## 0.1 Mathematics for SmartIoT, 2nd week

1. Find the limits as $k$ tends to infinity, if they exist, of the following sequences:

   (a) $x[k] = \frac{3k+2}{k^2+7}$, (b) $x[k] = k^3$, (c) $x[k] = \frac{2k+3}{4k+2}$, (d) $x[k] = \frac{k^2+k}{k^2+k+1}$.

2. An arithmetic series has a first term of 4 and its 30th term is 1000. Find the sum to 30 terms.

3. The sum to 20 terms of an arithmetic series is identical to the sum to 22 terms. If the common difference is -2, find its first term.

4. Find the sum to five terms of the geometric series with first term 1 and common ratio 1/3. Find the sum to infinity.

5. The sum to infinity of a geometric series is four times the first term. Find the common ratio.

6. Find the sum of series

   (a) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + ....$

   (b) $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k$.

   (c) $\sum_{n=1}^{\infty} \frac{1+2^{n+1}}{3^n}$.

7. Express the alternating harmonic series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

   in sigma notation.

8. **Bonus**

   The $z$ transform of a sequence $f[k]$, $k \in \mathcal{N}$, is defined by the formula:

$$F(z) = \mathcal{Z}\{f[k]\} = \sum_{k=0}^{\infty} f[k] z^{-k}.$$

   Use this definition and properties of the geometric series to find the $z$ transform of a sequence $f[k] = 1$, $k \in \mathcal{N}$.

   For what values of $z$ does this series converge?