NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Marine Technology

Examination paper for:
TMR4243 MARINE CONTROL SYSTEMS II

Academic contact during examination: Roger Skjetne and Hans-Martin Heyn

Phone: 92813672 (Roger) and 45763918 (Hans-Martin)

Examination date: June 1, 2016

Examination time (from-to): 09:00 – 13:00

Permitted examination support material: A – All printed and handwritten material allowed. All approved calculators allowed.

Other information:

The solutions to the 3 problems are given a maximum of 100 points. The exam counts 60% on the final grade.

Work fast; your answers should be short, clear, and concise. All statements should be explained; all mathematical answers should be derived.

Make qualified assumptions if:

- you cannot find an intermediate answer that is needed in further calculations, or
- there are missing (or obvious wrong) information in the problem text.

Language: English

Number of pages: 7

Number of pages enclosed:

Checked by:

Date Signature

TMR4243 - Marine Control Systems II

Exam

Spring 2016

Notation: Throughout this exam $|x|$ means the vector 2-norm, i.e. $|x| = \sqrt{x^\top x}$. For a scalar x, this corresponds to the absolute value.

1 Properties of nonlinear systems (30 pts)

1. Consider the nonlinear ordinary differential equation (ODE):

$$\dot{x} = h(x), \qquad x_0 = x(0),$$

where $x \in \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^n$.

(a) Suppose for each compact set $\mathcal{C} \in \mathbb{R}^n$ there exists a $k_\mathcal{C} > 0$ such that for any two vectors $u, v \in \mathcal{C}$ then $h(\cdot)$ satisfies that

$$|h(u) - h(v)| \leq k_\mathcal{C} |u - v|$$

What do you call this property for the system and what can you say about the solutions if this property is satisfied?

(b) Suppose there exists a constant $k > 0$ such that $h(\cdot)$ satisfies for all $u, v \in \mathbb{R}^n$

$$|h(u) - h(v)| \leq k |u - v|$$

What do you call this property for the system and what can you say about the solutions if this property is satisfied?

(c) Suppose $h(x) = Hx$ where $H \in \mathbb{R}^{n \times n}$ is a constant matrix.

- Show which of the above conditions the system now satisfies, and what this means for the properties of the solutions.
- Suppose $n = 3$ and the eigenvalues of H is $\{-1, 0, 1\}$. What type of equilibrium points does the system have?

2. For the scalar ODE

$$\dot{x} = -x^3$$

we propose the solution for $t \geq 0$ and initial condition x_0:

$$x(t, x_0) = \frac{x_0}{\sqrt{1 + 2x_0^2 t}}$$

   (a) Show that the proposed solution is indeed a solution to the ODE.

   (b) Explain the Lipschitz property of this ODE.

   (c) Discuss the properties of the solutions to this ODE and the stability of $x = 0$ in terms of class $\mathcal{K}$ and $\mathcal{L}$ functions.

   (d) Propose a Lyapunov function candidate for this system, and discuss stability of $x = 0$ in sense of Lyapunov.

3. Consider the scalar time-varying system

$$\begin{aligned} \dot{x}_1 &= -\phi(t)x_1 + x_2^5 \\ \dot{x}_2 &= -x_1^3 - \phi(t)x_2 \end{aligned}$$

for $t \geq t_0 \geq 0$.

   (a) Let $V(x_1, x_2) = x_1^4 + \frac{2}{3}x_2^6$ be a Lyapunov candidate. Differentiate this along the solutions of the system.

   (b) Suppose $\phi(t) := e^{-t}$. Discuss Uniform Global Asymptotic Stability of the equilibrium $x = 0$.

   (c) Suppose $\phi(t) := 10^{-3} + e^{-t}$. Discuss Uniform Global Asymptotic Stability of the equilibrium $x = 0$.

4. Consider the linear time-varying system:

$$\dot{x} = A\left(x - x_d(t)\right) + \dot{x}_d(t)$$

where A is Hurwitz, and $(x_d(t), \dot{x}_d(t))$ are bounded reference signals. Suppose the triple (P, A, Q) satisfies the Lyapunov equation, and let

$$V(t, x) = \frac{1}{2}\left(x - x_d(t)\right)^\top P\left(x - x_d(t)\right)$$

be a Lyapunov function candidate.

   (a) What does it mean that (P, A, Q) satisfies the Lyapunov equation?

   (b) Show how to differentiate $V(t, x)$ and make a stability conclusion from this.

2 Observer design (30 pts)

Consider the pendulum equations with unity mass, length, and friction coefficients $m = l = k = 1$:

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= -10\sin\theta - \omega + u\cos\theta \\ y &= \theta\end{aligned}$$

where θ is the angle from vertical hanging condition, ω is the angular rate, and u is a control torque.

1. Let $x = col(\theta, \omega)$, and assume small angular deviations $\theta \approx 0$. Write down the corresponding linearized model for the pendulum

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

   (a) Show that A is a Hurwitz matrix and that the pair (A, C) is completely observable.

   (b) What is the rank of the infinite time observability Gramian Q_∞ for this system? Is there an easy way to calculate the matrix Q_∞?

   (c) Letting $\hat{x}$ be the estimate of x, design a *Luenberger observer* for the linearized system.

   (d) What stability do you get for $\tilde{x} = x - \hat{x} = 0$ for the linearized error dynamics?

   (e) What stability can you claim for $\tilde{x} = 0$ for the real nonlinear error dynamics?

2. For the linearized pendulum model above, propose a state-feedback control that renders $x = 0$ UGES. How would you select your feedback gains?

3. Show that the *separation principle* holds for the linear closed-loop system with feedback taken from the estimated states by the Luenberger observer.

4. Aiming for a global observer, show now that the nonlinear system can be written on the form

$$\begin{aligned} \dot{x} &= Hx + \phi(x) + \psi(u, y) \\ y &= Cx \end{aligned}$$

where (H, C) is a completely observable pair.

   (a) Show that the nonlinearity $\phi(x)$ is Globally Lipschitz.

   (b) Propose a nonlinear observer, with linear injection term, for the nonlinear system, that takes advantage of the global Lipschitz property of $\phi(x)$.

   (c) Use a Lyapunov argument to find a parameter bound related to the global Lipschitz constant for $\phi(x)$ to ensure UGES of $\tilde{x} = 0$.

3 Control design (40 pts)

Consider the Nomoto steering model of a ship

$$
\begin{aligned}
\dot{\psi} &= r \\
\dot{r} &= -\frac{1}{\tau} r + \frac{\kappa}{\tau} (\delta + b) \\
y &= \psi + \psi_w
\end{aligned}
$$

where ψ is the ship yaw, r is the yaw rate, δ is the rudder angle, and (τ, κ) are model parameters. In the tasks below, the control objective is stabilization of $(\psi, r) = (\psi_{ref}, 0)$, where ψ_{ref} is constant.

1. The disturbances of the model are b, a slowly-varying rudder bias, and ψ_w, an oscillatory motion due to waves. What category of disturbances do we name these?

2. Assume $\psi_w = b = 0$:

   (a) Use *LgV backstepping* to design a state feedback control law for δ that solves the regulation control objective.

   (b) Suppose b is an unknown constant bias, let $\hat{b}$ be an estimate of b, and define the adaptation error $\tilde{b} = b - \hat{b}$. Redo Step 2 of the LgV backstepping design, by designing an adaptive update law for $\dot{\hat{b}}$ that renders $(z_1, z_2, \tilde{b}) = 0$ UGS and ensures the convergence $(\psi(t), r(t)) \to (\psi_{ref}, 0)$.

3. Provide a vectorial state-space model (A, B, C, D) for the Nomoto plant with $x = col(\psi, r)$:

   (a) Show that the pair (A, B) is controllable.

   (b) Assume $\psi_w = b = 0$. Let a linear control be

   $$\delta = -Kx + Ly_{ref}$$

   and give conditions on the feedback gain K and feedforward gain L such that in steady-state we achive stabilization of $(\psi - \psi_{ref}, r) = 0$.

   (c) Assume $\psi_w = b = 0$. Let a PID control law (with reference feedforward) be

   $$\begin{aligned} \dot{\xi} &= y - y_{ref} \\ \delta &= -Kx - K_i\xi + Ly_{ref} \end{aligned}$$

   Write down the closed-loop system, and state design conditions on K and K_i such that the closed-loop is stable.

   (d) Suppose $\psi_w = 0$, but b is constant nonzero. What is the steady-state solution for (ξ, ψ, r)? Is the regulation control objective met?

4. To compensate the wave motion ψ_w we can model this as a damped harmonic oscillator, and the slowly-varying bias b can be modeld as a Markov process:

   (a) Using the *internal model principle*, write up a model for the total system on state-space form.

   (b) Propose an observer for estimating the system state and the disturbances.

   (c) Design a linear control law, based on the estimated states, that solves the regulation objective while rejecting the influence of b and ψ_w.

   (d) Write down the overall closed-loop state-space system on vectorial form, and draw the block diagram of the system as a cascade of two subsystems. Discuss stability of the subsystems and if the separation principle is satisfied.