TMR4243 - Marine Control Systems II

Homework assignment 4

1 Input-to-State Stability

1.1 Task: ISS-Lyapunov function

For the following systems, assume that $\|u\| \leq u_{\max}$. Show that $V(x) = \frac{1}{2}x^2$ is an ISS-Lyapunov function for the following systems:

1. $$\dot{x} = -x + u \tag{1}$$

2. $$\dot{x} = -x^3 + x^2 u \tag{2}$$

3. $$\dot{x} = -x^5 + x^3 u \tag{3}$$

4. $$\dot{x} = -\left(1 + e^{|x|}\right) x + xu \tag{4}$$

1.2 Task: ISS

In Homework Assignment 3 we were given the (simplified) mechanical dynamics of a diesel-generator

$$\dot{\delta} = \omega_B (\omega - \omega_0) \tag{5}$$
$$2H\dot{\omega} = t_m - D\omega - t_L + w(t) \tag{6}$$

where ω is the frequency, δ is the load angle , t_m is the control input torque, t_L is an electric load torque, w a bounded disturbance torque, $(H, D, \omega_B) > 0$ are constants, and ω_0 is the frequency of the connected electric power bus. Controlling δ to δ_{ref} and ω to ω_0, we defined the error states $x = \mathrm{col}(e_\delta, e_\omega) = \mathrm{col}(\delta - \delta_{ref}, \omega - \omega_0)$, which gives

$$\dot{e}_\delta = \omega_B e_\omega \tag{7}$$
$$2H\dot{e}_\omega = t_m - De_\omega - D\omega_0 - t_L + w(t) \tag{8}$$

With the disturbance $\|w\| \leq w_0$ as input, show that the control law

$$u = t_m = -k_p e_\delta - k_d e_\omega + D\omega_0 + t_L, \tag{9}$$

with $k_p, k_d > 0$, renders the resulting closed-loop system ISS with respect to $x = 0$.

2 Feedback linearization

We will train on feedback linearization by considering the 3DOF horizontal vessel model

$$\dot{\eta} = R(\psi)\nu \tag{10}$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau, \tag{11}$$

where $\eta = col(x, y, \psi)$ is the position/heading; $\nu = col(u, v, r)$ the velocities; $C(\nu)$ the Coriolis/centripetal matrix; $D(\nu) > 0$ a nonlinear damping matrix; $M = M^\top > 0$; τ the control force input; and $R(\psi)$ the rotation matrix:

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

This has the properties that $R(\psi)^\top R(\psi) = R(\psi)R(\psi)^\top = I$ and $\dot{R} = R(\psi)S(r)$ where

$$S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -S(r)^\top. \tag{13}$$

2.1 Task: Feedback linearization

If the output to be controlled is $\eta \in \mathbb{R}^3$, do the following:

1. Explain the term "vector relative degree", and calculate the vector relative degree for the 3DOF vessel model.
2. Differentiate η according to the vector relative degree. What is the dimension of the zero dynamics?
3. Perform a full state feedback linearization design by differentiating η according to the vector relative degree.

2.2 Task: Feedback linarization by a nonlinear transformation

If the output to be controlled is $\eta \in \mathbb{R}^3$, define the transformation $z_1 := \eta$, $z_2 := R(\psi)\nu + C_1\eta$ where $C_1 = C_1^\top > 0$. This defines the state transformation $z = \mathrm{col}(z_1, z_2) = T(x)$, where $x := \mathrm{col}(\eta, \nu)$.

1. Show that this transforms the system into the controller form

$$\dot{z} = Az + B\Gamma(x)\left[u - \alpha(x)\right] \tag{14}$$

   where (A, B) is controllable, and $\Gamma(x)$ is nonsingular for all x.

2. Design a full-state feedback linearization control law, and prove UGES of $z = 0$ using Lyapunov's Direct Method.

2.3 Task: Zero dynamics

Assume for simplicity that $C(\nu) = 0$ and $D(\nu) = D > 0$ is a constant damping matrix.

For $\tau = \mathrm{col}(\tau_u, \tau_v, \tau_r) \in \mathbb{R}^3$, let

$$\tau_u = -k(u - u_0) = \gamma(u), \qquad k > 0, u_0 > 0 \tag{15}$$

$$\begin{bmatrix} \tau_v \\ \tau_r \end{bmatrix} = \begin{bmatrix} -Y_\delta \\ -N_\delta \end{bmatrix}\delta \tag{16}$$

$$\tau = B_1\gamma(u) + B_2\delta \tag{17}$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -Y_\delta \\ -N_\delta \end{bmatrix} \tag{18}$$

where $\delta \in \mathbb{R}$ is the new control input and (Y_δ, N_δ) are control gains. Let now the output to be controlled be the heading $\psi = h_\psi^\top \eta$, $h_\psi := \mathrm{col}(0, 0, 1)$.

1. What is the relative degree of the system?
2. Differentiate the output according to the relative degree and identify the controlled dynamics and the internal dynamics (keeping vector notation).
3. Perform a partial feedback linearization design that controls the output to a constant reference heading ψ_{ref}.

References