cdot\,)$ *is bounded and locally integrable. Suppose there exists a constant matrix* $P = P^\top > 0$ *such that*

$$PF(t) + F(t)^\top P \leq -N(t)N(t)^\top,$$

for some matrix function $N(\,\cdot\,)$ *and all* $t \geq 0$. *Then* $x = 0$ *of the above system is UGS. Moreover, if the pair* $[F(t), N(t)]$ *is UCO, then* $x = 0$ *of the above system is UGES.*

See (Anderson et al., 1986) for proof.

UCO is invariant under output feedback

Lemma

The pair $[F(t), N(t)]$ *is UCO if and only if the pair* $\left[F(t) - K(t)N(t)^\top, N(t)\right]$*, with* $K(\,\cdot\,)$ *bounded and locally integrable, is UCO.*

See (Anderson et al., 1986) for proof.

PE implies UGES

Theorem

Consider the dynamic linear system

$$\dot{x} = -LQ(t)Q(t)^\top x,$$

where $L = L^\top > 0$*. Suppose the matrix function* $Q(\,\cdot\,)$ *satisfies the following:*

- $t \mapsto Q(t)$ *is piecewise continuous,*
- $\exists \mu > 0$ *such that* $\|Q\|_\infty \leq \mu$*,*
- $Q(t)$ *is PE according to the above definition.*

Then $x = 0$ *of the above system is UGES.*

See (Anderson et al., 1986) for proof.

Parameter estimation

We are now ready to apply these results to estimation theory.
Suppose we have a measured signal $y(t) \in \mathbb{R}^n$ given by

$$y(t) = \Phi(t)^\top c$$

where $c \in \mathbb{R}^p$ is a constant unknown coefficient vector, and $\Phi(t) \in \mathbb{R}^{p \times n}$ for each $t \geq 0$.
The objective is to estimate c, and we will try our adaptive techniques.

Let $\hat{c}$ be an estimate of c and $\tilde{c} := \hat{c} - c$. Similarly, we define

$$\begin{aligned}\hat{y}(t) =& \Phi(t)^\top \hat{c} \\ \tilde{y}(t) :=& \hat{y}(t) - y(t) = \Phi(t)^\top \tilde{c}\end{aligned}$$

To design a continuous-time estimator, let an instantaneous cost function to minimize be

$$J(t, \hat{c}) := \frac{1}{2}\tilde{y}(t)^\top W \tilde{y}(t) = \frac{1}{2}\left(\hat{c} - c\right)^\top \Phi(t) W \Phi(t)^\top \left(\hat{c} - c\right),$$

where $W = W^\top > 0$ is a weight matrix putting cost on the individual measurements in y.

...Parameter estimation

Motivated by a gradient descent (or steepest descent), we propose an estimation algorithm that tries to minimize $J(t, \hat{c})$ along its fastest descent route. This happens to be in the negative direction of the gradient $J^{\hat{c}}(t, \hat{c})$.

The gradient is given by

$$J^{\hat{c}}(t, \hat{c}) = (\hat{c} - c)^\top \Phi(t) W \Phi(t)^\top = \tilde{y}(t)^\top W \Phi(t)^\top.$$

Then, letting $L = L^\top > 0$, the adaptive update law

$$\dot{\hat{c}} = -L J^{\hat{c}}(t, \hat{c})^\top$$

gives the estimation error system

$$\begin{aligned}\dot{\tilde{c}} = \dot{\hat{c}} &= -L J^{\hat{c}}(t, \hat{c})^\top \\ &= -L \Phi(t) W \tilde{y}(t) \\ &= -L \Phi(t) W \Phi(t)^\top \tilde{c}.\end{aligned}$$

We have then the result...

...Parameter estimation

Theorem

Suppose $\Phi(\cdot)$ is a bounded and piecewise continuous matrix function that satisfies the above PE condition. Then $\tilde{c} = 0$ of

$$\dot{\tilde{c}} = -L \Phi(t) W \Phi(t)^\top \tilde{c},$$

where $L = L^\top > 0$ and $W = W^\top > 0$, is UGES.

Proof: Follows from the earlier theorem.

Example 1

Consider the Nomoto steering model for a ship

$$T\dot{r} + r = K\delta$$

where $r = \dot{\psi}$ is the rotation rate of the ship. Suppose we can measure $\delta(t)$, $r(t)$, and $\dot{r}(t)$. Then we can represent this model as

$$y(t) = r(t) = \begin{bmatrix} -\dot{r}(t) & \delta(t) \end{bmatrix} \begin{bmatrix} T \\ K \end{bmatrix} = \Phi(t)^\top c.$$

A continous-time estimator is then

$$\begin{aligned} \dot{\hat{c}} &= -L\Phi(t)\left(\Phi(t)^\top \hat{c} - y(t)\right) \\ &= -L\Phi(t)\Phi(t)^\top \hat{c} + L\Phi(t)y(t) \\ &= -L\begin{bmatrix} \dot{r}(t)^2 & -\dot{r}(t)\delta(t) \\ -\dot{r}(t)\delta(t) & \delta(t)^2 \end{bmatrix} \hat{c} + L\begin{bmatrix} -\dot{r}(t) \\ \delta(t) \end{bmatrix} r(t). \end{aligned}$$

...Example 1

Obviously, $\dot{r}(t)$ and $\delta(t)$ are both continous and bounded. Hence, it follows that if there exists positive constants α, β such that

$$\alpha I \leq \int_t^{t+T} \begin{bmatrix} \dot{r}(\tau)^2 & -\dot{r}(\tau)\delta(\tau) \\ -\dot{r}(\tau)\delta(\tau) & \delta(\tau)^2 \end{bmatrix} d\tau \leq \beta I$$

holds for some $T > 0$, then we are ensured that the estimation error $\tilde{c}(t) \to 0$.

See simulation study.

Adaptive DP control

Consider the Dynamic Positioning system

$$\dot{\eta} = R(\psi)\nu$$
$$M\dot{\nu} = -D\nu + R(\psi)^\top b + \tau$$

$\eta = \mathrm{col}(x, y, \psi)$ the position/heading;
$\nu = \mathrm{col}(u, v, r)$ the velocities;
b is assumed a constant unknown bias load;
Damping matrix $D > 0$; mass matrix $M = M^\top > 0$; τ the control force input.
Notice in particular, for

$$R = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

that the following properties hold:

$$R(\psi)^\top R(\psi) = R(\psi)R(\psi)^\top = I, \qquad \det R(\psi) = 1$$
$$\dot{R} = R(\psi)S(r), \qquad \dot{R}^\top = -S(r)R(\psi)^\top, \qquad S(r) = -S(r)^\top.$$

Adaptive DP control

Control design model

$$\dot{\eta} = R(\psi)\nu$$
$$M\dot{\nu} = -D\nu + R(\psi)^\top b + \tau$$

We let

$$z_1 := R(\psi)^\top (\eta - \eta_d(t))$$
$$z_2 := \nu - \alpha_1,$$

and $\hat{b}$ an adaptive estimate of b with the error

$$\tilde{b} := b - \hat{b}.$$

Then we do an adaptive backstepping DP control design on the blackboard...

Preparations for next lecture

Adaptive backstepping:

- Lecture presentation.
- Skjetne (2005). Ch. 4.2

Bibliography

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