TMR4243 - Marine Control Systems II

Homework assignment 3

1 Stabilization

Consider the scalar system

$$\dot{x} = u + 2\,|x|\,x.$$

1. Let $V(x) = \frac{1}{2}x^2$ be a CLF for the system, and design a corresponding control for u that renders $x = 0$ UGES.
2. For the above quadratic CLF, derive the control law based on Sontag's formula, and discuss the achieved stability.
3. Show that the control law

   $$u = -kx, \qquad k > 0$$

   achieves:

   - Local stabilization.
   - Regional stabilization.
   - Semiglobal stabilization – but not global stabilization.

2 Practical stabilization

Consider the scalar system

$$\dot{x} = u + d(t), \qquad \|d\| \leq 1.$$

1. Show that the control law

   $$u = -kx, \qquad k > 0$$

   achieves global practical stabilization.

3 Diesel generator control

The (simplified) mechanical dynamics of a diesel-generator is given by

$$\begin{aligned}\dot{\delta} &= \omega_B\,(\omega - \omega_0)\\ 2H\dot{\omega} &= t_m - D\omega - t_e(t)\end{aligned}$$

where ω is the normalized (per-unit) electric frequency, δ is the load angle of the generator, t_m is the per-unit control torque from the cylinder combustion dynamics (our control input), t_e is the per-unit electric load torque, $H > 0$ is an inertia constant, $D > 0$ is a damping gain, $\omega_B = 120\pi$ [rad/s] is the base frequency constant, and ω_0 is the per-unit electric frequency of the connected electric power bus.

Suppose we want to control δ to δ_{ref} and ω to ω_0 and define the error states $e_\delta := \delta - \delta_{ref}$ and $e_\omega := \omega - \omega_0$. Assume that $t_e(t) = t_L + w(t)$ where t_L is a constant electric load torque and w a bounded disturbance torque.

1. Assume $w(t) \equiv 0$ and t_L is known. Write the system as a linear state-space vectorial system with $x = \mathrm{col}(e_\delta, e_\omega)$ and $u = t_m$.
2. State the control objective.
3. Let $P \in \mathbb{R}^{2\times 2}$ with $P = P^\top > 0$. For an appropriate choice of P, let $V(x) = x^\top P x$ be a CLF for the system and propose a corresponding control law for t_m that renders $x = 0$ UGES.
4. Let the bound for $\|w\| \leq w_0$. In presence of the disturbance w, show that your control law renders the closed-loop system Practically-UGES with respect to $x = 0$.

References