Structure and Behaviour

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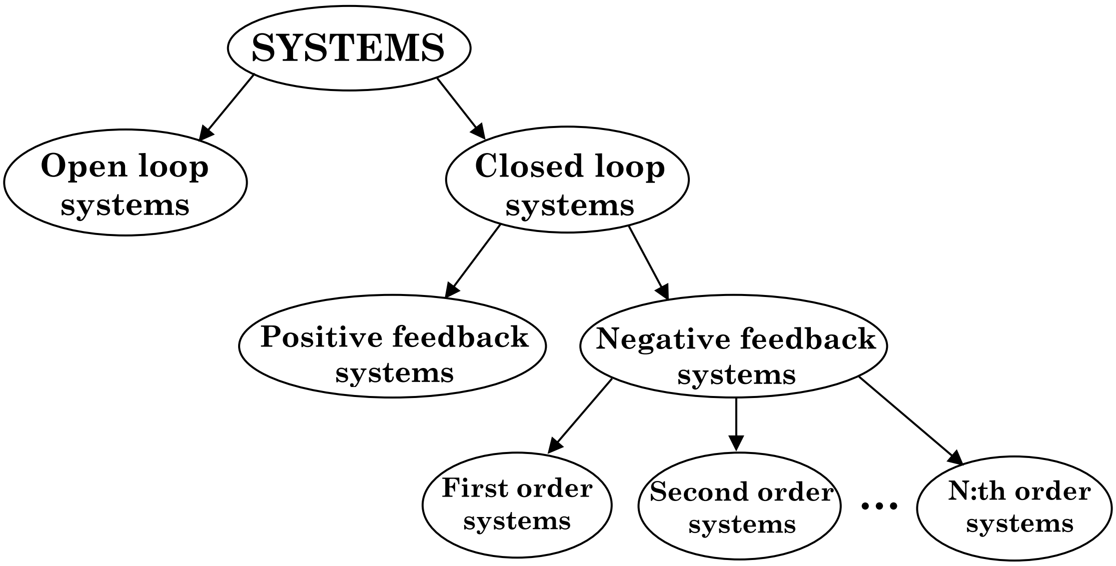
Contents: Common system structures are presented. Models of these structures are programmed in StochSD and simulated to see the behaviour a specific structure will generate. The following systems are studied:

The purposes of the exercise are:

- *To develop skills in modelling and simulation using StochSD.*
- *To provide understanding of the relation between structure and behaviour.*



"Everything you know is only a model. "

Name:	**Date:**
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1. Introduction

A system is made up by a number of interacting components. The behaviour of the system *cannot* be predicted by studying the individual components. The characteristics of a system depends on how the various components are interconnected into a structure. It is this structure that that creates the behaviour.

Classification of systems

Figure 1 shows one way of classifying systems.

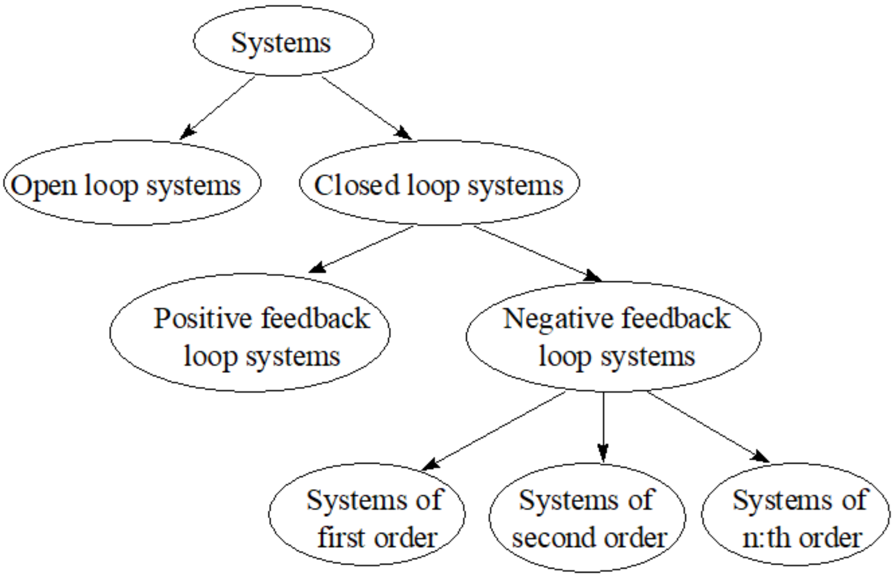

Figure 1. Classification of systems.

In a *closed-loop* system the output from the system is connected to its input so that the output has a renewed impact on the system. In an open-loop system there is no such connection. See Figure 2.

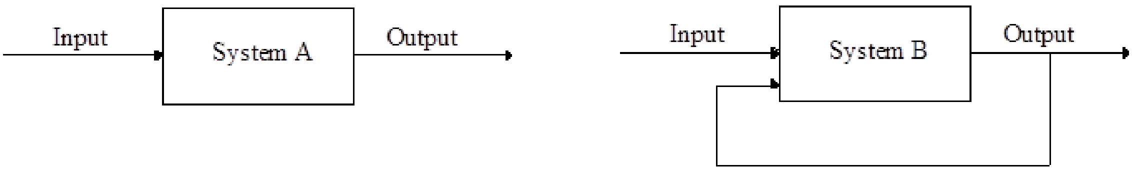

Figure 2. A) Structure of an open loop system. The output does not affect the system.
B) Structure of a closed loop system. The output does affect the system.

The *order* of a system is equal to the *number of compartments* (also called *state variables*) in the system. In StochSD compartments are called Stocks.

2. Open-loop systems

Exercise 1

An example of an open loop system is a bath tub which you fill with water. The present water level depends on the original water level, the flow rate, and the time you had the inflow.

A causal-loop diagram and a StochSD diagram with a Time Plot is shown in Figure 3.

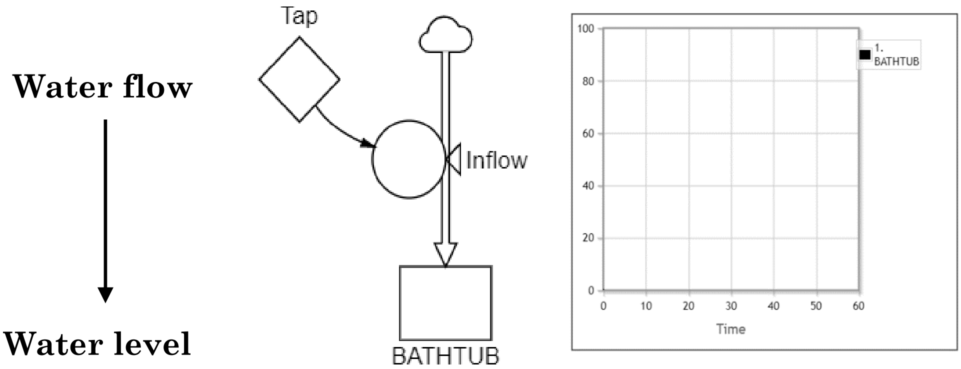

Figure 3. Causal-loop and StochSD diagram for filling a bathtub.

Build the StochSD model. At time zero the BATHTUB is empty and the Tap gives a constant Inflow of 1 litre/second. Simulate the system for 60 seconds and plot the water level (BATHTUB) in a Time Plot. Sketch the result in Figure 3. ■

Exercise 2

Another example is about the relations between distance, velocity and acceleration for a falling body (where we assume no air resistance) as described in Figure 4.

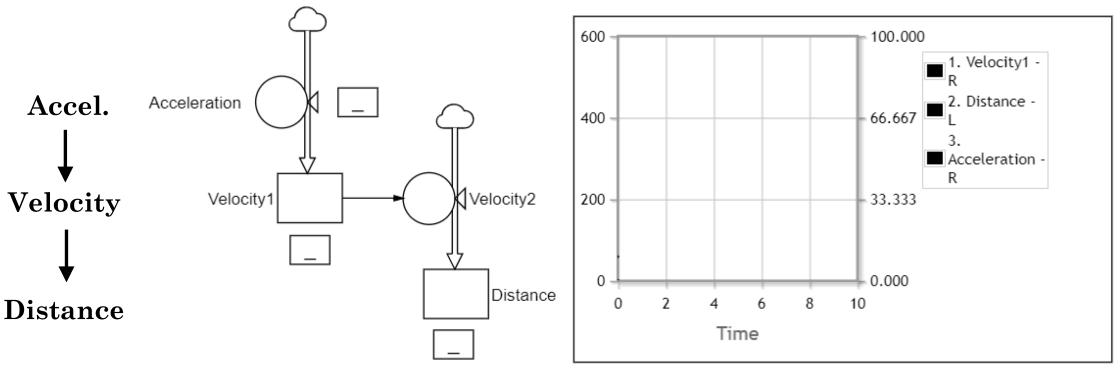

Figure 4. Acceleration, Distance and Velocity. Acceleration is equal to g = 9.81 m/s^2 (free fall).

Comment: StochSD is designed for systems where it's natural to think in terms of Stocks and Flows. In this particular example the StochSD diagram of the system does not come naturally. However, it makes sense, because Velocity is a Stock in relation to Acceleration but a flow in relation to Distance. Therefore, we need two symbols for Velocity in this example.

Build the StochSD model from Figure 4. Set the initial values for Velocity and Distance to zero. Include a Time-Plot, a Table and Number Boxes. Simulate and sketch Acceleration, Velocity and Distance during 10 seconds in Figure 4.

How far has a free-falling body travelled in 10 seconds?

Answer: ....

The accuracy of a simulation depends on the time-step (DT) used. Test with DT=1, 0.1 0.01 and 0.001, respectively. What is the shortest proper DT if you want the accuracy of the free-falling body to be **a)** better than ± 5 meters and **b)** better than ± 1 meter? (**Hint:** Consider DT=0.001 giving an almost correct answer.)

Answer: a) ....

b) ....

At what time does the speed reach 100 km/h (≈ 28 m/s)?

Answer: ....

What is the downside of using an unnecessarily small DT?

Answer: ....

[**EXTRA:** You can also use the Runge-Kutta method (RK4) with DT=1 to see its efficiency for *deterministic* models. (Unfortunately, this efficiency is usually lost for *stochastic models*, i.e. models where random numbers are used.)] ■

3. Closed-loop systems (feedback)

A closed-loop system is characterised by some kind of causal loop in the system. For instance, if **A** affects **B** and **B** affects **A**, then the system containing **A** and **B** is a closed-loop system.

Closed loop systems can be divided into *positive* and *negative* feedback systems.

3A. Positive feedback

If a component **A** *increases* the value of another component **B**, and **B** in turn *increases* the value of **A**, then you have a positive loop creating and *increase* in both **A** and **B**. This structure creates *growth* and the system is *unstable*.

Exercise 3

An example of a positive feedback system is the growth of a population. See Figure 5.

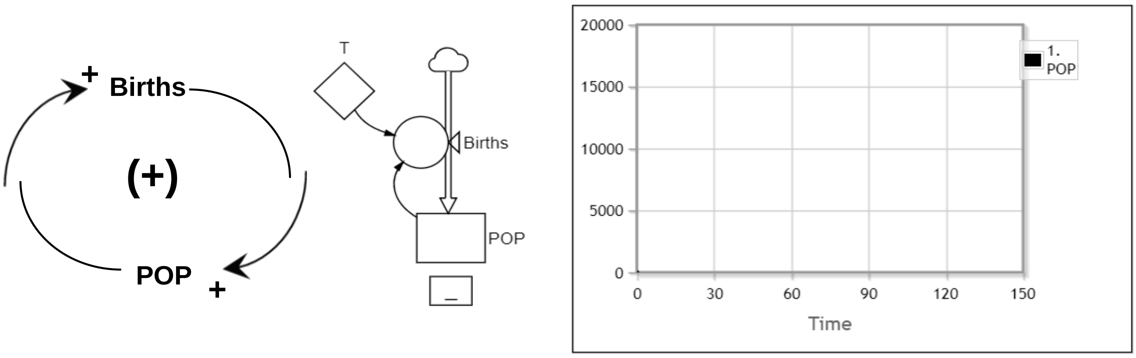

Figure 5. Positive feedback loop. The larger the population, the more births. The more births the more the population increases. T is a time constant which determines the rate of growth (thus Births=POP/T).

Build a StochSD model with the initial value of the population at 100 individuals (humans, flies, plants, …) and the time constant for growth T = 30 time units. Simulate the system for 150 days and sketch the population in Figure 5. ■

Comment: A positive feedback loop with a single Stock generates an *exponential growth*. The analytical solution of the problem above is: $POP(t) = POP(0)\cdot e^{t/T}$ (POP(0)=100 is the initial value). □

Examples of positive feedback: Saving account growing because of interest, the chain reaction in an atomic bomb, or the growth of a tumour are other examples of positive feedback loops.

3B. Negative feedback

A negative feedback system is characterised by a *declining*.

Exercise 4

An example of negative feedback system is the decay of a radioactive material. See Figure 6.

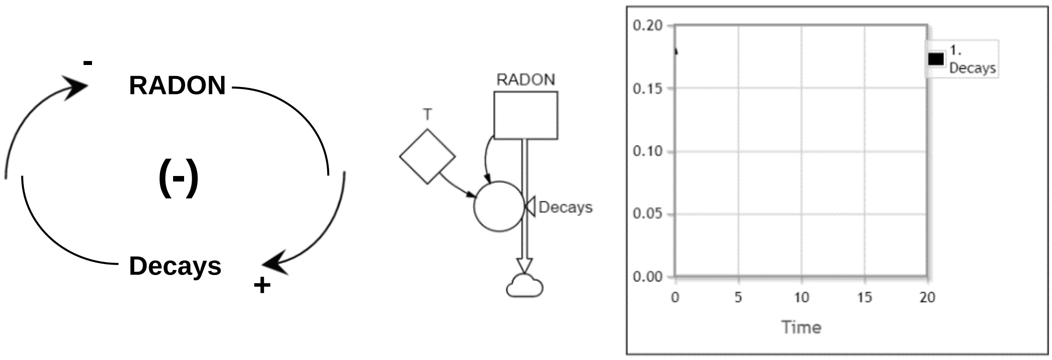

Figure 6. A negative feedback system. The more radioactive material, the higher the rate of decay (disintegration). The higher the decay rate, the less radioactive material left. T is a time constant which decides the decay rate.

Make a StochSD model which shows the radioactive decay of 1 unit (Bq/m^3) of radon. The half-life $T_{½}$ of radon is 3.823 days. The time constant $T = T_{½} / \ln 2$. Hence T = 5.52 days. Simulate the system for an appropriate time interval.

According to your simulation, how long does it take until the radioactivity decreases to 50% of its initial level? (Here, you can use a Table.)

Answer: .... ■

3C. Control

Control is closely related to negative feedback. In fact, negative feedback is the very foundation for all control. It is the method of nature itself, and you will find it in all kinds of technical and societal systems.

In order to control a system, you need a *Goal*, often just a *Reference value*. The human body keeps a constant body temperature of 37°C. The difference between the reference value and an the actual (observed) value can then be calculated and used for control. See Figure 7.

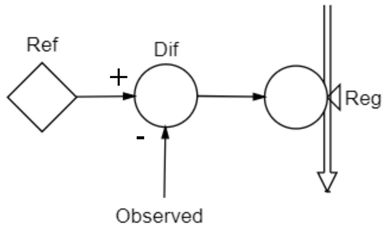

Figure 7. Calculation of the difference between Reference and Observed value.

Control is based on negative feedback where the Observed value is compared with a Reference value. A Regulator acts on this difference by forcing the Observed value towards the Reference value. The effect of the difference can be increased by a factor K. See Figure 8.

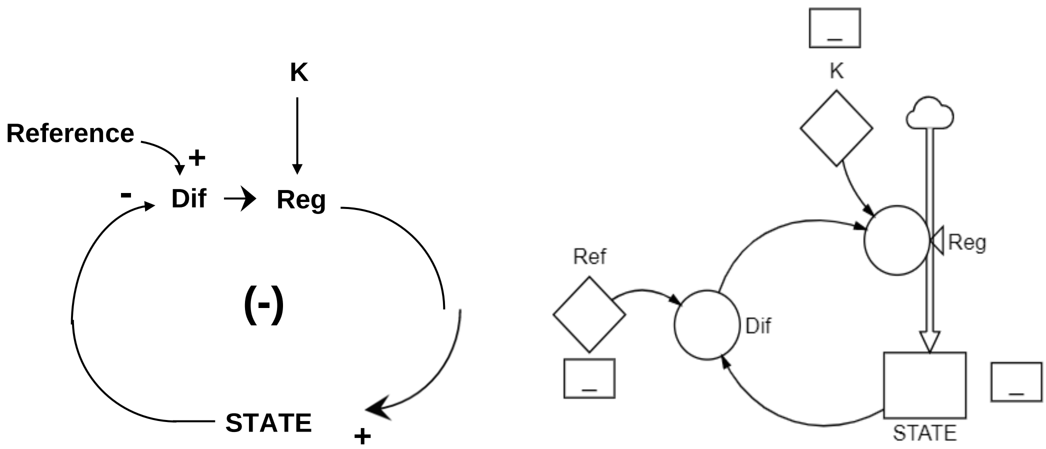

Figure 8. By comparing Reference and State values, the Regulator drives the State towards the Reference value. By multiplying the Difference by a factor, K, the speed of the regulation of the STATE towards the Reference value can be adjusted.

Note that the constant K (Reg=K·Dif) amplifies the regulation. You are just about to investigate the influence of this constant.

Exercise 5

Make a StochSD model for the system above. Choose a reference value which is different from the initial value of STATE. Simulate for 30 time units with K = 0.1, 1, and 2. Let the time-step be DT = 0.1. What is the influence of K?

Answer: ....

....

Is the regulator able to get the State value equal to the reference value?

Answer: ……... ■

3D. Loaded system

The system in the example above is completely unloaded. We will now study a loaded system.

Exercise 6

In this example we assume that we are controlling the temperature in a house. The indoor temperature is affected by a sudden drop e.g. because of heat leakage when a window is opened. (This is a simplified model to show the principle. In physical terms it is energy that enters and leaves the house, and the amount of energy corresponds to a certain indoor temperature.)

A StochSD diagram of the loaded system is shown in Figure 9.

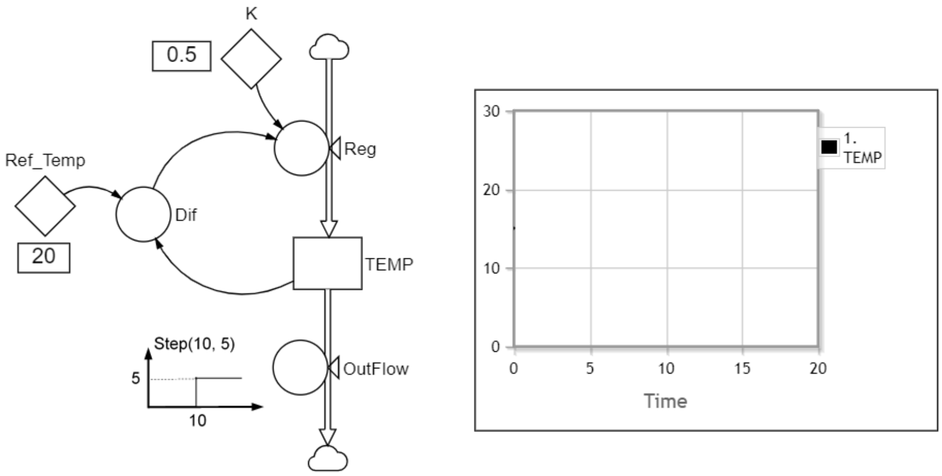

Figure 9. The indoor temperature will be regulated towards its reference value for 10 hours – then a window is opened.

Build a StochSD model to simulate the indoor temperature. Start with an indoor temperature of 15°C and set the reference to 20°C. Choose K = 0.5. After 10 hours a window is opened that leaks heat 5 ‘units of heat’ per hour. (OutFlow = STEP(10, 5)). What happens? Is the regulator able to hold the temperature? Increase the amplification K to 1, and then to 2. What happens now? What is the impact of K?

Answer: ....

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Comment: This kind of control where the effort (here supplied heat) is proportional to the difference is called *proportional control* and the regulator is a *P-controller*. The drawback with P-control is that a load on the system gives a remaining error, which can be made smaller but not eliminated by increasing the value of K. (However, there are more powerful control strategies!)

3E. Systems of higher order (Two or more Stocks in a loop)

In a system containing a negative feedback loop over two (or more) Stocks, oscillations may be generated. These oscillations may decline to a final level in time. If so, the system is *stable*. However, the oscillations may increase in time. The system is then *unstable*. Depending on structure and constants, 2:nd order systems behaves differently.

Exercise 7

Study the system in Figure 10 with two Stocks and two Flows. Start with Stock1(0) = 1 and Stock2(0) = 0, and Flow1 = Stock2 (which has a positive effect on Stock1 since Flow1 is an inflow) and Flow2 = Stock1 (which has a negative effect on Stock2 since Flow2 is an outflow).

Choose DT=0.1 and use the RK4 method. Simulate for 20 time units, and sketch the result in Figure 10.

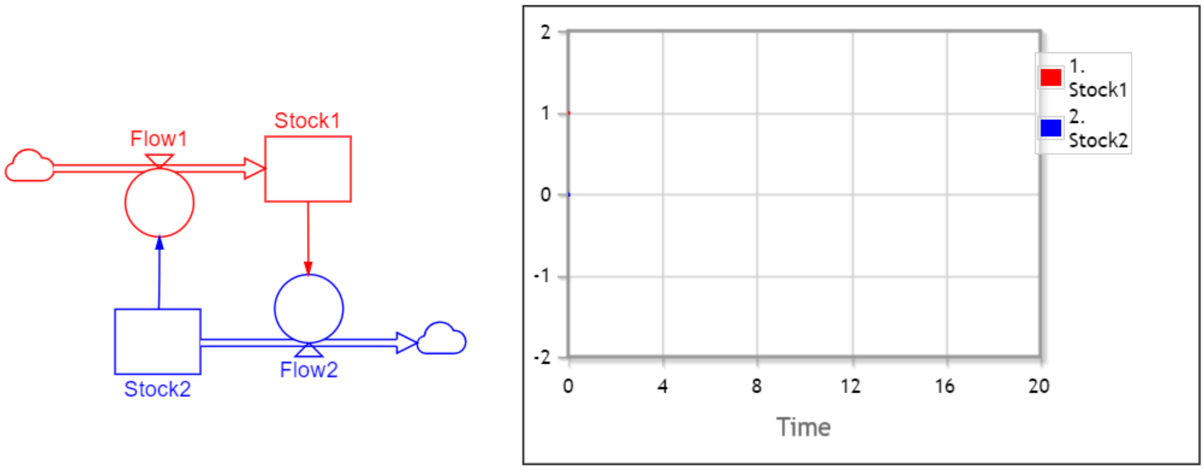

Figure 10. Behaviour of an oscillator constructed by a negative feedback loop of second order.

Describe what you see!

Answer: ....

.... ■

Exercise 8

Now, add an outflow from Stock1, named Flow3. Here, Flow3=c*Stock1, so for c=0 the model should behave the same as in the last exercise. For c > 0 Stock1 will have an extra output from Flow3, and for c < 0 Stock1 will have an extra input (a negative outflow).

Set c = 0, 0.2 and -0.2, respectively and simulate. What do you find?

Answer: ....

....

.... ■

4. Stage and compartment (The sojourn time and its distribution)

One important concept when describing real processes is the *sojourn time* or *delay time*. For example, when 1000 persons are simultaneously infected, they will not recover at the same time.

In modelling we talk about *stages*. The time between getting sick and getting well can be denoted the *Illness stage*.

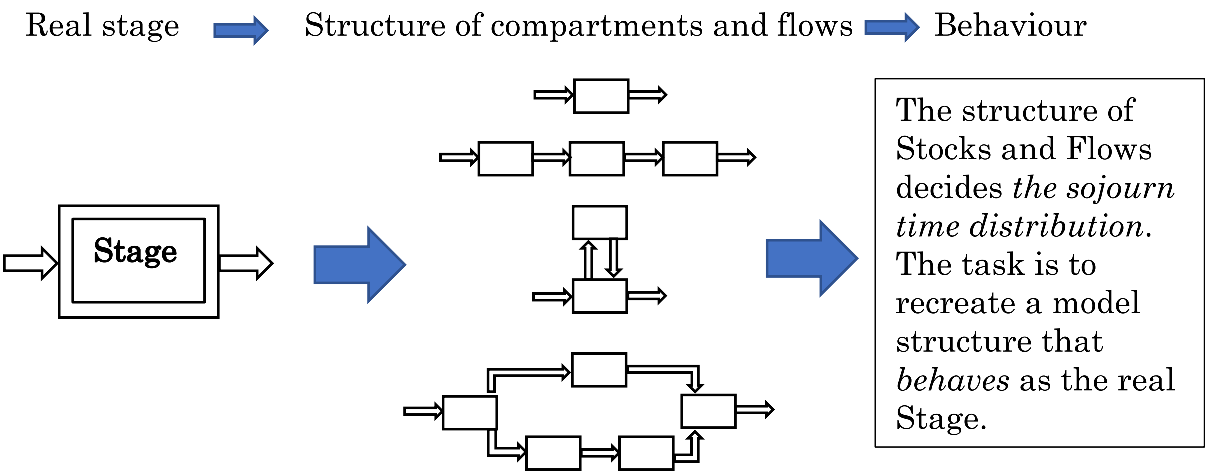

Figure 11. A Stage must be modelled with one or more compartments (Stocks) and Flows in series and/or parallel in order to preserve the sojourn time distribution of the Stage. We denote a Stage by double frames and a compartment with a single frame.

Another example is the delivery time of letter or commodity. Although *'delivery stage'* has to be treated in the same way (i.e. modelled as a structure of compartments), we usually prefer to use the term *delay* here.

A *stage*/*delay* has two aspects:
1) The *average time* in a stage/delay, the '*sojourn time'*, has to be preserved in the model.
2) Also, the *statistical distribution* of the sojourn time/delay time must be realistically modelled.

A sojourn time/delay time can be constant in the meaning that the process or transport always takes the same fixed time, or it may vary according to some statistical distribution. For example, for radioactive material, the sojourn time is exponentially distributed. (See Exercise 4, above.)

However, for most natural processes a stage/delay can **NOT** be modelled by a single Stock. **This is probably the most common and destructive misconception in CSS modelling.**

When modelling a stage or delay the two aspects: The *average time* and the *statistical distribution* must both be preserved in the model!

However, a realistic sojourn/delay time distribution can always be achieved by a structure of compartments (Stocks) and Flows arranged in series and/or parallel. (Here we will only treat the serial case.)

We will now explore how the *sojourn time/delay time* <u>*distribution*</u> will vary for a stage composed of 1, 2 or 3 Stocks in a series.

Exercise 9

Figure 12 shows three examples of a *stage/delay* that delays the input from 'In' before it leaves the stage through Out1, Out2 or Out3.

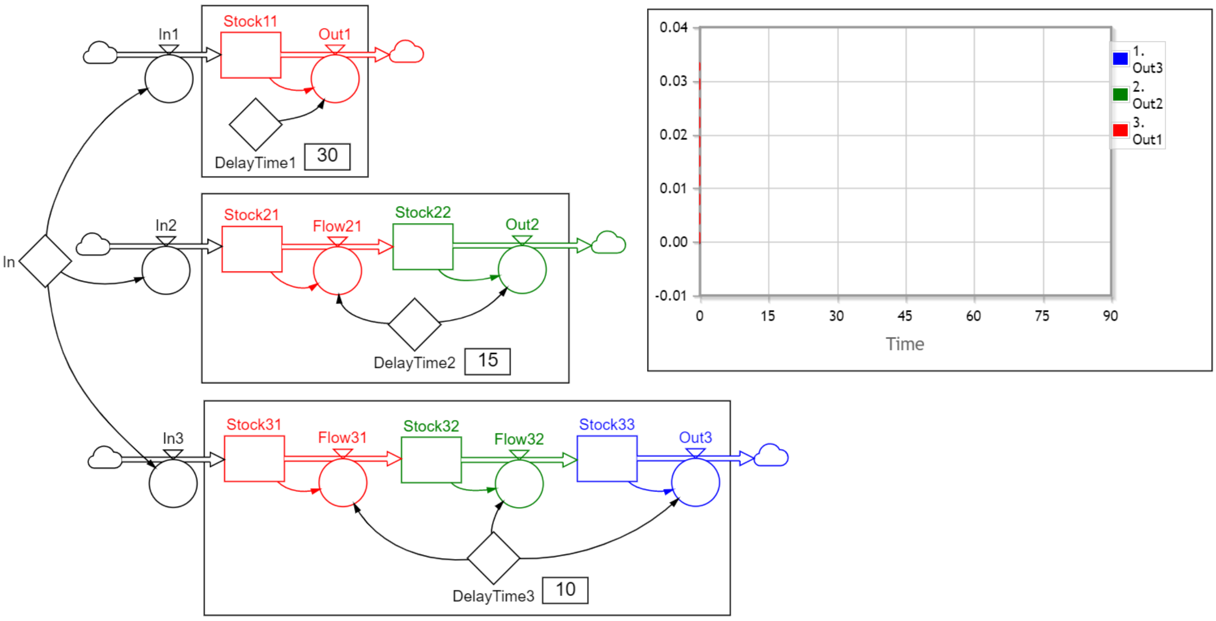

Figure 12. The structure and effects of a stage/delay modelled by 1, 2 or 3 serial Stocks and Flows.

Construct these three models in StochSD. The input is an Impulse that enters the first Stock at time zero: In = PulseFcn(Time, Volume, Interval); where 'Time' of the impulse is zero, the 'Volume' is one unit, and the 'Interval' is set to a large number so that the impulse is not repeated during the simulation.

The Stocks are all initially empty, and the Flows are drained as StockXY/DelayTimeX.

Finally, the overall sojourn/delay time should be 30 time units. This means that DelayTime1 = 30, DelayTime2 = 15 (used twice), and DelayTime3 = 10 (used three times).

Use DT = 0.1 time unit and Length=90 time units.

Task: Sketch the three outputs of the impulse passing the stages/delays in figure 12**.**

The number of Stocks of the Stage or Delay is its order. Above, we saw Stages/Delays of order 1, 2 and 3, respectively. When the order increases the shape of the output approaches a Normal distribution with smaller and smaller standard deviation. Theoretically, an infinite order Stage/Delay is a pulse.

Comment: Also, i*nformation can be processed in stages (e.g. in computer calculations).*

A FINAL COMMENT*: It's the structure and parameter values that decide the behaviour of the model. It does NOT depend on whether the modelled system is a biological, technical, economical or other kind of system.*

5. Summary

Exercise 10

Write a very short summary about open and closed systems, positive and negative feedback and about the order of systems. You should also mention a few words about control. Also describe the difference between a stage and a compartment. Half a page should be enough.

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