Marine Control Systems II

Lecture 9: Disturbance rejection and integral action

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Goals of lecture

- Understand that any practical system is exposed to different disturbances: *Input*, *output*, and *process disturbances*.
- Be able to design integral action for a linear system and show how this compensates constant disturbances.
- Learn to design a nonlinear integral action based on a CLF, to compensate a constant disturbance matched to the control.
- Be able to design PID control based on the nonlinear action method.
- Understand the *internal model principle* for more general time-varying disturbances.
- Be able to reject disturbances by estimation techniques, with DP as an example.

Literature

Disturbance rejection and integral action:

- Khalil (2015).
  - Chapters: 13.1-13.4
- Lecture presentation.

Disturbances

Consider the nominal system

$$\begin{aligned} \dot{x} &= f(x, u), \qquad x \in \mathbb{R}^n, u \in \mathbb{R}^q \\ y &= h(x), \qquad y \in \mathbb{R}^m \end{aligned}$$

A disturbance $w \in \mathbb{R}^p$ can enter several places in the plant:

- Output disturbance:
$$y = h(x) + w$$
- Input disturbance:
$$\dot{x} = f(x, u + w)$$
- Process disturbance:
$$\dot{x} = f(x, u, w)$$

State feedback for linear plant

Consider the nominal system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and regulation $y \to y_{ref}$. Assume (A, B) is a controllable pair, and let a state feedback control law be

$$u = -Kx + Ly_{ref}$$

where K renders $(A - BK)$ Hurwitz. Then

$$\begin{aligned} \dot{x} &= (A - BK)\,x + BLy_{ref} \\ x_{ss} &= -\left(A - BK\right)^{-1} BLy_{ref} \\ y_{ss} &= -C\left(A - BK\right)^{-1} BLy_{ref} \end{aligned}$$

where the feedforward gain L must be chosen such that

$$-C\left(A - BK\right)^{-1} BL = I$$

to obtain correct steady-state value.

...Augmented integral action

We now introduce the integral state

$$\xi(t) = \int_0^t \left(y(\tau) - y_{ref}\right) d\tau$$

and augmented integral action

$$u = -Kx - K_i\xi + Ly_{ref}$$

The closed-loop system now becomes

$$\begin{aligned} \dot{\xi} &= y - y_{ref} = Cx - y_{ref} \\ \dot{x} &= (A - BK)\,x - BK_i\xi + BLy_{ref} \end{aligned}$$

for which we assume (K, K_i) is designed such that

$$\begin{bmatrix} 0 & C \\ -BK_i & A - BK \end{bmatrix}$$

is Hurwitz.

...Steady-state

Closed-loop system:

$$\begin{aligned}\dot{\xi} &= y - y_{ref} = Cx - y_{ref} \\ \dot{x} &= (A - BK)\, x - BK_i \xi + BL y_{ref}\end{aligned}$$

In steady-state, with $-C\,(A - BK)^{-1}\,BL = I$ again, we get

$$\begin{aligned}y_{ss} &= Cx_{ss} = y_{ref} \\ (A - BK)\, x_{ss} &= BK_i \xi_{ss} - BL y_{ref} \\ y_{ss} &= C\,(A - BK)^{-1}\, BK_i \xi_{ss} + y_{ref} = y_{ref}\end{aligned}$$

Assuming $C\,(A - BK)^{-1}\, BK_i$ is full rank, this implies

$$\xi_{ss} = 0$$

...Constant output disturbance

Consider now a **constant output disturbance**

$$y = Cx + w$$

Closed-loop:

$$\begin{aligned}\dot{\xi} &= y - y_{ref} = Cx + w - y_{ref} \\ \dot{x} &= (A - BK)\, x - BK_i \xi + BL y_{ref}\end{aligned}$$

Steady-state equilibrium:

$$\begin{aligned}y_{ss} &= y_{ref} \\ y_{ss} &= Cx_{ss} + w = C\,(A - BK)^{-1}\, BK_i \xi_{ss} + y_{ref} + w\end{aligned}$$

Assuming $C\,(A - BK)^{-1}\, BK_i$ is full rank, this implies

$$\xi_{ss} = -\left[C\,(A - BK)^{-1}\, BK_i\right]^{\dagger} w$$

...Constant input disturbance

Consider now a **constant input disturbance**

$$\begin{aligned} \dot{x} &= Ax + B\left(u + w\right) \\ y &= Cx \end{aligned}$$

Closed-loop:

$$\begin{aligned} \dot{\xi} &= y - y_{ref} = Cx - y_{ref} \\ \dot{x} &= \left(A - BK\right)x - B\left(K_i \xi - w\right) + BLy_{ref} \end{aligned}$$

Steady-state equilibrium:

$$\begin{aligned} y_{ss} &= y_{ref} \\ y_{ss} &= Cx_{ss} = C\left(A - BK\right)^{-1} B\left[K_i \xi_{ss} - w\right] + y_{ref} \end{aligned}$$

Assuming $C\left(A - BK\right)^{-1} B$ is full rank, this implies

$$v_{ss} = K_i \xi_{ss} - w$$

is either the zero vector or converges to the nullspace of $C\left(A - BK\right)^{-1} B$. Is this always possible to find?

Example 1: Linear plant

Consider the plant with $A = 0$ and $B = I$ and an input disturbance:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ y = x_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} x = Cx \end{aligned}$$

Select the control law

$$\begin{aligned} \dot{\xi} &= y - y_{ref} = Cx - y_{ref} \\ u &= -Kx - P\xi + Ly_{ref} = -\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} x - \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \xi + Ly_{ref} \end{aligned}$$

To ensure unity regulation gain we get

$$CK^{-1}L = 1 \quad \text{or} \quad L = KC^{\top} = \begin{bmatrix} k_1 \\ 0 \end{bmatrix}$$

...Example 1: Linear plant

Closed-loop:

$$\begin{aligned}\dot{\xi} &= y - y_{ref} = Cx - y_{ref} \\ \dot{x} &= -Kx - (P\xi - w) + KC^\top y_{ref}\end{aligned}$$

Steady-state:

$$\begin{aligned}y_{ss} = Cx_{ss} &= -CK^{-1}(P\xi - w) + CK^{-1}KC^\top y_{ref} \\ &= -\begin{bmatrix} \frac{1}{k_1} & 0 \end{bmatrix}\begin{bmatrix} p_1\xi - w_1 \\ p_2\xi - w_2 \end{bmatrix} + y_{ref} = y_{ref}\end{aligned}$$

implying $CK^{-1}(P\xi - w) = 0$.

Let $v(t) = P\xi(t) - w$. Then $v(t)$ will converge to a vector in the nullspace of CK^{-1}, that is, $v_{ss} = K\begin{bmatrix} 0 & c \end{bmatrix}^\top = \begin{bmatrix} 0 & ck_2 \end{bmatrix}$ and $P\xi_{ss} = v_{ss} + w$, that is,

$$\begin{bmatrix} p_1\xi_{ss} \\ p_2\xi_{ss} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 + ck_2 \end{bmatrix}$$

or $\xi_{ss} = \frac{w_1}{p_1}$ and $c = \frac{p_2 w_1}{p_1 k_2} - \frac{w_2}{k_2}$.

Integral action

Consider the system

$$\begin{aligned}\dot{x}_1 &= G_1(x_1)x_2 \\ \dot{x}_2 &= f(x_1, x_2) + G_2(x_1, x_2)u + G_3(x_1, x_2)w,\end{aligned}$$

where $x = (x_1, x_2) \in \mathbb{R}^n \times \mathbb{R}^n$, $u \in \mathbb{R}^n$, $w \in \mathbb{R}^p$ **is a constant disturbance**, and $G_1 : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $G_2 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$, and $G_3 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times p}$ are always nonsingular.
Let the control objective be regulation $x_1 \to x_{ref}$ where x_{ref} is constant.

Letting $e_1 = x_1 - x_{ref}$ and $e_2 = x_2 - 0$, we get

$$\begin{aligned}\dot{e}_1 &= G_1(e_1 + x_{ref})e_2 \\ \dot{e}_2 &= f(e_1 + x_{ref}, e_2) + G_2(e_1 + x_{ref}, e_2)u + G_3(e_1 + x_{ref}, e_2)w,\end{aligned}$$

For the equilibrium condition $e_1 = e_2 = 0$, assume there exists u_{ss} s.t.

$$f(x_{ref}, 0) + G_2(x_{ref}, 0)u_{ss} + G_3(x_{ref}, 0)w = 0.$$

The problem now reduces to finding u that stabilizes $(e_1, e_2) = 0$.

...Integral action

Let a nominal control law, with $w = 0$, be

$$u_0 = G_2(e_1 + x_{ref}, e_2)^{-1}\psi(e_1, e_2).$$

The steady state condition then requires

$$\begin{aligned} 0 &= f(x_{ref}, 0) + \psi(0,0) \\ \psi(0,0) &= \psi_{ss} = -f(x_{ref}, 0) \end{aligned}$$

Assume further there is a global and radially unbounded Lyapunov function $V_0(e_1, e_2)$ such that

$$\begin{aligned} &V_0^{e_1}(e_1, e_2)G_1(e_1 + x_{ref})e_2 \\ &\quad + V_0^{e_2}(e_1, e_2)\left[f(e_1 + x_{ref}, e_2) + \psi(e_1, e_2)\right] \le -\alpha_3(|e|), \end{aligned}$$

where $e := (e_1, e_2)$, and α_3 is positive definite.

This means that the nominal control u_0 renders $e = 0$ UGAS.

...Integral action

We include integral action, and define the integral state

$$\dot{\xi} = \gamma(e) \qquad \text{s.t.} \qquad \xi(t) = \int_0^t \gamma(e(\tau))d\tau,$$

where $\gamma(\,\cdot\,)$ is a nonlinear function to be defined.

Consider the control law

$$u = G_2(e_1 + x_{ref}, e_2)^{-1}\left[\psi(e_1, e_2) - \Gamma(e)K_i\xi\right]$$

where $K_i = K_i^\top > 0$ and Γ is a nonlinear matrix function to be defined.

...Integral action

This gives

$$\begin{aligned}\dot{\xi} &= \gamma(e)\\ \dot{e}_1 &= G_1(e_1 + x_{ref})e_2\\ \dot{e}_2 &= f(e_1 + x_{ref}, e_2) + \psi(e_1, e_2) - \Gamma(e)K_i\xi + G_3(e_1 + x_{ref}, e_2)w\end{aligned}$$

The equilibrium condition now becomes $e_1 = e_2 = 0$, and $\xi = \xi_{ss}$ is such that

$$\Gamma(0)K_i\xi_{ss} = G_3(x_{ref}, 0)w.$$

If $\Gamma(e)$ is invertible $\forall e$, then $\xi_{ss} = K_i^{-1}\Gamma(0)^{-1}G_3(x_{ref}, 0)w$ is unique.

...Integral action

In presence of $w \neq 0$, let a CLF be

$$V = V_0 + \frac{1}{2}\left(\xi - \xi_{ss}\right)^\top P\left(\xi - \xi_{ss}\right).$$

We will use this to define the functions γ, Γ, and $P = P^\top > 0$. Differentiating gives

$$\begin{aligned}\dot{V} &= V_0^{e1}G_1(e_1 + x_{ref})e_2\\ &+ V_0^{e2}\left[f(e_1 + x_{ref}, e_2) + \psi(e_1, e_2) - \Gamma(e)K_i\xi + G_3(e_1 + x_{ref}, e_2)w\right]\\ &+ \left(\xi - \xi_{ss}\right)^\top P\gamma(e)\\ &= V_0^{e1}G_1(e_1 + x_{ref})e_2 + V_0^{e2}\left[f(e_1 + x_{ref}, e_2) + \psi(e_1, e_2)\right]\\ &+ V_0^{e2}\left[-\Gamma(e)K_i\xi + G_3(e_1 + x_{ref}, e_2)w\right]\\ &+ \left(\xi - K_i^{-1}\Gamma(0)^{-1}G_3(x_{ref}, 0)w\right)^\top P\gamma(e).\end{aligned}$$

...Integral action

$$
\begin{aligned}
\dot{V} &\leq -\alpha_3(|e|) \\
&\quad + V_0^{e2}\left[-\Gamma(e)K_i\xi + G_3(e_1 + x_{ref}, e_2)w\right] \\
&\quad + \left(\xi - K_i^{-1}\Gamma(0)^{-1}G_3(x_{ref}, 0)w\right)^\top P\gamma(e) \\
&= -\alpha_3(|e|) - V_0^{e2}\left[\Gamma(e)K_i\xi - G_3(e_1 + x_{ref}, e_2)w\right] \\
&\quad + \gamma(e)^\top P K_i^{-1}\left(K_i\xi - \Gamma(0)^{-1}G_3(x_{ref}, 0)w\right)
\end{aligned}
$$

Choose now

$$
\begin{aligned}
\Gamma(e) = G_3(e_1 + x_{ref}, e_2) \implies \Gamma(0) = G_3(x_{ref}, 0) \\
P = K_i
\end{aligned}
$$

Then

$$
\begin{aligned}
\dot{V} &\leq -\alpha_3(|e|) - V_0^{e2}G_3(e_1 + x_{ref}, e_2)\left[K_i\xi - w\right] + \gamma(e)^\top\left(K_i\xi - w\right) \\
&= -\alpha_3(|e|) - \left[V_0^{e2}G_3(e_1 + x_{ref}, e_2) - \gamma(e)^\top\right]\left(K_i\xi - w\right)
\end{aligned}
$$

...Integral action

The final choise is then

$$
\gamma(e) = G_3(e_1 + x_{ref}, e_2)^\top V_0^{e2}(e_1, e_2)^\top
$$

which renders

$$
\dot{V} \leq -\alpha_3(|e|) \leq 0
$$

proving by LaSalle-Yoshizawa that $(e_1, e_2, \xi - \xi_{ss}) = 0$ is UGS and $\lim_{t\to\infty} e(t) = 0$. Why?

The final control law, with nonlinear integral action, becomes

$$
\begin{aligned}
\dot{\xi} &= G_3(e_1 + x_{ref}, e_2)^\top V_0^{e2}(e_1, e_2)^\top \\
u &= G_2(e_1 + x_{ref}, e_2)^{-1}\left[\psi(e_1, e_2) - G_3(e_1 + x_{ref}, e_2)K_i\xi\right].
\end{aligned}
$$

...Integral action

Letting $\tilde{\xi} := \xi - K_i^{-1} w$, the closed-loop system becomes

$$\begin{aligned}
\dot{\tilde{\xi}} &= G_3(e_1 + x_{ref}, e_2)^\top V_0^{e_2}(e_1, e_2)^\top \\
\dot{e}_1 &= G_1(e_1 + x_{ref})e_2 \\
\dot{e}_2 &= f(e_1 + x_{ref}, e_2) + \psi(e_1, e_2) - G_3(e_1 + x_{ref}, e_2)K_i\tilde{\xi}.
\end{aligned}$$

Can you show by Krasovskii-LaSalle's Invariance principle (or Matrosov's Theorem) that the equilibrium $(e_1, e_2, \tilde{\xi}) = 0$ is indeed UGAS?

Example 2: Linear plant

Consider the linear system

$$\begin{aligned}
\dot{x}_1 &= A_{12}x_2 \\
\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + B_2 u + E_2 w,
\end{aligned}$$

where A_1, B_2, and E_2 are invertible. We recognize the system from our development as $G_1 = A_{12}$, $G_2 = B_2$, $G_3 = E_2$, and $f(x_1, x_2) = A_{21}x_1 + A_{22}x_2$.

Letting $e_1 := x_1 - x_{ref}$ and $e_2 := x_2$ we get with $e = (e_1, e_2)$

$$\dot{e} = Ae + Hx_{ref} + Bu + Ew$$

$$A = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ A_{21} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ E_2 \end{bmatrix}$$

and we assume that the pair (A, B) is controllable.

...Example 2: Linear plant

Let $K_{pd} := \begin{bmatrix} K_p & K_d \end{bmatrix}$ be such that $A - BK_{pd}$ is Hurwitz, and assign the nominal PD+FF control law

$$u_0 = -K_{pd}e - B_2^{-1}A_{21}x_{ref} = -\left(K_pe_1 + K_de_2 + B_2^{-1}A_{21}x_{ref}\right).$$

With $w = 0$ we get

$$\dot{e} = \left(A - BK_{pd}\right)e,$$

and we define the nominal Lyapunov function

$$V_0(e_1, e_2) = e_1^\top P_{11}e_1 + 2e_1^\top P_{12}e_2 + e_2^\top P_{22}e_2 = e^\top Pe$$

where $P = P^\top > 0$ satisfy $PA + A^\top P = -I$. This ensures that $\dot{V}_0 = -|e|^2$ for $w = 0$.

The full PID-type control law then becomes

$$\begin{aligned} \dot{\xi} &= E_2^\top V_0^{e_2}(e_1, e_2)^\top = 2E_2^\top (P_{12}e_1 + P_{22}e_2) \\ u &= -K_pe_1 - K_de_2 - B_2^{-1}E_2K_i\xi - B_2^{-1}A_{21}x_{ref} \end{aligned}$$

Example 3: DP control system

Consider the DP control design model

$$\begin{aligned} \dot{\eta} &= R(\psi)\nu \\ \dot{\nu} &= -M^{-1}D\nu + M^{-1}\tau + M^{-1}R(\psi)^\top b, \end{aligned}$$

We recognize the system from our development as $x_1 = \eta$, $x_2 = \nu$, $u = \tau$, and $w = b$, and the following matrices $G_1(x_1) = R(\psi)$, $G_2(x_1, x_2) = M^{-1}$, $G_3(x_1, x_2) = M^{-1}R(\psi)^\top$, and $f(x_1, x_2) = -M^{-1}D\nu$.

A nonlinear PID+FF control law follows directly from the above development.

Disturbance modeling

Another approach to disturbance rejection is to model the disturbances and then aim for asymptotic output tracking/regulation. When an exogenous disturbance is generated by a known model, then output tracking and disturbance rejection can be achieved by including this model in the control design model. This is referred to as the **“internal model principle”**.

Some classical disturbance models are:

- Constant:

$$\dot{w} = 0 \quad \Rightarrow \quad w(t) = a = \text{constant}$$

- Ramp:

$$\ddot{w} = 0 \quad \Rightarrow \quad w(t) = a + bt = \text{ramp}$$

- Harmonic oscillator:

$$\ddot{w} + \omega_0^2 w = 0 \quad \Rightarrow \quad w(t) = a\sin(\omega_0 t + b) = \text{sinusoid}$$

- Harmonic oscillator with damping:

$$\ddot{w} + 2\lambda\omega_0\dot{w} + \omega_0^2 w = 0 \quad \Rightarrow \quad \text{damped sinusoid}$$

Disturbance estimation

Having an internal model of the disturbance opens up for another strategy to achieve disturbance rejection, by **disturbance estimation**.
Suppose we have a linear system

$$\begin{aligned} \dot{x} &= Ax + B(u + w_1) + Ew_2 \\ y &= C_1 x + w_3 \\ z &= C_2 x \end{aligned}$$

where (w_1, w_2, w_3) are input, process, and output disturbances, respectively, y is the measured output, and z is the output to be controlled.
Suppose we can model these disturbances:

$$\begin{aligned} \dot{\xi}_1 &= F_1\xi_1, & w_1 &= H_1\xi_1 \\ \dot{\xi}_2 &= F_2\xi_2, & w_2 &= H_2\xi_2 \\ \dot{\xi}_3 &= F_3\xi_3, & w_3 &= H_3\xi_3 \end{aligned}$$

...Disturbance estimation

The augmented model now becomes

$$\dot{X} = \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} F_1 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 \\ 0 & 0 & F_3 & 0 \\ BH_1 & EH_2 & 0 & A \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ B \end{bmatrix} u = \mathcal{A}X + \mathcal{B}u$$

$$y = \begin{bmatrix} 0 & 0 & H_3 & C_1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ x \end{bmatrix} = \mathcal{C}X$$

Suppose $(\mathcal{A}, \mathcal{C})$ is an observable pair. Then we can design an observer (Kalman filter, Luenberger, etc.) to reconstruct the states and outputs:

$$\begin{aligned} \hat{X} &= \mathrm{col}(\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \hat{x}) \\ \hat{y} &= C_1 \hat{x} + H_3 \hat{\xi}_3 \\ \hat{z} &= C_2 \hat{x} \end{aligned}$$

Disturbance rejection in linear system

Suppose for our system that w_3 only produces an acceptable high frequency oscillatory motion that we cannot compensate within our actuator's limitations. We don't want this oscillation to enter our feedback loop to reduce wear and tear. However, w_1 and w_2 are low-frequency disturbances that we must compensate – w_1 is constant and w_2 is a zero-mean slowly-varying disturbance.

Given our state estimates $\hat{X} = \mathrm{col}(\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3, \hat{x})$ we can now design a state feedback control law

$$u = -K\hat{x} - H_1\hat{\xi}_1 - PH_2\hat{\xi}_2 + Lz_{ref}$$

This control law rejects directly the influence of w_1 and w_2, while the influence of w_3 is filtered out (it depends only on low-frequency components).

K is designed to make $(A - BK)$ Hurwitz, L to make the tracking gain $-C_2 (A - BK)^{-1} BL = I$, and P to minimize $\|E - BP\|$.

...Disturbance rejection in linear system

The closed-loop system becomes

$$\dot{x} = (A - BK)\,x + BLz_{ref} + BK\tilde{x} + B\tilde{w}_1 + EH_2\tilde{\xi}_2 + (E - BP)\,H_2\hat{\xi}_2$$

If $\tilde{X} = 0$ and $E - BP = 0$ we get our desired closed-loop

$$\begin{aligned} \dot{x} &= (A - BK)\,x + BLz_{ref} \\ z &= C_2 x \end{aligned}$$

Dynamic positioning

The low-frequency DP model is given by

$$\begin{aligned} \dot{\eta} &= R(\psi)\nu \\ M\dot{\nu} + D\nu &= \tau + R(\psi)^{\top} b \\ \eta_y &= \eta + \eta_w \end{aligned}$$

where $\eta = (x, y, \psi)$ is the low-frequency position/heading, $\nu = (u, v, r)$ is the low-frequency velocity vector, b is a slowly-varying bias force, and η_w is high-frequency oscillations due to 1st order wave loads.

DP control designed has successfully applied the internal model principle, by the disturbance models

$$\left.\begin{aligned} \dot{\xi} &= A_w\xi \\ \eta_w &= C_w\xi \end{aligned}\right\} \qquad \text{damped harmonic oscillator}$$

and

$$\left.\dot{b} = -F^{-1}b\right\} \qquad \text{slowly varing Markov process}$$

(considering a Kalman filter for the disturbance estimator, we also add zero-mean white noise to the right-hand sides of all equations).

...Dynamic positioning

A DP observer is implemented to estimate the augmented state vector $X = \mathrm{col}(\xi, b, \eta, \nu)$.

DP control design is implemented to control the low-frequency estimate $\hat{\eta}$ to the desired position/heading η_d while rejecting the influence of b and filtering out the wave-frequency oscillations η_w.

Preparations for next lecture

Adaptive control:

- Lecture presentation.
- "Extract from Stability of Adaptive Systems (1977)" – for deeper insight (technical)